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CONTENTS

NUMBER I.

NUMBER II.

NUMBER III.

NUMBER IV.

NUMBER V.

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THE THEORY OF THE OCULAR SPECTROSCOPE.

By F. L. O. Wadsworth.

The forms of spectroscope in common use in astronomical spectroscopy may be classified under three general heads: (1) the compound slit spectroscope, (2) the objective spectroscope, and (3) the ocular (*sic*) spectroscope. Each class of instruments has its own peculiar merits and defects which determine the class of work to which it can be applied with best advantage. Thus, the compound slit spectroscope is best suited to determinations of absolute or relative wave-length, but is wasteful of light. The objective spectroscope is very efficient in this latter respect,[1] but in the usual form in which it is used, that of an objective prism, it is unsuited to accurate determinations of wave-length,[2] and is, moreover, very expensive.[3] The ocular

[1] "Theory of the Objective Spectroscope," Astrophysical Journal, 4, 56, June 1896.

[2] In some of the forms described by Professor Hale and the writer this objection may not apply. See paper on "The Objective Spectroscope," Astrophysical Journal, 4, 64–78, June 1896.

[3] The cost of the objective prism can be considerably decreased without loss of resolving power or efficiency by use of the sectional prism or the fluid prism with free surfaces described by the writer. See Astrophysical Journal, 4, 245, March 1895, and 4, 274, November 1896.

spectroscope is the least expensive of any and retains many of the advantages of the objective prism as respects light efficiency, but as it has been heretofore made and used it is inferior in definition and resolving power to either of the forms first mentioned. The object of the present paper is to investigate the cause of this inferiority, and to indicate the manner in which this type of instrument may be improved in this respect.

In the case of the ocular spectroscope the dispersing train is placed directly in the path of the cone of rays from the objective either just inside or just outside the focus. In the latter case the train may be placed either in front of or behind an eyepiece or an equivalent eye lens. It is therefore first of all necessary to investigate the effect of aberration due to the introduction of a dispersing system into the cone of rays from the objective. The amount of aberration thus produced will set a limit to the possible resolving power of the system. We will consider first the simplest case, in which the dispersing train consists of a single prism placed at minimum deviation.

The aberration produced in such a case has been already investigated by Lord Rayleigh, who finds [1] for the longitudinal aberration in the primary plane

$$\delta v = \text{longitudinal aberration} = \frac{3y \sin \phi (\mu^2 - 1)}{\mu^2 \cos^2 \phi'} , \tag{1}$$

where y is the semi-linear aperture (measured along the face) of the prism, μ its index of refraction for the ray passing at minimum deviation, ϕ and ϕ' the angles of incidence of the axial pencil on the first and second prism faces respectively. The angular width of the resulting image of an infinitely thin slit under the best conditions of focus is therefore

$$\omega = \delta v \frac{\theta}{u} , \tag{2}$$

where θ is the angular semi-aperture of the cone of rays and u is the distance of the apex of the cone from the prism face. Hence, since

$$\theta = \frac{y \cos \phi}{u} ,$$

[1] "Investigations in Optics with Special Reference to the Spectroscope," *Phil. Mag.*, 3d ser., 9, 40–49.

we have
$$\omega = \frac{3\theta^2(\mu^2 - 1)\sin\phi}{\mu^2 \cos\phi \cos^2\phi'} , \qquad (3)$$

The effect of this in a continuous spectrum would be to practically obliterate the images of absorption lines whose angular width was less than ω. In the case of a $60°$ prism of light flint glass, $\mu = 1.6$, $\phi' = 30°$ and $\phi = \sin^{-1} 0.8 = 53° 8'$.

Hence
$$\omega = 3.25\theta^2 . \qquad (4)$$

In the case of the objective spectroscope unaffected by aberration, the purity of the spectrum is equal to the theoretical resolving power R of the instrument.[1] This corresponds to an angular separation of the images of the two resolved lines equal to[2]
$$\Omega = \frac{4}{7} D\Delta\lambda + \frac{a^2}{D\Delta\lambda + a} , \qquad (5)$$
where D is the dispersion of the spectroscope train, $\Delta\lambda$ the "width" of the lines, and a the resolving power of the spectroscope aperture. For a single prism at minimum deviation[3]
$$D = \frac{2 \sin\phi'}{\sqrt{1 - \mu^2 \sin^2\phi'}} \cdot \frac{dn}{d\lambda} . \qquad (6)$$

For the case just considered $\phi = 30°$, $\mu = 1.6$,
$$\therefore D = \frac{5}{3} \cdot \frac{dn}{d\lambda} .$$

For light flint $\frac{dn}{d\lambda} \cong 1 \times 10^{-5}$ (where λ is expressed in tenth meters) and $D = 0.000017$. For solar lines the average "width" of the line is probably less the 0.1 tenth-meters, while for the broad hydrogen lines in star spectra the "width" is perhaps 1.0 tenth-meter or more.[4] Assuming further that the prisms

[1] "The Objective Spectroscope," ASTROPHYSICAL JOURNAL, 4, June 1896. See p. 60.

[2] "On the Conditions of Maximum Efficiency in the Use of the Spectrograph," ASTROPHYSICAL JOURNAL, 3, 335, May 1896.

[3] "General Conditions Respecting the Design of Astronomical Spectroscopes," ASTROPHYSICAL JOURNAL, 1, 55, January 1895.

[4] "On the Resolving Power of Telescopes and Spectroscopes for Lines of Finite Width," *Phil. Mag.*, 5th ser., **43**, 317. Also *Mem. Spec. Ital.*, **26**, 15, and *Wied. Ann.* **61**, 622 *et seq.*

have a clear aperture $a = 2y \cos \phi = 25$ mm we have

$$\alpha \cong 0.00002$$

and

$$\Omega \begin{cases} = 0.000019 & \text{for } \Delta\lambda = 0.1 \\ = 0.000021 & \text{for } \Delta\lambda = 1 \end{cases} \quad . \tag{7}$$

In order that the effect of aberration may not seriously affect the resolving power of the instrument, ω must not exceed $\frac{1}{2}\ \Omega$.[1] Hence we must have from (4) and (7)

$$\theta^2 > 0.0000031 \quad \text{or} \quad \theta \gtreqless 0.00176$$

i. e., the angular aperture of the cone of rays must not be larger than $\frac{1}{280}$, if the effect of aberration is not to be prejudicial to the resolving power of a single 60° prism of 1 inch clear aperture. The angular apertures of astronomical telescopes are very much larger than this, and the use of the prism in the direct cone of rays from the objective is therefore inadmissible, if apertures of any size are to be made use of.

Conversely, if the angular aperture of the telescope be given, we can easily determine the maximum permissible aperture and resolving power of a prism placed in the cone of rays. For astronomical telescopes θ varies from 0.035 to 0.025. Assume $\theta = 0.03$. From (4) and (5) we have at once

$$0.0029 = \frac{2}{7} D\Delta\lambda + \frac{\alpha}{2\left(\frac{D\Delta\lambda}{a} + 1\right)} \quad . \tag{8}$$

From what has preceded it will be at once evident that if θ is to be increased, we can only preserve definition by decreasing D or increasing a. In either case the quantity $\frac{D\Delta\lambda}{a}$ will become small, and may, in the denominator of the second term, be neglected. Hence, we have at once as a limiting relation between D and a

$$\alpha_{\text{max.}} \cong 0.006\,(1 - 100\, D\Delta\lambda) \quad . \tag{9}$$

Hence, for a single 60° prism of light flint for which $D\Delta\lambda =$ 0.000017 to 0.0000017, we must have

$$\alpha \gtreqless 0.00599 \quad \text{or} \quad \alpha \cong 20' \quad .$$

[1] "Conditions of Maximum Efficiency in the Use of the Spectrograph," ASTROPHYSICAL JOURNAL, 3, May 1896. See p. 336.

That is, the aperture of the prism must be considerably less than 0.1 mm.

Somewhat more favorable conditions may be realized by decreasing ϕ' or μ, or both. For with small values of ϕ', and therefore of ϕ, the aberration diminishes very nearly as $\sin\phi$, while, as seen from (6), the dispersion, and therefore the resolving power, diminishes somewhat less rapidly. We can find the maximum permissible value of r, the resolving power, by expressing directly the necessary ratio between ω and Ω, as defined by (3), (5), and (6). This gives

$$\frac{6\theta^2(\mu^2-1)\sin\phi}{\mu^2\cos\phi\cos^2\phi'}=\frac{4}{7}\frac{2\sin\phi'}{\sqrt{1-\mu^2\sin^2\phi'}}\cdot\frac{dn}{d\lambda}\cdot\Delta\lambda+\frac{a}{\dfrac{D\Delta\lambda}{a}+1}$$

or

$$3\theta^2\frac{\mu^2-1}{\mu}\cdot\frac{1}{\cos^2\phi'}=\frac{4}{7}\cdot\frac{dn}{d\lambda}\Delta\lambda+\frac{a}{D}\frac{dn}{d\lambda}\left(\frac{1}{\dfrac{D\Delta\lambda}{a}+1}\right). \tag{10}$$

From the well-known relation[1]

$$r=\frac{D\lambda}{a}$$

we get at once by substitution and transposition

$$\frac{1}{\dfrac{r\Delta\lambda}{\lambda}+1}\cdot\frac{\lambda}{r}\left(\frac{dn}{d\lambda}\right)=3\theta^2\frac{\mu^2-1}{\mu}\cdot\frac{1}{\cos^2\phi'}-\frac{4}{7}\frac{dn}{d\lambda}\cdot\Delta\lambda\ , \tag{11}$$

or neglecting as before the quantity $\dfrac{r\Delta\lambda}{\lambda}$,

$$\frac{1}{r}\cong\frac{1}{\lambda}\left[3\theta^2(\mu-\mu^{-1})\frac{1}{\cos^2\phi'\,\dfrac{dn}{d\lambda}}-\frac{4}{7}\Delta\lambda\right], \tag{12}$$

which shows, as already stated, that as ϕ' decreases, r increases, although not very greatly.

Assuming the values of λ, $\dfrac{dn}{d\lambda}$, $\Delta\lambda$ (max. = 1 tenth-meter) already used, and taking θ as 0.03, we have for r

$$\frac{1}{r}\cong\frac{1}{5600}\left(\frac{270}{\cos^2\phi'}-\frac{4}{7}\right).$$

[1] *Loc. cit.*

For the maximum value of $\cos\phi' = 1$ (which is, of course, unattainable, we would therefore have for r

$$r \cong 20$$

or the closest lines that could be resolved would be separated by an interval of about 300 tenth-meters — over fifty times the distance between the two D lines. Such an arrangement is, therefore, inadmissible for work in the visible part of the spectrum.[1]

For particular portions of the spectrum the amount of aberration may be considerably reduced by the use of a direct vision prism instead of a simple prism. As in the case of the achromatic objective, we may, by properly proportioning the angles of the different component prism surfaces, completely correct for aberration for one wave-length λ_o in the spectrum. The definition can, however, only be good over a very limited range of spectrum; all other parts will suffer from the effect of aberration to an extent depending, as in the case of the simple prism, on the dispersion and aperture, *i. e.*, on the resolving power, of the prism system. The exact effect of aberration on a cone of rays passing through the combination of prisms necessary to form the direct vision system can be determined in a manner similar to that employed in the case of a simple prism, but in the case of a combination of five prisms such as is commonly used it becomes very complicated. A simpler, and at at the same time fairly approximate, method of determining its effect is to consider such a system as optically equivalent to a simple prism of the same aperture, and of a refracting angle such as will produce a deviation of the rays under consideration equal to the difference in deviation between these rays and the rays of wave-length λ_o, for which the aberration is zero in the direct vision system. Thus, with a dispersion of 2° between C

[1] This is not the case in work in the infra-red with the spectrobolometer, where λ is very large and θ is small. In such a case, a collimator may be dispensed with, not only without detriment, but with advantage, because the loss of energy due to absorption in the collimator lens is thus avoided. See article on "Fixed Arm Spectroscopes," *Phil. Mag.*, 38, October 1894; also *Report of the Smithsonian Astrophysical Observatory*, 1893.

and E, the aberrational effects on rays of wave-length λ_E (assum-it to be zero for wave-length λ_C) will be the same, to the above degree of approximation, as is produced by a prism of an index μ and a refracting angle $2\phi'$ equal to

$$\frac{2}{\mu - 1},$$

since the equivalent prism is thin.

For $\mu = 1.6$ as assumed before

$$2\phi' = 3^\circ\ 20' \quad \text{and} \quad \phi = 2^\circ\ 40',$$
$$\therefore\ \omega \cong 0.085\,\theta^2, \tag{13}$$

or only about one thirty-seventh as great as with the 60° prism already considered.

The dispersing power of such a system, $D = \frac{d\theta}{d\lambda}$, is about equal to two 60° prisms of the same index $\mu = 1.6$. The effect of aberration on a system of given resolving power, $Da = Const.$, is, therefore, very considerably less than when simple prisms are used, and we may, therefore, use apertures considerably larger than is possible in the latter case without injury to definition. Even the very considerable gain thus resulting is not, however, sufficient to render the use of any considerable resolving power possible, as for the value of θ assumed before we find at once from (13) and (5)

$$\begin{aligned} a_{max.} &= 0.00015 - 0.58\, D\Delta\lambda \cong 0.00013, \\ &\cong 26'', \end{aligned} \tag{14}$$

or b, the limiting linear aperture of the prism train, is about 4 mm, corresponding to a resolving power in the prism train of 1360 units, a power barely sufficient to resolve, but not distinctly separate, the D lines under the best conditions.

As in the preceding case, something more may be gained by diminishing the dispersive power of the combination and increasing the aperture, but the gain will not be nearly so great, because the possible range in the value of ϕ and ϕ' is much

smaller. The maximum value of r will, as before, be found by equating (3) and (5). This gives as before

$$\frac{6\theta^2(\mu^2-1)\sin\phi'}{\mu\cos\phi\cos^2\phi'}=\frac{4}{7}D\Delta\lambda+a\left(\frac{1}{\frac{D\Delta\lambda}{a}+1}\right).$$

But in this case, by our assumption of equivalent prisms,[1]

$$\phi'=(\mu-1)\frac{\delta}{2}=\frac{\mu-1}{2}\cdot D(\lambda_C-\lambda_E)\ , \tag{15}$$

and since ϕ', as just shown, is small for the most favorable case, we have by substitution and reduction

$$3\theta^2\frac{(\mu^2-1)(\mu-1)}{\mu}(\lambda_C-\lambda_E)=\frac{4}{7}\Delta\lambda+\frac{\lambda}{r}\left(\frac{1}{\frac{r\Delta\lambda}{\lambda}+1}\right), \tag{16}$$

or, assuming the quantity $\frac{r\Delta\lambda}{\lambda}$ to have a maximum value 0.5, we get

$$\frac{1}{r}\cong\frac{1}{\lambda}\left[4.5\theta^2(\mu-\mu^{-1})(\mu-1)(\lambda_C-\lambda_E)-\frac{6}{7}\Delta\lambda\right], \tag{17}$$

and assuming the values of λ, θ, and $\Delta\lambda$ (max. = 1 tenth-meter) used before, we get

$$r_{\text{max.}}\cong\frac{5600}{3.13-\frac{6}{7}} \tag{18}$$

$$r_{\text{max.}}\cong 2450\ ,$$

about 120 times that obtainable with a simple prism, and nearly equal to that obtainable with a single 60° prism of white flint of about 15mm aperture.

As the value of θ increases the value of $r_{\text{max.}}$ rapidly decreases. Thus for a reflecting telescope $\theta=0.09$ we would have

$$r_{\text{max.}}=\frac{5600}{27.13}\cong 200\ . \tag{19}$$

It is interesting to compare the effects of aberration due to the direct vision prism system just considered with that due to a simple lens. From (13) we have for the E line

$$\omega=0.085\,\theta^2\ . \tag{20}$$

[1] Assuming the dispersion uniform throughout the range λ_C to λ_E.

The longitudinal aberration in a simple lens having faces of the most favorable curvatures is $0.83u\theta$. The angular width ω' of the image of a line in the best focus is therefore

$$\omega' = 0.83\,\theta^3\ . \tag{21}$$

Hence for a value of $\theta = 0.03$ we have

$$\frac{\omega}{\omega'} = 3\frac{1}{3}\ ,$$

or the effect of the prism is over three times as great as the effect of a simple lens of glass. For the portion of the spectrum above E the effect of aberration becomes rapidly worse; at G, for example, the effect will be about six times as great in the case of the direct vision prism as in the case of the lens.

In view of these results it is evident that the general statement made by Scheiner[1] and others that when direct vision prism systems are placed directly in a cone of rays from the telescope objective, the loss in definition is "slight," and that "the errors resulting from imperfect parallelism of the rays are of about the same order of magnitude as those which result from disregarding the thickness in the treatment of simple lenses," are incorrect, or at least incomplete and therefore misleading. It is only true for very small resolving powers which would be practically useless in most lines of modern spectroscopic and spectrographic research. Secchi used this form of instrument, but only recommended it for small apertures.[2] He soon discarded it for the objective prism of 16 cm aperture which, he states, gave spectra which were both far brighter and far better defined than those obtained with the ocular spectroscopes he first used.[3]

The preceding considerations apply to the ocular spectroscope whether the prism system be placed inside or outside the focus. In the latter arrangement some advantage may be gained in the way of diminishing aberration by placing the prism system outside the eye lens of the telescope, as the angu-

[1] *Astronomical Spectroscopy* (Frost's translation), p. 30.

[2] *C. R.* **65**, 389 and 979. [3] *C. R.* **69**, 1054.

lar divergence of the cone of light, *i. e.*, the value of θ in (17) is here considerably less than that of the main cone of rays inside the focus. It is obvious, however, that this advantage is secured only at the sacrifice of resolving power (owing to the necessarily small dimensions which the prism system must then assume), and at the sacrifice of another important advantage, *i. e.*, the possibility of using the system either in micrometric or spectrographic work in the principal focal plane.

In the preceding investigation the effect of the thickness of the prism system has not been considered. The effect of this will be to increase the aberration to an extent depending on the distance of the system from the focal plane. When the prism is placed inside the focus this distance may be made large in comparison with the thickness and the effect of the latter element will be small. But when the prism system is placed outside the eye lens, and thus of necessity near the vertex of the cone of rays, the magnitude of the effect is increased. In this respect, therefore, the latter form of ocular spectroscope is inferior (in theory at least) to the one first considered.

To sum up briefly, it is evident from the preceding results: (1) That the use of simple prisms of any angle directly in the cone of rays from the objective is only possible when the aperture is so small as to be practically useless for the large majority of spectroscopic investigations. (2) That the effect of aberration due to the prism system may be reduced and the use of somewhat larger apertures rendered possible by the use of direct vision prism systems instead of simple prisms, but that this gain is not large and is confined to a comparatively narrow range of spectrum, and is secured moreover only at a great sacrifice of light efficiency, especially in the upper part of the spectrum, owing to the greatly increased absorption of the direct vision combination, the flint glass element of which must necessarily be quite dense or quite thick. (3) That aberration can also be reduced by placing the prism system outside the eyepiece (or what amounts to the same thing, a single eye lens), but that this improvement is secured by a still further sacrifice of resolving power, and

what is still worse, a sacrifice of the possibility of using the system in focal plane work.

The general conclusion to be drawn is, that in its present form the ocular spectroscope is unsuited to, and incapable of satisfying the demands of modern spectroscopic and spectrographic work. I shall show in a subsequent paper how this type of instrument may be so modified as to be greatly improved in optical performance and applied with advantage to the investigations of a number of interesting and important researches in astronomical spectroscopy and spectral photometry.

ALLEGHENY OBSERVATORY,
1899.

DESCRIPTION OF A NEW TYPE OF FOCAL PLANE SPECTROSCOPE AND ITS APPLICATION TO ASTRONOMICAL SPECTROSCOPY.

By F. L. O. WADSWORTH.

In the preceding paper I have investigated the effect of aberration in limiting the resolving power of the ocular spectroscope as usually constructed, and have shown that this limitation is so serious as to prohibit the use of this form of instrument in the great majority of cases. The advantages of this type of spectroscope over the compound slit spectroscope, on the score of light efficiency, and over both the compound spectroscope and the objective spectroscope, on the score of greatly diminished first cost, are, however, so great that I was induced to devote my attention to the question of overcoming or avoiding the difficulties due to aberration without sacrificing resolving power. In the course of the preceding investigation a method of doing this suggested itself which is extremely simple and effective, but which does not seem to have been previously used. Referring to equation (3) of the preceding paper, it is obvious that the only quantities in that equation which can be varied to any degree are ϕ, ϕ', and θ. As was there pointed out, a most decided improvement can be effected by decreasing the value of θ; but we are limited in this direction by mechanical considerations, as it is impossible to indefinitely, or even very greatly, increase the focal length of our present equatorial telescopes. But there is another very simple way of accomplishing the same result, and this is to insert in front of the prism system a negative collimator lens. By making the latter of suitable focal length, the angular divergence of the cone of rays passing through the prism system may be made as small as we please, and, over a considerable range of wave-length, reduced to zero. It is then only necessary to place behind the system a second positive lens of whatever focal length desired to form the spectral image. The form and con-

struction of these lenses will be determined by the angular aperture of the objective and the nature of the color curve. Simple lenses are, of course most desirable if they can be employed without themselves introducing an undue amount of either spherical or chromatic aberration. We will first examine the conditions which determine the former requirement.

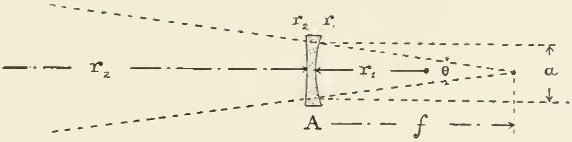


FIG. 1.

The form of simple lenses which introduces the minimum amount of spherical aberration is defined by the relations

$$\frac{1}{r_1} = \frac{1}{2(\mu - 1)(\mu + 2)}\left[(2\mu^2 + \mu)\frac{1}{f''} + (2\mu^2 - \mu - 4)\frac{1}{f'}\right], \qquad (1)$$

$$\frac{1}{r_2} = \frac{1}{2(\mu - 1)(\mu + 2)}\left[(2\mu^2 + \mu)\frac{1}{f'} + (2\mu^2 - \mu - 4)\frac{1}{f''}\right], \qquad (2)$$

where r_1 and r_2 are the radii of curvature of the first and second surfaces, and f', f'', the conjugate focal distances of the pencil of rays traversing the lens. If we make the focal length of the lens such that the pencil of light under consideration is rendered parallel, or, what amounts to the same thing, if we place the lens at its own focal distance from the focal plane of the objective, we have at once

$$\frac{1}{f'} = 0 \quad \text{and} \quad \frac{1}{f''} = \frac{1}{f} .$$

Then

$$\frac{r_2}{r_1} = \frac{2\mu^2 + \mu}{2\mu^2 - \mu - 4} , \qquad (3)$$

the well-known relation defining the ratio of the radii of curvatures of the so-called "crossed lens," the face of maximum radius of curvature, r_2, being presented to the incident cone of light from the large objective, as in Fig. 1.

Under the above conditions the expression for the longitudinal aberration becomes[1]

[1] LORD RAYLEIGH, "On the Minimum Aberration of a Single Lens for Parallel Rays," *Proc. Camb. Phil. Soc.*, 8, 373, April 1880.

$$\delta\nu = -\frac{a^2}{32f}\mu\left[\frac{4\mu - 1}{(\mu - 1)^2(\mu + 2)}\right], \tag{4}$$

where a is the aperture of the objective. The value of $\delta\nu$ decreases as μ increases. It is, however, unadvisable to employ very dense glass, on account of the relatively greater absorption and loss by reflection at the refracting surfaces. If we assume $\mu = 1.6$, corresponding to the ordinary light flint, we have

$$\delta\nu = 0.21\frac{a^2}{f}. \tag{5}$$

If we assume, as before, that in order to avoid injuring the definition the effect of the longitudinal aberration must not exceed one-half the resolving power of the aperture, then

$$\frac{\delta\nu\theta}{f} \leqq \frac{\lambda}{b} \quad \text{or} \quad \delta\nu \leqq \frac{\lambda}{\theta^2} = \frac{4\lambda}{a^2}f^2. \tag{6}$$

From (5) and (6) we have at once, for the limiting relation between θ and f,

$$\theta \gtreqqless \sqrt[4]{\frac{\lambda}{0.83f}}. \tag{7}$$

If, as before, we assume $\theta = 0.03$, which corresponds to the usual construction of refracting object-glasses, we have for f

$$\begin{aligned} f_{\text{max.}} &= \frac{\lambda}{0.67} \times 10^8, \\ &= 83.6\,\text{cm} \quad \text{for} \quad \lambda_v = 5600 \text{ tenth-meters}, \\ &= 64.2\,\text{cm} \quad \text{for} \quad \lambda_p = 4300 \text{ tenth-meters}. \end{aligned} \tag{8}$$

That is, for an ordinary telescope the focal length of the negative collimator must not exceed 84 cm for the visual rays, or 64 cm for the photographic rays. This condition at once determines the possible linear aperture a, and hence the resolving power of a given prism train. For, under the conditions above imposed,

$$\begin{aligned} a &= 2f\theta, \\ &= 5.02\,\text{cm} \quad \text{for visual rays}, \\ &= 3.85\,\text{cm} \quad \text{for photographic rays}. \end{aligned} \tag{9}$$

The possible maximum values of f and a decrease very rapidly

as the value of θ increases. For photographic telescopes and for reflecting telescopes the value of θ is frequently as large as 0.10. For this value the limiting values of f and a for a single lens become

$$\left.\begin{aligned} f_{\text{max.}} &\cong 0.8\,\text{cm}\ ; \quad a_{\text{max.}} \cong 1.6\,\text{mm} \quad \text{for } \lambda = 5600\ , \\ f_{\text{max.}} &\cong 0.6\,\text{cm}\ ; \quad a_{\text{max.}} \cong 1.2\,\text{mm} \quad \text{for } \lambda = 4300\ . \end{aligned}\right\} \tag{10}$$

For such cases, therefore, the possible dimensions of a single lens used as a negative collimator alone become very small.

If, however, we impose certain conditions with reference to the second positive lens used behind the prism train to form the spectral image, we may considerably reduce the effects of aberration due to the first lens alone, and thus considerably increase the possible aperture of the simple lens system. Thus, if we agree to make this second lens of the same material and focal length as the first (as may often be done) the aberration in the two lenses will be equal but of opposite sign, and, if the two were in contact, the resultant effect would be zero. The compensation will not be quite exact, on account of the separation of the two lenses and the intervention of the intermediate prism train, but the residual effect due to the latter will be small, and may, up to apertures equal to those already considered, be safely neglected. With such a construction as indicated above we may, therefore, safely employ single lenses up to 5 cm or more in diameter, even when the angular aperture of the objective is as large as 1 to 5, provided it has no large amount of chromatic aberration.

When the large objective itself has a considerable amount of chromatic aberration the use of a single lens is not, however, so satisfactory, as it is only possible then to collimate the rays over a limited range of wave-lengths. In order to increase the range over which the spectrum may be simultaneously brought to a sharp focus by the second positive lens we must construct the collimator so as to correct, as far as possible, for the chromatic aberration of the large objective. The degree to which this result may be secured will be next investigated.

In Fig. 2 let $AF = d =$ distance of the negative collimator from focal plane of the visual rays, $\lambda \cong 5600$; let $AF' = d' =$ distance of the negative collimator from focal plane for photographic rays, $\lambda \cong 4300$.

For the focal length of the negative lens we have the usual formulae

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \left(\mu_c - 1\right)\left[\frac{1}{r_1} + \frac{1}{r_2}\right] + (\mu_f - 1)\left[\frac{1}{r_3} + \frac{1}{r_4}\right], \quad (11)$$

f_1, f_2, being the individual focal lengths of the component flint and crown elements and r_1, r_2, r_3, r_4, the radii of curvatures of the successive surfaces measured from the right toward the left.

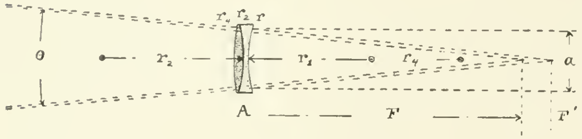


FIG. 2.

In order to render both the visual and photographic rays, λ_v and λ_p, simultaneously parallel, we must have

$$\frac{dF}{d\lambda}(\lambda_v - \lambda_p = \Delta\lambda) = d' - d = \Delta F,$$

which gives at once

$$d' - d = \left\{ \frac{d\mu_c}{d\lambda}\left[\frac{1}{r_1} + \frac{1}{r_2}\right] + \frac{d\mu_f}{d\lambda}\left[\frac{1}{r_3} + \frac{1}{r_4}\right] \right\} F^2 (\lambda_v - \lambda_p),$$

or

$$\frac{\Delta F}{F^2} = \frac{d\mu_c}{d\lambda}(\Delta\lambda)\frac{1}{(\mu_c - 1) f_1} + \frac{d\mu_f}{d\lambda}(\Delta\lambda)\frac{1}{(\mu_f - 1) f}, \quad (12)$$

For the usual soft crown and dense flint glasses we have

$$\left.\begin{array}{l} \mu_c = 1.5163 \text{ for } \lambda_v \\ \mu_f = 1.6256 \text{ for } \lambda_v \end{array}\right\} (\lambda = 5600).$$

Also

$$\frac{d\mu_c}{d\lambda} \cdot \frac{\Delta\lambda}{(\mu_c - 1)} = \frac{\delta\mu_c}{\mu_c - 1} \cong 0.01995$$

$$\frac{d\mu_f}{d\lambda} \cdot \frac{\Delta\lambda}{(\mu_f - 1)} = \frac{\delta\mu_f}{\mu_f - 1} \cong 0.03133.$$

Introducing these values and solving for f_1 and f_2 we get

$$\frac{1}{f_1} = \frac{1}{F}\left(87.87\frac{\Delta F}{F} + 2.753 + \right)$$
$$-\frac{1}{f_2} = \frac{1}{F}\left(87.87\frac{\Delta F}{F} + 1.753 + \right) . \qquad (13)$$

When ΔF is zero these equations become of course the usual formulae for a compound lens achromatic for the two rays λ_v and λ_p.

As a practical example let us consider the construction of a negative collimator of 3 cm aperture for the Yerkes 40-inch telescope. In this case

$$\theta \cong \tfrac{1}{2}\left(\frac{1}{19}\right) ,$$
$$\therefore F = 3 \times 19 = -57 \text{ cm} ,$$
$$\Delta F = -6 \text{ cm}\ ^{1} .$$

With these values we obtain at once

$$f_1 = -\frac{F}{12} = -4.75 \text{ cm}$$
$$f_2 = +\frac{F}{11} = +5.18 \text{ cm} , \qquad (14)$$

i. e., the desired chromatic collimation will be secured by using a negative lens of crown glass of 4.75 cm (virtual) focal length in combination with a positive lens of flint glass of a focal length of 5.18 cm.

In order to determine the best radii of curvature of the successive surfaces we have, as the condition of aplanatism for parallel rays,

$$\frac{m_1}{f_1}\left(\epsilon^2 + \frac{n_1}{f_1^2}\right) + \frac{m_2}{f_2}\left[\epsilon'^2 - \frac{1}{f_1}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + \frac{n_2}{f_2^2}\right] = 0 , \qquad (15)$$

where m_1, n_1, m_2, n_2, are constants defined by the relations

$$m_1 = \frac{\mu_c}{\mu_c + 2} \qquad m_2 = \frac{\mu_f}{\mu_f + 2}$$
$$n_1 = \frac{4\mu_c - 1}{4(\mu_c - 1)^2} \qquad n_2 = \frac{4\mu_f - 1}{4(\mu_f - 1)^2} . \qquad (16)$$

[1] From an unpublished determination of the color curve of the 40-inch objective made by the writer in 1897.

If we assume, as a second condition, that the inner faces of the two lenses shall be cemented (to avoid loss of light), then

$$r_2 = r_3 \ ,$$

and we have, as a second equation,

$$\frac{q_1}{f_1} + m_1\epsilon = \frac{q_2}{f_1} + p_2\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + m_2\epsilon' \ , \tag{17}$$

where

$$\left.\begin{aligned} q_1 &= \frac{2\mu_c^2 - \mu_c - 4}{2(\mu_c - 1)(\mu_c - 2)} \\ p_1 &= \frac{2\mu_c^2 + \mu_c}{2(\mu_c - 1)(\mu_c + 2)} \ , \end{aligned}\right. \tag{18}$$

and q_2 and p_2 have similar values in terms of μ_f .

If we assume as before the usual soft crown and dense flint construction, we have for μ_c and μ_f

$$\left.\begin{aligned} \mu_c &= 1.5163 \\ \mu_f &= 1.6256 \end{aligned}\right\} \text{ for } \lambda_v = 5600 \ .$$

Substituting these values in (16) and (18), and solving (15) and (17) for ϵ and ϵ' we find:

$$\begin{aligned} p_1 &= 1.6840 & p_2 &= 1.5324 \\ q_1 &= -0.2528 & q_2 &= -0.0750 \\ m_1 &= 0.4312 & m_2 &= 0.4484 \\ m_1 &= 4.7498 & m_2 &= 3.5146 \ , \end{aligned}$$

and for ϵ and ϵ'

$$\epsilon = +0.3148 \qquad \epsilon' = +0.4444 \ .$$

For the radii of curvature of the four surfaces we have

$$\frac{1}{r_1} = \frac{p_1}{f_1} + m_1\epsilon \qquad \text{or} \qquad r_1 = -4.56$$

$$\frac{1}{r_2} = \frac{1}{r_3} = \frac{q_1}{f_1} + m_1\epsilon \qquad r_2 = r_3 = +5.305$$

$$\frac{1}{r_4} = \frac{p_2}{f_1} + q_2\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + m_2\epsilon' \qquad r_4 = -8.326 \ .$$

The crown lens is therefore a double concave and the flint lens a double convex form, and the crown lens is turned toward the prism train as in Fig. 2.

When a compound lens of the form above described is used

as a collimator we may without restriction use a lens of any material and focal length desired for the image-forming lens beyond the prism train. If simple lenses are used for this purpose the limiting apertures and focal lengths which it is possible to employ are determined by equations (7) and (9) as before. In most cases simple lenses answer very satisfactorily in this place for the reason that the angular aperture θ_1 of this lens may be made small, thus reducing spherical aberration, and chromatic aberration need not be considered in the formation of a spectral image.

The first instrument of this kind which I constructed and tested was a small direct prism form, designed at the request of Professor Stone for the 26-inch Leander McCormick telescope of the University of Virginia.[1] The mechanical and optical construction is shown in detail in the cross-sectional view, Figs. 1 and 2, Plate I. *A* is the negative collimating lens, which in this case is of 16 mm clear aperture and 76 mm focal length. *B* is the direct-vision prism system, consisting of 5 prisms as shown, with a clear aperture of 13 mm and a total dispersion, C to H, of about 6°. *C* is the positive image-forming lens, which in this case is of the same focal length, 76 mm, as the negative collimator. *D* is a scale in the focal plane, and *E* and *F* the plane parallel glass reflector and lamp for illuminating the latter. *G* is an ordinary positive eyepiece of about 45 mm equivalent focus.

The negative collimator *A* and prism system *B* are mounted together in a short tube, *b*, which slips tightly into a second tube, *c*, carrying the positive lens, *C*. The tube *c* slides rather tightly in the third tube *d* which fits into the draw tube of the great telescope. The tube *d* carries at one end a bracket which supports the lamp box, *f*, and is threaded at the outer end to receive the fourth eyepiece tube, *g*. A fifth tube, *e*, fitting tightly into *g*, carries the reflector and scale. The tubes *g* and *e* are slotted at the point *i* to receive a metal diaphragm or slide, *k*.

The instrument is adjusted before being attached to the large

[1] See my *Report of the Allegheny Observatory*, 1898–9, p. 9.

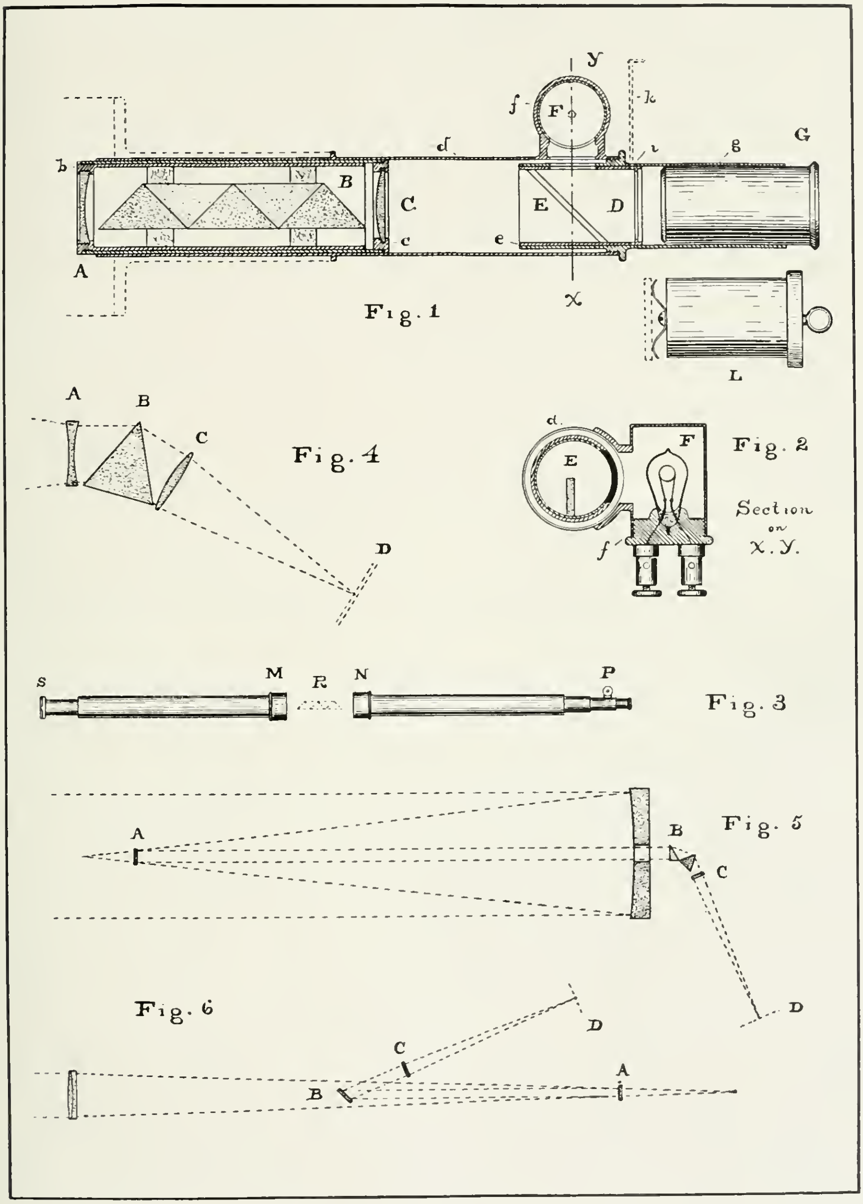
telescope, as follows: The tube *b*, containing the negative collimator and prism system, is first removed and the eyepiece is focused on the scale *D*. The positive lens *C* is then focused for parallel rays by means of the tube *c*, either by looking at a distant object, or better, by placing the instrument in front of the object glass of a transit and focusing on the cross-wires of the same. The tube *b* is then replaced, taking care not to disturb the relative positions of *c* and *d*. The whole instrument is then placed in the draw tube of the large telescope and focused on any particular line in the spectrum by moving the latter in or out as with an ordinary eyepiece.

It is evident that with this form and construction of instrument, micrometric and wave-length observations and comparisons may be made directly at the focal plane by the aid of the scale *D* in the same manner as with the objective spectroscope. The instrument is also adapted to photographic work. To carry this out the eyepiece is removed and a small photographic plate substituted, the sensitive surface of the latter being held against the scale *D* by means of the wooden plug and spring *L*. The diaphragm *k* then serves as a slide, and the tube *g* with diaphragm *k* and plug *L* as a plate-holder, which is readily detached from the instrument by unscrewing it from *d*. The spectrum of the object under examination and the reference scale *D* can thus be photographed simultaneously on the sensitive plate.

The optical resolving power of the instrument above described is small, being about equivalent to one 60° white flint prism of 1-inch clear aperture, a power about three times that required to separate the D lines.

In order to subject it to a more critical and severe test than that involved in the examination of stellar spectra, the arrangement shown in Fig. 3 of the same plate was used. In this figure *M* is a large collimator (7 cm aperture) with slit at *s* accurately adjusted for parallel rays by the usual method. *N* is an ordinary observing telescope with the above described instrument inserted at *P* in the place of the usual eyepiece. When the slit *s* is illuminated by sunlight an image of the solar spec-

PLATE I.



trum is, of course, formed at the focal plane of the instrument.

The definition of the prism train alone can be tested by removing it from the instrument and placing it at *R*, between the two telescope objectives, as in the usual spectroscope. Tested carefully in this way the definition and resolving power was found to be equally good with both arrangements. The nickel line between the D's was sharp and well resolved. When, however, the negative collimator lens and positive lens, *C*, was removed and the same direct-vision prism alone used at *P*, as in the usual form of ocular spectroscope, all the lines became very hazy and ill defined, and it was found impossible to separate even the two D lines.

The second instrument of this new form which has been constructed was one for the Crossley 36-inch reflector of the Lick Observatory. Professor Keeler, just before his death, had planned to use an ocular spectroscope consisting of a single 50° prism of quartz placed just inside the focus on the Crossley reflector, for the purpose of photographing the spectra of faint stars. As was shown in my previous paper, such an arrangement would be very bad optically, and this was found by experiment to be the case. The idea of using a negative collimator to improve the definition then independently suggested itself to Professor Campbell, who entered into correspondence with Mr. Brashear in regard to the matter in September 1900.

In the case of the Crossley reflector the ratio of aperture to focal length is 3 : 17.5 or 1 : 5.84. Hence $\theta = 0.086$.

From (7) we have, therefore, in this case,

$$f_{\text{max.}} = \frac{\lambda \times 10^6}{45} \,,$$

$$= 1.25 \,\text{cm} \quad \text{for the visual rays} \,, \qquad \lambda = 5600 \,,$$

$$= 0.95 \,\text{cm} \quad \text{for the photographic rays} \,, \quad \lambda = 4300 \,.$$

The permissible aperture of the negative collimator therefore is $a = \dfrac{f}{5.83} \cong 2 \,\text{mm}$ for the photographic region of the spectrum.

It was desired to use an aperture about twelve times as large as this, the aperture of the quartz prism being about 1 inch.

In order to make this possible without introducing compound lenses, the plan described in an earlier paragraph of making the second positive lens of the same material and focal length as the negative collimator was adopted. As the large objective was itself perfectly achromatic (being a reflector), chromatic aberration did not have to be considered. In order to avoid absorption the lenses were made of quartz (like the prism), the plane of the lens being cut perpendicular to the axis of the crystal (to avoid double refraction).

The dimensions and computed radii of the lenses were as follows:

$$\left.\begin{array}{ll}\text{negative collimator} & f = -\ 152.4\,\text{mm (6 in.)} \\ \text{positive lens} & f = +\ 152.4\,\text{mm (6 in.)}\end{array}\right\}\ \text{for G}\ .$$

For G ($\lambda = 4307$) μ_0, the ordinary index of quartz, is 1.55425. Hence for a "crossed" lens we have from (3)

$$r_1 \cong 9r_2\ ,$$

and with this relation we find at once

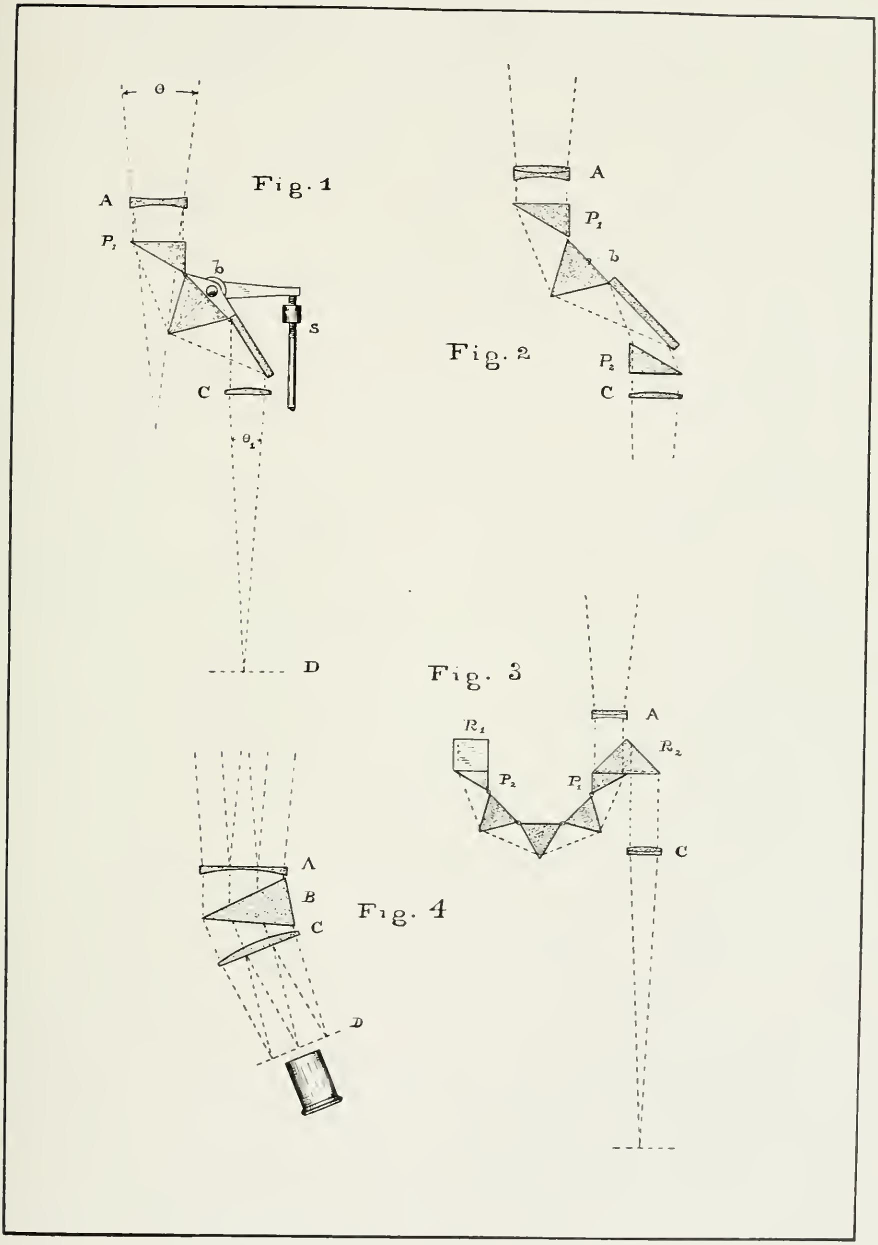
$$r_1 = 845\,\text{mm}\ (33.25\ \text{in.})\ , \qquad r_2 = 93.8\,\text{mm}\ (3.695\ \text{in.})\ .$$

The arrangement of the parts of this instrument with reference to each other and to the large speculum is shown diagrammatically in Fig. 4, Plate I.

It is frequently very necessary or desirable that the angular aperture of the positive image-forming lens should be much smaller than the large objective and negative lens. Thus, to obtain full photographic resolution on a photographic plate placed at *D*, Figs. 1 and 4, the focal length of the lens *C* should be from twenty-five to forty times its aperture.[1] In such cases it is impossible to compensate the aberration of the first lens in the manner adopted in the case of the instrument last described, and, if large linear apertures of the collimator are to be used in conjunction with the main objective, it becomes necessary to correct the aberration of the negative collimator independently

[1] "Further Notes on Astronomical Spectroscopes," ASTROPHYSICAL JOURNAL 3, 187 *et seq.*, March 1896; "Conditions of Maximum Efficiency in the Use of the Spectrograph," ASTROPHYSICAL JOURNAL 3, 327 *et seq.*, May 1896.

PLATE II.



of the second positive lens. This may be done, as already pointed out, by the use of a compound lens of glass. For work in the ultra-violet where glass (especially dense flint) is objectionable on account of absorption, we may construct the compound lens of quartz and fluorite. Or better still, especially in the case of a reflecting telescope, we may use in place of a lens a convex mirror, as in the Cassegrainian type of instrument, the small mirror being in this case parabolic (not hyperbolic) in cross-section and placed at its own focal length from the focus of the large objective. The prism train and image-forming train may then be placed at any point in the parallel beam of light reflected from the small mirror; most conveniently, perhaps, just behind the large objective, as shown in diagram in Fig. 5, Plate I. The reflecting collimator may also be used where the large objective is a refractor in such a way as illustrated in Fig. 6 (same plate). In such cases, however, it is not so convenient as the negative lens, and the latter, moreover, has the added advantage already pointed out that we may by its use correct also for the chromatic aberration of the large objective.

It is obvious that any desired form or arrangement of prism train may be used with these instruments as readily and easily as with the compound slit spectroscope. A number of other trains of different form and arrangement of prisms which seem well adapted to this type of instrument are shown in the various figures of Plate II. Other modifications of these or of the usual simple prism trains will readily suggest themselves. In Fig. 1 of this plate the dispersing system consists of a fixed arm combination of type 4 described several years ago by the writer,[1] with an added half-prism *P*, which secures about 50 per cent. added resolving power and at the same time reduces the horizontal width of the emergent pencil of rays, and consequently reduces the horizontal angular aperture θ_1 of the image-forming lens *C*, thus necessitating a smaller focal length of the latter to obtain full photographic resolving power at *D*. If the latter

[1] "Fixed Arm Spectroscopes," *Phil. Mag.*, 38, 348, October 1894; also *A. and A.*, December 1894.

advantage is unimportant we may add still more to the resolving power and secure symmetry of train by placing a second half-prism, P_2, in front of the lens C as shown in Fig. 2. It is obvious that, when these half-prisms are placed as shown in these figures, with their short sides perpendicular to the axes of the respective lenses A and C, they themselves require no adjustment for minimum deviation, and do not interfere in any way with the performance of the fixed-arm system B, by means of which any desired part of the spectrum can be brought to the center of the field of view by simply rotating the prism mirror system about the axis b by means of the screw s or any equivalent device. Such half-prisms therefore might be used with advantage in other cases where the "fixed-arm" systems have been employed.[3]

In Fig. 3 is shown another type of "fixed-arm" dispersing train which is particularly adapted to those lines of work in which it is necessary to change the resolving power of the instrument without disturbing the position of the collimator or eyepiece.

The beam from the negative collimator A falls first on the lower part of a half-prism P_1, thence passes through the train B to a second half-prism P_2 at the end, thence is returned by a double-reflection right-angled prism R_1 through the upper half of the prisms P_2, B, and P_1. On emergence from the upper half of P_1 it is caught and returned to the image-forming lens C by a second double-reflection right-angled prism R_2 placed with its axis at right angles to the axis of R_1. Any desired dispersion and resolving power down to one (two half) prism can be secured by changing the number of prisms in the intermediate train B.

This new form of focal plane spectroscope would seem to be particularly well adapted to certain lines of astronomical investigation which require a large instrument, either refractor or reflector, and which cannot well be undertaken with the com-

[3] *Loc. cit.;* also ASTROPHYSICAL JOURNAL 1, 232, March 1895; *ibid.*, 2, 264, November 1895; *ibid.*, 3, 169, March 1896; etc.

pound slit spectroscope, because of its nature, nor with the objective spectroscope, because of its cost. Among these may be mentioned:

1. The photographic survey of the spectral type of very faint stars. This is a field of work which, as Professor Pickering has demonstrated, can be carried on most rapidly and satisfactorily with the objective prism. But in the study of very faint stars very large apertures and consequently very large prisms are necessary. The expense of the latter may be avoided by using in place of the objective prism one of the forms of focal plane spectroscope above described. In order to secure the spectra of a number of stars at once, as is possible with the objective spectroscope, the aperture of the spectroscope should be made considerably larger than that required to receive the cone of rays from the large objective. In this case, in order to avoid the effects of aberration and astigmatism in the lateral images and secure as large as a field of good definition as possible, the two lenses are made of the same material and focal length and are each concavo-convex in form as shown in Fig. 4, Plate II.

2. Photographic study of the spectra of faint individual stars, particularly of the ultra-violet region. This is the field of work to which Professor Keeler proposed to apply the instrument which has already been described. It is easy to obtain quartz prisms, lenses, etc., of the size required for the focal plane instrument, while it would be quite impossible to obtain them of a size required for an objective spectroscope.

3. Photometric studies of the relative spectral intensity of faint stars and variables. In this field again, as well as in the others, the focal plane instrument has the advantage over the objective prism in that the spectroscopic resolving power can be readily and inexpensively varied to suit the requirements of the investigation in hand by the use of a form of instrument similar to that shown in Fig. 3, Plate II, and already described.

The instruments are applicable to many other lines of work which it is unnecessary to mention in detail. In the improved

form they will occupy, I think, a much more important field of usefulness than has been filled in the past by the ocular spectroscope.

In addition to the two instruments for the Lick and the Leander McCormick Observatories described in this paper, a third and much larger one, of 70 mm or 2¾-inch aperture, has been designed for use with the 13-inch (33 cm), 18-inch (46 cm), and 30-inch (76 cm) telescopes of the new Allegheny Observatory[1] which is now approaching completion. This last instrument is particularly designed for use in the fields of work (1) and (3) above described.

Allegheny Observatory,
December 1900.

[1] See "Report of Director for Year Ending December, 1900," *Misc. Scientific Papers of the A. O., No. 1*, Murdoch Kerr Press, Pittsburg.

THE ARC SPECTRUM WITH HEAVY CURRENTS.

By W. B. Huff.

At the suggestion of Professor Ames, the writer undertook a study of the effects produced on the arc spectrum of various substances by the use of heavy currents. The arrangement of resistances was such that the 110-volt direct current used could be varied from two amperes, the smallest for which the discharge would pass, to over two hundred amperes. The exposures for the two extreme currents in a given case were made on the same plate, all possible precautions being taken to prevent accidental disturbances of the camera.

Metals of low fusing points—such as lead, zinc, and cadmium—when placed in carbon holders in such quantities as practically to constitute one pole of the arc, showed fairly definite absorption spectra, which became sharper with heavier currents and longer exposure. A comparison of the plates obtained by using extreme values of the currents gave no definite results.

The spectra of more refractory substances, such as calcium, were brought out very strongly by a current of one hundred amperes, though the relative intensities were about the same as for a current of but a few amperes.

When a large amount of calcium was used, the H and K lines were reversed so sharply that it was possible, by using much less of the metal, to obtain these same lines, fine and clear, in the reversals. These finer lines were not symmetrically placed as to the reversals, illustrating the "density-shift" noted by various observers.

Because of the importance of the iron spectrum in astrophysical work, it was deemed advisable to obtain some plates of this spectrum in the region of the violet. The extreme values of the currents were two amperes and about two hundred and fifty amperes. The times of exposure were about three

seconds for the strong currents and ten to twenty minutes for the weak ones. But the discharge with heavy currents took place in a series of sharp explosions which usually blew out the arc and made time estimation difficult.

As would be expected, the spectrum of such a violent discharge showed a good deal of continuity, as well as many reversals and shaded lines. The results from these two extreme currents were compared in order to detect possible shifts, and for change of relative intensities. Though the sharpest plates were chosen for measurement, the character of the heavy-current spectrum made accurate settings difficult. Various pairs of lines of the two spectra were found slightly displaced with respect to each other, but the shift was not uniform, and in no case was it more than a few thousandths of a unit. Errors of measurement and the probability of the lines being shifted differently, owing to differences of pressure and of density at different times of exposure, as well as at different parts of the arc, make it impossible to say anything more definite. The relative intensities of the two plates were not greatly different, though in neither case did they agree with those assigned by other observers; *e. g.*, with those of Kayser and Runge.

The most interesting difference in the spectra of the arcs from these extreme currents was in the case of the bands appearing in the spectrum of ordinary carbon poles, one group of which is generally ascribed to carbon or one of its oxides; the other, to cyanogen. The different bands of the "cyanogen" group seemed to come out with about the same relative intensities among themselves, and a similar remark applies to the "carbon" group. But the two groups come out very differently in arcs from small and from large currents. Many plates were taken, and all showed that, as the current was increased, the carbon bands came out more and more strongly as compared to those of cyanogen. The latter came out clearly for very small currents, even when those of carbon are scarcely to be detected. Of course the cyanogen bands come out more strongly with increasing currents, but their increase of intensity by no means

equals that of the carbon bands for the same increase of current. The extreme currents used were two amperes and one hundred and eighty amperes.

In order to investigate more closely the structure of the arc, it was studied objectively with the flat grating, in the manner suggested to me by Mr. L. E. Jewell. For this kind of work the plane grating is obviously superior to the concave, avoiding, as it does, the integrating effect of the latter.

Studying the arc as a whole, the observations of previous observers[1] were confirmed in many particulars. Using the arc between ordinary commercial carbons, the lines due to metallic impurities are seen only in the immediate neighborhood of the negative pole; the bands of carbon and of cyanogen appear to have their origin at the positive or hot pole. If larger amounts of metal are put into either pole of the arc, the metallic lines may extend across to the positive pole. But in no case was a short metallic line observed originating at the positive pole. Increasing current also causes these lines, which appear as short eruptions of metallic vapor, to extend entirely to the hot pole. With sudden changes of current, these prominences lengthen or shorten, but so slowly that the change in length is readily followed with the eye. The heads of the carbon bands can be traced as extremely brilliant lines on the continuous spectrum coming from the hot pole. The cyanogen bands were not traced so far. It seems possible, therefore, that the radiation giving rise to the cyanogen group is to be thought of as coming from the surface of the arc discharge, although showing up more brilliantly nearer the hot pole. The ordinary quiet discharge usually takes the well-known curved path. If the poles are brought closer together, the discharge changes into the hissing form and appears to take the shortest path between the terminals. As would be expected from this shortening of the path, the resistance of the hissing arc is less than that of the quiet one. For a given separation of the poles, the discharge may be unstable, taking first one form, then the other. But even then the discharge with the shortest path has the least resistance.

[1] BALDWIN, *Phys. Rev.*, 3, 1895; FOLEY, *ibid.*, 5, 1897; THOMAS, *C. R.*, 119, 1894.

For the hissing arc there is considerable continuous spectrum, and the carbon bands flash out strongly at the hot pole; more strongly, indeed, than the same current shows when it is passing quietly. If a blast of air is directed against the silent arc in such a direction as to shorten the path of the discharge, the arc assumes the hissing form and the resistance may be decreased as much as 20 per cent. As a result of the increased current, there is an increase in the intensity of the carbon bands in the hissing arc. But, in the arc subjected to the air blast, the cyanogen bands seem at times to come out more strongly. This goes to support the idea that the cyanogen bands are a surface effect in the arc. When the arc is studied objectively, it is possible to get the carbon bands much more sharply defined than those of cyanogen. It is as if the latter came from the outer sheath of the arc.

If an alternating discharge is used, both groups of bands extend entirely across the space between the poles, and appear to be equally strong at the two poles. The prominences, due to a metal which is present in the arc in considerable quantity, also extend across this space. Analysis with a revolving mirror would probably show that such lines are shot out from the poles alternately.

The analogy between this discharge of particles proceeding from the negative pole to the intensely heated positive pole, and the kathode discharge heating the anti-kathode of a vacuum tube, is at least worth noting. Both the arc discharge and that in the vacuum tube are deflected by a magnet, though Röntgen rays have not been obtained from the arc.

In conclusion, then, it may be said that heavy currents bring out the carbon bands much more strongly than they do those of cyanogen; and that, in an arc between carbon poles, small amounts of metal show metallic lines only at the cool pole, while the bands of carbon and cyanogen appear strongest near the hot or positive pole.

PHYSICAL LABORATORY,
JOHNS HOPKINS UNIVERSITY,
May 1902.

THE SPECTRA OF CATHODO-LUMINESCENT METALLIC VAPORS.

By Percival Lewis.

Recent investigations have in general tended to confirm the view advanced by Hittorf [1] and elaborated by E. Wiedemann,[2] that the luminosity of gases and vapors, particularly in the case of vacuum-tube phenomena, is of the nature of "phosphorescence;" that is to say, it is directly dependent, not upon high temperature, but upon chemical, electrical, or unknown processes. All such phenomena of radiation, to which Kirchhoff's law does not quantitatively apply, may, until our knowledge permits of further differentiation, be grouped under the general title "luminescence," suggested by E. Wiedemann.

Since fluorescence is one of the most striking luminescent phenomena, it was of interest to discover whether substances possessing a finite number of definite free periods of vibration, such as gases and vapors, could be made to fluoresce. This question was answered affirmatively by E. Wiedemann and G. Schmidt,[3] who showed that sodium and potassium vapors fluoresce under the action of sunlight, giving characteristic spectra, in part corresponding to the ordinary flame spectra.

Cathode rays are far more effective than sunlight in producing fluorescence of solids, and it seems reasonable to expect similar effects in the case of vapors, independent of the direct effect of the current passing through them. Some instances substantiating this expectation are well known. The negative glow from a plane cathode extends not only toward the anode, or in the direction of the current, but from the opposite side of the cathode as well, even when all precautions are taken to

[1] Hittorf, *Pogg. Ann.*, 7, 587, 1879.

[2] E. Wiedemann, *Phil. Mag.*, 28, 149, 248, 1889; *Wied. Ann.*, 37, 177, 1889.

[3] E. Wiedemann and G. Schmidt, Astrophysical Journal, 3, 207, 1896; *Wied. Ann.*, 57, 447, 1896.

avoid a flow of current in that direction; the length of this negative column increases as the pressure diminishes, and when it reaches the walls of the tube they begin to fluoresce. Hertz,[1] in 1883, isolated the effects of the cathode rays from those of the current by constructing a vacuum tube in which the anode was concentric with the cathode, and almost in the same plane, so that the stream lines were limited to a small space, while the cathode rays were projected to the end of the tube, 30 cm distant. Mercury vapor at the end of the tube glowed under the action of the cathode rays, and showed the characteristic strong lines of the mercury spectrum. Lenard[2] found that cathode rays would pass through a thin aluminum window, into gases at atmospheric pressure; these gases glowed, but too faintly to show any spectral lines. In this case also electric currents could have played only a very small part, as they could have arisen only from small static charges.

Some investigations on the vacuum tube spectra of metals have been published,[3] but, with the exception of the one experiment by Hertz, no attempt seems to have been made to isolate the cathodo-luminescence from the direct effects of the current. The writer has investigated the effects of cathode rays upon the vapors of such volatile metals as were available, and has observed a characteristic luminosity in a number of cases. With sodium and potassium these effects seemed somewhat different from those observed by Wiedemann and Schmidt when these vapors were fluorescing under the action of sunlight. The cathode rays may merely produce a stronger fluorescence, or the effects may be excited by the mechanical impact of the cathode corpuscles, or by their electrical charges, in a manner altogether different from fluorescence, so it may be safer to apply the more general term "luminescence" to the observed phenomena. It seems evident, however, that this radiation can depend neither upon

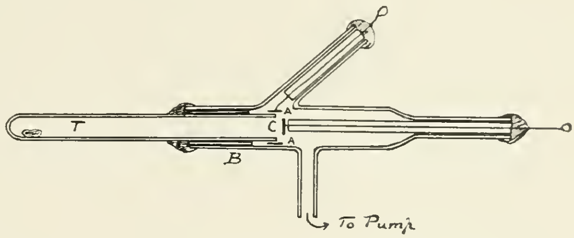
[1] H. Hertz, *Wied. Ann.*, **19**, 809, 1883.

[2] P. Lenard, *Wied. Ann.*, **51**, 229, 1894.

[3] See, for example, E. Wiedemann and G. C. Schmidt, *Wied. Ann.*, **57**, 454, 1896; A. C. Jones, *Wied. Ann.*, **62**, 30, 1897.

high temperature nor upon an electric current in the ordinary sense, and we might expect to see only the more fundamental spectral lines.

The vacuum tube used was somewhat like that used by Hertz, and is shown in the accompanying diagram. The substance to be investigated is placed in the hard glass tube *T*, which is then telescoped into the tube *B*, and joined to the latter with sealing wax. From the disk-shaped cathode *C* the cathode rays are projected to the bottom of the tube *T*, at a distance of 25 cm. The ring-shaped anode *A* lies outside of *T*, at a distance of about 5 mm from *C*, and almost in the same plane. This arrangement gives a compact system of current lines, and shields the bottom of *T* from cathode rays when the current is reversed. As a further precaution against stray currents and electrostatic effects, a wire gauze cylinder or a long wire loop was sometimes inserted throughout the entire length of *T*, without sensibly altering the observed phenomena. The tube was exhausted with a Töpler-Hagen pump, and excited with a coil giving a spark about 15 cm in length. The luminous vapors were examined with a Browning pocket spectroscope or a chemical spectroscope with calibrated scale for identifying the lines. In the case of some lines too weak to see with the latter instrument, a rough identification was made by comparison with the position of the hydrocarbon bands in the spectrum of the Bunsen flame used to heat the tube. These estimates are of course liable to error, but such lines probably correspond to known strong lines of the metal.



Below is a brief statement of the results obtained. None of the metals used were chemically pure. The atmosphere within the tube was chemically prepared nitrogen.

Sodium. — At a vapor pressure corresponding to a temperature below red heat, an orange-colored glow was observed, showing the D lines ; at red heat the glow became greenish-yellow, and the citron-green lines $\lambda 5683-88$ appeared ; also faint lines or bands in the red and blue-green, probably the pairs $\lambda 6154-61$ and $\lambda 4979-83$. This luminescence and that in the other cases described, appeared only when the exhaustion had reached such a point that the cathode rays struck the bottom of T and produced fluorescence of the glass. It disappeared when the cathode rays were deflected with a magnet, but was not affected by the introduction of a gauze cylinder or long wire. It did not appear when the current was reversed or when, at pressures slightly above cathode-ray pressure, a small conduction current was transmitted through the vapor by touching the end of T with an earthed conductor. These facts indicate that the observed effects were due to the cathode rays alone. The color and spectrum of the luminescence were different from those of the fluorescence seen by Wiedemann and Schmidt, which was green, and showed green and red flutings.

Potassium. — A light purple glow. The yellow sodium lines were the brightest in the spectrum. In addition, there were visible the yellow potassium lines λ 5783, 5802, and 5832, and several faint lines in the green, not bright enough to be identified. The red and violet lines were not visible, which is not surprising, considering how near they lie to the limits of the spectrum, but the color of the luminescence left little doubt that their radiations were present.

Magnesium. — Magnesium powder was heated in the tube. It did not melt, but decrepitated actively and sublimed. At dull red heat a bright green glow filled the end of the tube, showing the triplet λ 5183, 5172, 5167.

Mercury. — The glow was pale green, not pink, as described by Hertz. A difference of atmosphere may account for this. The yellow, green, and blue lines were observed, the green line being strongest and most persistent.

Zinc. — Just below red heat, a faint lilac glow was seen, giv-

ing the triplet λ4811, 4722, and 4680. These lines are sometimes seen in the flame spectrum of the chloride. At red heat a weak red line (probably 6383) also appeared, and the glow changed to a rich purple. These lines are the only ones which appeared in a feeble spark spectrum, without condenser, and are the longest lines of the spark spectrum. Sometimes λ4912 was faintly seen with the pocket spectroscope.

Cadmium. — Below red heat the glow was pale lavender; at red heat, violet. The lines observed, in the order of their intensity, were λ5086, 4413, 4800, and 4678; at very high temperatures a red line also appeared (probably 6431). The lines 5086, 4800, and 4678 can be seen in flame spectra of the chloride. It is remarkable that 4413, which is almost invisible with a simple spark, and relatively weak with small condenser, was next to the green line in intensity, while 4678 was very weak. A. C. Jones[1] states that in the vacuum tube spectrum of cadmium the color is green without a spark-gap, blue with it, and that, without a spark-gap, λ4800 and 4678 are strong, and 4413 weak. The longest lines in the spark spectrum are λ5086, 4800, and 4678. These differences in color and spectrum are additional evidence that the effects observed here are essentially different from those attending the passage of the current through the vapor.

Thallium. — At comparatively low temperatures a bright green luminescence was produced, which showed the green line λ5380.

Bismuth, lead, antimony, tin, and aluminum showed no appreciable luminescence.

Sulphur, selenium, and tellurium gave an almost inappreciable blue glow, which showed a very weak spectrum, apparently continuous in the green and blue.

In most cases, the lines observed were those which are seen in flame or weak spark spectra, and seem, therefore, to correspond to fundamental types of vibration.

Further investigation of this subject will be carried on.

University of California,
Berkeley, May 1902.

[1] A. C. Jones, *Wied. Ann.*, 62, 30, 1897.

A REPLY TO THE RECENT ARTICLE BY LOUIS BELL.

By A. Perot and C. Fabry.

In an article recently published in this Journal[1] Dr. Louis Bell criticises our interference methods for the measurement of wave-lengths, and in particular our work on the corrections to Rowland's scale. His conclusions may be summed up as follows: (1) Interference methods do not give results which are more precise than those obtained with gratings; (2) our correction curve to Rowland's tables seems to be based on inadequate evidence. We desire to reply briefly to these criticisms.

So far as absolute values (comparison with the meter) are concerned, we have naturally taken as the point of departure the result of Michelson and Benoit, which is unquestionably the most precise. Reference to old values is only of historical interest. We are, therefore, concerned only with comparisons, or relative measures, which are much easier and more precise than absolute measures. Thus the quantity one-millionth, given as the probable error of the results obtained by Michelson and Benoit, applies to the absolute values; the relative wave-lengths are much more precise, as we have frequently had occasion to verify for the green and red lines.

In two places in his article Dr. Bell insists upon the discordance between the values found by Michelson and by Hamy for the relative wave-lengths of the green and red lines of cadmium. But it must not be forgotten that two different sources were employed (Hamy used the discharge of a condenser through a tube without electrodes); the wave-lengths are probably different, and although this point calls for further researches, it is unfair to attribute the differences to the interference methods. It is none the less improper to condemn the green cadmium line as a standard of wave-length, as Dr. Bell seems to do. As a matter of fact, the use of the lines of metallic vapors at low pressures,

[1] Astrophysical Journal, 15, 157, April 1902.

as standards, constitutes a marked advance in spectroscopy. During an experience of five years in spectroscopic and metrological measurements, we have never observed an error which could be attributed to a variation of the cadmium lines employed as a standard. The discovery of a brighter luminous source is certainly to be desired, but it is difficult to conceive of one giving better defined wave-lengths.

All comparisons between the values found in the arc and in the solar spectrum seem to us incompetent to solve the problem of the precise comparison of wave-lengths. Mr. Jewell's direct comparisons show the displacements, usually in the direction of an increase of wave-length for the solar spectrum, but occasionally in the opposite direction. It is certain that these variations demand further investigation, but it does not seem reasonable to endeavor to solve a simple and clearly defined problem through another which is much more complex.

There is no occasion for surprise in the following fact, alluded to by Dr. Bell : If we start with the values found by us for the lines of *Zn* or *Na*, and apply our corrections to reduce them to Rowland's scale, values about 0.02 tenth-meters smaller than Rowland's are obtained. Rowland's values are relative to the solar spectrum, while ours refer to artificial sources at low pressure (a vacuum for *Zn* and atmospheric pressure for *Na*). The differences between Rowland's values and our own is in the direction and of the order that one would expect.

Finally, if our corrections to Rowland's scale were due to ordinary experimental errors, the points of our diagram would be irregularly distributed on both sides of a horizontal line ; that this is not the case may be seen at a glance. It appears incontestable that, if two lines are selected in Rowland's tables, one near λ 5300 and the other near λ 5600, the ratio of the two values given by Rowland is in error by about seven thousandths; in other words, if one were correct, the other would be in error by about 0.03 or 0.04 tenth-meter. This can in no wise diminish our admiration for the immortal work of Rowland, but the importance of this work should be a further reason for seeking to derive from it the best possible results.

NOTE ON THE NEBULA SURROUNDING *NOVA PERSEI.*

By Louis Bell.

The photographs by Ritchey, and the notes by Perrine, in the April Astrophysical Journal seem to afford a new basis for the consideration of the apparent very rapid motion in the nebular appendages of the *Nova*. The chief nebular ring in Ritchey's photograph of September 20, having a radius of about 12′ 30″, indicates, if its expansion rate be taken as that of electromagnetic waves for the 210 days following the outburst, a parallax of 0.″02, and a distance of 159 light years. The contour of the ring suggests that it lies nearly in the normal plane. If it does not so lie, then the radius observed shows either a propagation-speed greater than V, or a parallax greater than 0.″02.

Perrine's photograph of March 29 previous gives the principal ring a radius of about 2′, which also leads to a parallax of almost exactly 0.″02 on the same assumptions as before. Hence, if the one ring is an expansion of the other from whatever cause, there is no evidence of acceleration, and any hypothesis of projected matter of ionic or other dimensions gaining velocity under a repulsive force, becomes untenable. The case is no better for projected matter, if the velocity is assumed to be due to an initial explosion, since, aside from the magnitude of the forces involved, matter cannot be driven with or without constant acceleration to a velocity V by a force having a propagation-speed less than V under such circumstances.

The expansion of the inner rings on the dates mentioned is evidently not in the normal plane, but unites in showing non-accelerated motion. In the September photograph there is a nebulous patch 20′ from the *Nova*, which may or may not be part of the system. If it is, then the parallax would be nearly 0.″03, or the expansion speed proportionately greater. Now

Chase, Aitken, and others unite in a very small value of the parallax of the *Nova*, little, if at all, larger than 0″.01. Hence V is certainly as small a propagation-speed as agrees with the facts, and there is at least a possibility of having to account for a larger value. Of propagation-speeds greater than V, but two are known even hypothetically: that of change of gravitational potential, and that of a compressional wave in the ether. Of the former nothing is known; of the latter, only that it would equal $\sqrt{\frac{k+\frac{4}{3}n}{\rho}}$. As we have only inferential knowledge of the bulk-modulus of the ether, speculation seems uncalled for, until the parallax is finally shown to be below 0″.02.

Of hypotheses to account for the propagation of nebulosity outward with velocity V, the most obvious is that of Kapteyn, independently devised by Seeliger and by one or two others. A modification of this fundamental idea based on the secondary effects of an electro-magnetic wave-front was suggested by the writer to Professor Hale in December last; and, as recent data have some bearing on the matter, it may be worth a brief discussion. There are three rather serious objections to the hypothesis of pure reflection. First, reflected light, whether reflected in the ordinary way from heterogeneous surfaces or from small particles, would be polarized, and Perrine's report on this feature of the case indicates absence of polarization. Second, reflection does not adequately explain the very remarkable persistence of some regions of strong nebulosity at a small angular distance from the *Nova*. Especially the nebular peak nearly south of the *Nova* has an intensity all out of proportion to that of the outer ring, while both on the reflection hypothesis should be at similar radial distances. If they are, then the ring must represent a condition of matter having a very small albedo compared with that in the other region. Third, at the radius of 210 light days denoted by the radius of the ring of September 20, reflection does not adequately account for the brightness of the nebular matter observed.

Taking the *Nova* as of zero magnitude at its maximum, and of

surface brilliancy equal to the Sun, it would have from the computed parallax about 2500 times the total solar light. A material body at $R = 36{,}400$ would receive from such a source a light a little stronger than full moonlight on the the Earth, and at even the high albedo of *Mars* would reflect light about as intense as $\frac{1}{2 \times 10^6}$ of terrestrial sunlight. Since the distance implied by a parallax of $0''.02$ is about 10^7 units, even a solid body of the dimensions of the *Nova* itself would be far below any magnitude within reach of existing instruments; and it is extremely doubtful whether even a matter-charged region of the extent of the nebular wisps observed would, at any admissible value of the mean albedo, be within the present reach of photography, and still less give images of the density found.

If, however, one considers the luminous disturbance as a secondary result of radiated electro-magnetic strains, somewhat similar in character to those observed in solar storms, the case is more promising. Every electro-magnetic wave-front carries an orthogonal system of electric stresses. When of sufficient magnitude, these are capable of exciting powerful luminosity in masses of tenuous gas. Air at a pressure *circa* 0.01 mm becomes a rather good conductor — as good as the best electrolytes — and currents set up in such a medium give spectra of the gases involved very much modified from their usual forms, and nearly or quite free from the lines due to any intermingled metallic particles.

In comparatively trivial solar storms the envelope of tenuous gas 100 km or so above the Earth becomes the seat of powerful disturbances of this sort manifest as widespread auroras and as the source of violent changes in the magnetic elements accompanied by great local variations in Earth potential.

Admitting that a solar disturbance involving, say, $\frac{1}{500000}$ of the visible solar area is capable of such effects, then a disturbance of similar intensity involving the whole surface of a body of the magnitude of the *Nova* would set up, even at $R = 36{,}400$, electric forces about equal to those known to act on the Earth and its gaseous envelope.

Matter-charged space filled with tenuous gas dissipated from the matter is the necessary seat of currents and the transformation of energy when subjected to such stresses. Such is the normal result in the upper atmosphere, although the luminous effects are of only moderate brilliancy, owing to the small thickness of the strata in which the action takes place. In cosmic masses there may well be very brilliant results. For example, even so small a volume as a cube 1000 km on the edge, subject to a potential gradient of 10 volts per km, would transform an amount of energy sufficient to give the mass a surface brilliancy not less than 50 times as great as that which could be due to pure reflection at any reasonable value of the albedo. The effect may be compared to that of an "end-on" Geissler tube of colossal dimensions. The light, moreover, would be unpolarized, and would give a relatively simple gaseous spectrum of which the lines would not necessarily be identical in number or relative intensity, even were the gases in the spaces affected of similar tenuity and composition. These discharges in spaces not possessing definite electrodes are quite freaky in the matter of spectrum produced.

This view of the case would strongly support Seeliger's hypothesis, since it requires a similar distribution of matter, not in dust, which would produce relative opacity, but in masses of moderate dimensions. According to it, any considerable electric disturbance in matter-charged space would produce luminosity, which should then be an ordinary concomitant of the aggregation or disintegration of masses in or near such space. It permits relatively rapid changes of nebular luminosity, and considerable variations in the spectra in various parts of a single nebula.

If, in the case of the *Nova*, one is dealing with three superposed spectra—viz., a continuous spectrum with absorption bands, due to the main body concerned ; a true gaseous spectrum from surrounding incandescent gases, and this secondary gaseous spectrum—some of the changes observed become more comprehensible. Moreover, masses not gathered up by the principal

body, and carrying off their disturbing action in hyperbolic orbits through the nebular region, are competent to explain the persistence of luminosity in the line of the *Nova's* path. If there were present near the *Nova* a sufficiently retarding envelope of matter to convert these orbits into ellipses for any considerable swarm, the somewhat irregular light-variations observed become explicable, and must be greatly modified or disappear as the primary disturbances and the resulting secondary ones here noted die away. This electro-magnetic hypothesis in no way controverts the existence of cosmic nebular masses at the temperature of true incandescence, but it does avoid the necessity of such an explanation for every tenuous nebular wisp that gives a bright-line spectrum, and it does represent a condition which is not unknown in the laboratory.

THE SPECTRA OF POTASSIUM, RUBIDIUM, AND CÆSIUM, AND THEIR MUTUAL RELATIONS.[1]

By HUGH RAMAGE.

THE spectra of this group of metals have already been considered in a paper by the author on "A Comparative Study of Spectra," etc.[2] It was there shown that the differences between the principal series of lines in these metals depended on the atomic mass alone; and also that there was a close connection between the subordinate series and the atomic mass. A further study of the latter series was impossible at the time of writing the paper, owing to the fewness of the lines which had been observed and measured in them; practically no lines were known in the second subordinate series of rubidium and cæsium.

Some lines belonging to the subordinate series have been measured in Bunsen-flame and spark spectra by Lecoq de Boisbaudran, and in the arc spectrum by Liveing and Dewar, and by Kayser and Runge. Lehmann has measured some lines in the arc spectra in the red region. Lines recorded by these observers were found by the writer, with considerable intensity, in the oxyhydrogen flame spectra of the metals; and other lines, weaker than the above, were present which had never been recorded. Photographs of these high-temperature flame spectra were taken with a spectrometer designed by Professor Liveing, fitted with a Rowland plane grating ruled with 14,438 lines to the inch. The quartz lenses were plano-convex, with a focal length for the D lines of about 778 mm. The spectra in the first and second orders were photographed, and some measurements were made in the red region by eye observations. Spark spectra were photographed, superimposed on the flame spectra, of iron and titanium principally, but other metals were also employed.

[1] From advance proofs, sent by the author, of a paper read before the Royal Society, on June 5, 1902.

[2] *Roy. Soc. Proc.*, 70, 1, 1902.

These furnished the numerous fiducial lines required for the accurate determination of the wave-lengths.

The lines in the subordinate series are generally more diffuse

CÆSIUM.

Wave-length	Oscillation frequency, in vacuum,	Intensity	Previous measurement	Observer	Series and number
6984	14314	6			
74	335	9	6973.9	K. and R.	I_4
6869	554	2			
29	639	2			
6722	873	9	6723.6	"	I_4
6630	15079	2			
6590	171	8	....	....	II_4
6472	447	2			
6433	540	2			
6354	733	8	....	....	II_4
6217.6	16078.7	2			
13.33	89.7	8	6213.4	K. and R.	I_5
6034.43	16566.7	4	....	....	II_5
10.59	16632.4	8	6010.6	K. and R.	I_5
5847.86	17095.6	2			
45.31	102.7	8	5845.1	"	I_6
39.33	120.2	2	....	....	II_5
5746.37	397.2	1	....	....	II_6
5664.14	649.7	7	5664.0	K. and R.	I_6
35.44	739.6	5	35.1	"	I_7
5574.4	933.9	1	5572	L. de B.	II_7
68.9	951.6	1	....	....	II_6
03.1	18166.2	3	5501.9 ?	K. and R.	I_8
5466.1	289.2	4	5465.8	"	I_7
14.4	463.8	1	....	....	I_9
07.5	487.3	1	....	....	II_7
5351	682	1	....	....	I_{10}
5341.15	717.0	3	5345	L. de B.	I_8
5304	848	<1	....	....	I_{11}
5256.96	19016.8	1	5257	L. de B.	I_9
5209	192	<1			
5199	228	<1	....	....	I_{10}
5154	396	<1	....	....	I_{11}
4593.30	21764.8	8	4593.34	K. and R.	P_2
55.46	945.6	10	55.44	"	P_2
3888.75	25707.9	2	3888.83	"	P_3
76.31	790.4	4	76.73	"	P_3
3617.49	27635.7	<1	3617.08	"	P_4
11.70	680.0	2	11.84	"	P_4
3477.25	28750.3	1	....	....	P_5
3398.40	29417.3	1	....	....	P_6
48.72	29853.7	<1	....	....	P_7
3314	30166	<1	....	....	P_8
3287	30414	<1	....	....	P_9

than those in the principal series. Some of the weaker lines, notably those of cæsium, are very broad with diffuse edges; very accurate measurement of these is impossible.

Particulars of the spectra are recorded above; the oscillation frequencies are reduced to their values in a vacuum. The lines

RUBIDIUM.

Wave-length	Oscillation frequency, in vacuum	Intensity	Previous measurement	Observer	Series and number
			7950.46	Lehmann	P_1
7799	....	Very strong	7805.98	"	P_1
6306.8	15851.3	1			
6299.19	870.5	9	6297	L. de B.	I_4
6206.74	16106.8	8	6203	"	I_4
6160.04	228.9	5	6159	"	II_4
6071.04	466.8	4	....	....	II_4
5724.62	17463.2	8	5724.41	K. and R.	I_5
5654.16	680.9	3	5654.22	"	II_5
48.19	699.6	7	48.18	"	I_5
5579.3	918.1	2	....	....	II_5
5432.05	18403.9	6	5431.83	K. and R.	I_6
5391.3	543.0	1	....	....	II_6
63.15	640.3	5	5362.94	K. and R.	I_6
22.83	781.5	1	....	....	II_6
5260.51	19004.0	4	5259.8	K. and R.	I_7
34.6	098	1	....	....	II_7
5195.76	240.7	3	5194.8	K. and R.	I_7
65.35	354.1	2			
51.20	407.2	2	....	....	I_8
5132	480	<1	....	....	II_8
5089.5	642.5	1	....	....	I_8
76.3	693.6	1	....	....	I_9
37	847	1			
23	902	1	5021.8	K. and R.	I_{10}
17	926	<1	....	....	I_9
4983	20062	<1	....	....	I_{11}
67	127	<1			
4215.68	23714.4	9	4215.72	K. and R.	P_2
02.04	791.4	10	01.98	"	P_2
3591.86	27832.8	3	3591.74	"	P_3
87.27	868.4	4	87.23	"	P_3
3350.98	29833.5	1	3351.03	"	P_4
48.84	852.6	2	48.86	"	P_4
3229.26	30958.0	1	....	....	P_5
28.18	968.4	1	....	....	P_5

The isolated line λ 5165.35 is narrow and sharp; it differs, in these respects, from the lines in the series.

POTASSIUM.

Wave-length	Oscillation frequency, in vacuum	Intensity	Previous measurement	Observer	Series and number
7697	....	Very strong	7701.92	Lehmann	P_1
7664	....	"	7668.54	"	P_1
6939	14407	8	6938.8	K. and R.	II_3
13	462	7	11.2	"	II_3
5832.25	17141.3	6	5832.23	"	I_4
12.53	199.5	5	12.54	"	I_4
02.12	230.8	7	02.01	"	II_4
5782.74	288.1	6	5782.67	"	II_4
5359.96	18651.8	4	5359.88	"	I_5
43.38	709.6	2½	43.35	"	I_5
40.17	720.9	3	40.08	"	II_5
23.68	778.9	2	23.55	"	II_5
5112.76	19553.1	2	5112.68	"	I_6
5099.83	602.7	1	5099.64	"	II_6
97.64	611.1	1½	97.75	"	I_6
85.07	659.4	1	84.49	"	II_6
4965.61	20132.5	1	4965.5		I_7
4957	167	<1	56.8		II_7
51.46	190.1	1	52.2	"	I_7
4870	528	<1	4870.8	L. and D.	I_8
62	562	<1	63.8	"	II_8
57	583	<1	56.8	"	I_8
29	7c2	<1			
03	814	<1	4803.8	"	I_9
01	823	<1	......	....	II_9
4798	836	<1	4796.8	L. and D.	
67	972	<1			
60	21002	<1	4759.8	"	I_{10}
4642.35	21534.4	2	4642	H. and R.	
38.6	51.8	<1			
4047.39	24700.3	9	4047.36	K. and R.	P_2
44.33	719.0	10	44.29	"	P_2
3447.56	28997.8	3	3447.49	"	P_3
06.55	29006.3	4	06.49	"	P_3
Present	....	<1	3217.76	"	P_4
3217.36	31072.7	2	00.27	"	P_4

Other lines, very feeble indeed, appear on the strong continuous spectrum in the region near 4642. The line λ4642.35 was first observed in the spectrum of the Bessemer flame; Hartley and Ramage, *Phil. Trans.*, **196**, 491, 1901.

have been sorted into the principal, and the first and second subordinate series, and marked P, I, or II, with the number of the line, according to Rydberg's formula, in the sixth column. The

wave-lengths of the lines which have been observed before are given in the fourth column.

In the column of observers, L. and D. represent Liveing and Dewar; K. and R., Kayser and Runge; L. de B., Lecoq de Boisbaudran.

Diagrams of these spectra were drawn, as described in my former paper, to scales of oscillation frequencies for abscissæ, and (1) atomic masses, (2) squares of atomic masses for ordinates. The conclusions previously deduced from the less complete data were thereby amply confirmed. There is undoubtedly a very close connection between the spectra and the atomic masses; and the lines, which connect the corresponding members of homologous doublets in diagram (2), do intersect on the line of zero atomic mass.

The two limits in each spectrum toward which the two subordinate series appeared to converge were determined by a slight modification of Rydberg's method combined with graphical methods. These were inserted in the diagrams, and curves were drawn through the points. In diagram (1) the curves were turned away from each other and the points of bisection of the lines between the limits lay on a straight line; so also did the points of bisection of the lines between the two more refrangible and corresponding doublets of the second subordinate series. In diagram (2) the curves through the limits of the series, when produced, intersected on the line of zero atomic mass. This fact indicates that the difference between the two limits of the series, while not proportional to the square of the atomic mass, is a simple function of it. Rydberg, Kayser and Runge, and Rummel[1] have each shown that the differences between the convergence points of the subordinate series are approximately proportional to the squares of the atomic masses.

A diagram of the spectra and limits of the series was also drawn for the three metals to scales of wave-lengths and atomic masses. The more refrangible limits of the subordinate series and the more refrangible members of the second series now lay

[1] *Proc. Roy. Soc. Victoria*, 9 and 10, 1897.

on straight lines; the change in wave-length was thus proportional to the atomic mass.

After a careful study of the facts and many computations, it was found possible to calculate the subordinate series with considerable accuracy by the following formulæ:

THE FIRST SUBORDINATE SERIES.

The two convergence points (n_∞) of this series are obtained as follows: $n_\infty = 22830 - 21.633W \pm \frac{A}{2}$, where W is atomic mass and A is the average difference between the doublets. The latter quantity, as determined from the lines which are best suited for accurate measurement, is for potassium 57.8, for rubidium 236.4, and for cæsium 547.6. These values, as shown above, are simple functions of the atomic mass; but the best method of expressing them is not yet clear. This formula gives the following values for n_∞ belonging to the doublets of the first subordinate series: potassium, 21953.9 and 22011.7; rubidium, 20861.8 and 21098.2; and cæsium 19677.2 and 20224.8.

When these values are substituted for n_∞ in Rydberg's formula

$$n = n_\infty - \frac{N_0}{(m+\mu)^2};$$

in which $n = 10^8\lambda^{-1}$, $N_0 = 109675$, $m = 3 \cdot 4 \cdot 5 \cdot \cdot \cdot \cdot$ and when we also substitute for μ (assuming it to have a constant value for the series) the value

$$\mu = 0.7869 - 1466W^2 \times 10^{-8},$$

we obtain the following results:

POTASSIUM.

m	Oscillation Frequencies		Differences	m	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	....	14214.6	......	3	....	14272.4	......
4	17141.3	17122.4	−18.9	4	17199.5	17180.2	−19.3
5	18651.8	18653.3	+ 1.5	5	18709.6	18711.1	+ 1.5
6	19553.1	19557.1	+ 4.0	6	19611.1	19614.9	+ 3.8
7	20132.5	20134.7	+ 2.2	7	20190.1	20192.5	+ 2.4
8	20528	20526.1	− 1.9	8	20583	20583.9	+ 0.9
9	20814	20803.6	−10.4	9	....	......	......
10	21002	21007.4	− 5.4	10	....	......	......

RUBIDIUM.

m	Oscillation Frequencies		Differences	m	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	....	12763.2	....	3	....	12999.6	....
4	15870.5	15854.4	−16.1	4	16106.8	16090.8	−16.0
5	17463.2	17462.3	− 0.9	5	17699.6	17698.7	− 0.9
6	18403.9	18404.0	+ 0.1	6	18640.3	18640.4	+ 0.1
7	19004.0	19002.3	− 1.7	7	19240.7	19238.7	− 2.0
8	19407.2	19406.1	− 1.1	8	19642.5	19642.5	0
9	19693.6	19691.3	− 2.3	9	19926	19927.7	+ 1.7
10	19902	19900.3	− 1.7	..	....	....	....
11	20062	20057.9	− 4.1	..	....	....	....

CÆSIUM.

m	Oscillation Frequencies		Differences	m	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	10852.1	10865.7	+13.6	3	11404.1	11413.3	+9.2
4	14335	14327.9	− 7.1	4	14873	14875.5	+2.5
5	16089.7	16088.2	− 1.5	5	16632.4	16635.8	+3.4
6	17102.7	17103.6	+ 0.9	6	17649.7	17651.2	+1.5
7	17739.6	17741.9	+ 2.3	7	18289.2	18289.5	+0.3
8	18166.2	18169.2	+ 3.0	8	18717.0	18716.8	−0.2
9	18463.8	18469.1	+ 5.3	9	19016.8	19016.7	−0.1
10	18682	18687.7	+ 5.7	10	19228	19235.3	+7.3
11	18848	18851.9	+ 3.9	11	19396	19399.5	+3.5

THE SECOND SUBORDINATE SERIES.

In this series

$$n_\infty = 22850 - 21.812\,W \pm \frac{B}{2},$$

where B is, for potassium, 57.8; for rubidium, 238.0; and for cæsium, 553.6; and

$$\mu = 0.7990 + 7984\,W^2 \times 10^{-9}.$$

It will be observed that the doublets in the second subordinate series are more widely separated than those in the first series. It would appear also that the two series do not converge toward the same limit; the difference between the limits, however, diminishes in the different metals as the atomic mass increases. This is true on the supposition that μ is constant, and

not variable, as in the formula given for the principal series. Kayser and Runge hold the view that there are different limits for the two series, while both Rydberg and Rummel favor the view that the limits are the same.

The observed and calculated oscillation frequencies are as follows:

POTASSIUM.

m	Oscillation Frequencies		Differences	m	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	14407	14417.4	+ 10.4	3	14462	14475.2	+ 13.2
4	17230.8	17229.9	− 0.9	4	17288.1	17287.7	− 0.4
5	18720.9	18720.3	− 0.6	5	18778.9	18778.1	− 0.8
6	19602.7	19603.9	+ 1.2	6	19659.4	19661.7	+ 2.3
7	20167	20170.5	+ 3.5	.	....	....	......
8	20562	20555.3	− 6.7	.	......	....	......
9	20823	20828.6	+ 5.6	.	......	....	......

RUBIDIUM.

m	Oscillation Frequencies		Differences	m	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	13498	13496.7	−1.3	3	13738	13734.7	−3.3
4	16228.9	16219.6	−9.3	4	16466.8	16457.6	−9.2
5	17680.9	17671.4	−9.5	5	17918.1	17909.4	−8.7
6	18543.0	18535.8	−7.2	6	18781.5	18773.8	−7.7
7	19098	19091.8	−6.2	.	....	....	.....
8	19480	19470.3	−9.7	.	....	....	.....

CÆSIUM.

m	Oscillation Frequencies		Differences	m	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	......	12609.3	......	3	....	13162.9	......
4	15171	15180.2	+9.2	4	15733	15733.8	+0.8
5	16566.7	16566.1	−0.6	5	17120.2	17119.7	−0.5
6	17397.2	17397.3	+0.1	6	17951.6	17950.9	−0.7
7	17933.9	17934.7	+0.8	7	18487.3	18488.3	+1.0

The convergence points of the series as deduced in different ways are given in the following table:

Element	From above Formulæ			By calculation from observed lines. p.	From formula[1] for principal series	Numbers calculated by Rydberg
	First Series	Second Series	Mean of two series			
				Mean		
Potassium (1)	21953.9	21968.0	21960.95	21960	21969.4	21955.46
" (2)	22011.7	22025.8	22018.75	22018	22024.3	22013.31
Rubidium (1)	20861.8	20868.3	20865.65	20865	20868.6	20869.15
" (2)	21098.2	21106.3	21102.25	22101	21112.3	21098.83
Cæsium (1)	19677.2	19674.2	19675.7	19672	19686.7	
" (2)	20224.8	20228.0	20226.4	20226	20234.2	

The numbers in the sixth column were obtained by the law, discovered by Rydberg and independently by Schuster, which connects the principal and subordinate series: the convergence points of the subordinate series are given by the differences between the convergence points and first lines (for which $m = 1$) of the principal series. One set of the numbers was obtained from the expression

$$\frac{N_0}{(1 + 1.19126 + 0.00103\,W)^2},$$

and the other set from the expression

$$\frac{N_0}{(1 + 1.19126 + 0.00103\,W + 182\,W^2 \times 10^{-8})^2}.$$

The figures in this column agree best with those of the second subordinate series in the third column; and it will be remarked, as confirming the closer connection between the principal and second subordinate series, that the results calculated for the latter series of rubidium differ by about nine units, whereas those given by the formula for the principal series differ by about 27.5 units from the observed numbers. The connection between the first subordinate series and the atomic mass is apparently simpler than between the other two series and the atomic mass.

The numbers in the last column were taken from Rydberg's[2]

[1] Author, *loc. cit.* [2] *Paris Congress Reports*, **2**, 212, 1900.

paper. He calculated them by means of an empirical formula from the observed lines.

All the strong lines and nearly all the weak lines which have been observed in the flame and arc spectra of these elements, are included in the three harmonic series of lines. The empirical formulæ given show that the differences in the corresponding series depend wholly on the atomic masses of the three elements.

ST. JOHN'S COLLEGE,
Cambridge.

Minor Contributions and Notes.

A DETERMINATION OF THE WAVE-LENGTHS OF THE BRIGHTER NEBULAR LINES.[1]

In the course of a recent investigation of the spectrum of *Nova Persei* (*Bulletin* No. 8) there was developed the necessity for a more accurate knowledge of the wave-lengths of the brighter nebular lines than then existed. The wave-lengths of the two chief nebular lines had been determined with great accuracy by Dr. Keeler in his classical work on the motions in the line of sight of the brighter gaseous nebulæ,[2] but the uncertainties in the positions of the remaining lines were of the order of a tenth-meter or more, according to the brightness of the line and its position in the spectrum. With a view of obtaining a more accurate knowledge of these wave-lengths, the spectra of a number of the brighter nebulæ have been photographed, and the results are given in the following pages.

Table I contains a list of the negatives secured, and some of the details of the exposures. The first column contains the plate number; the second, the name of the nebula whose spectrum was photographed; the third, the Pacific Standard Time of the middle of the exposure, and the fourth, the length of exposure. The letter in the fifth column refers to the spectrograph employed, as described below. The sixth column specifies the region covered by the photograph, and the seventh records the elements used for comparison spectra.

These spectra were obtained with spectrographs attached to the thirty-six-inch telescope. The small photographic correcting lens was used in photographing Nos. 2204, 10, 13, 34, 36, 96, and 97. Except in the cases of very small nebulæ, the twelve-inch telescope is optically more efficient for such work, but the superior appointments of the larger instrument more than compensate for its disadvantages.

[1] *Lick Observatory, University of California, Bulletin* No. 19.

The results given in this paper were presented before the December meeting of the Astronomical and Astrophysical Society of America.

[2] *Publications of the Lick Observatory*, III.

TABLE I.

Plate No.	Object	Date	Exp. time	Inst	Region of spectrum	Comparison
2202F	*G. C.* 4390	1901 July 22 10^h	3^h 00^m	*b*	λ 4000—λ 6000	*He*
04F	*G. C.* 4390	24 10	3 10	*b*	3700— 5700	*Fe, H, He*
10C	*N. G. C.* 7027	Aug. 2 13	7 10	*b*	3700— 5700	*Fe, H, He*
13D	*G. C.* 4390	5 11	3 37	*b*	3700— 5700	*Fe, H, He*
22D	*N. G. C.* 7027	9 10	3 59	*b*	4000— 6000	*He*
34C	Ring Nebula in *Lyra*	17 12	3 40	*b*	3700— 5700	*Fe, He*
36C	*G. C.* 4964	18 13	7 20	*b*	3700— 5700	*Fe, He, H*
68E	*N. G. C.* 7027	Sept. 15 8	2 04	*c*	4700— 5100	*He, H*
96B	*G. C.* 4964	Nov. 4 11	3 20	*b*	3700— 5700	*He, H*
97E	*Orion* Nebula	4 15	4 00	*b*	3700— 5700	*He, H*
2313B	*Orion* Nebula	Dec. 8 12	5 02	*a*	4200— 4500	*Fe, H*
14A	*Orion* Nebula	12 12	4 30	*b*	3700— 5700	*Fe, He, H*
15E	*Orion* Nebula	16 12	4 40	*c*	4700— 5100	*Fe, He, H*
16A	*Orion* Nebula	17 12	4 30	*c*	4700— 5100	*Fe, Pb, He, H*
2317A	*Orion* Nebula	18 12	4 20	*a*	4200— 4500	*Fe, H*

The spectrographs used were as follows:

a. The Mills spectrograph.

Focal length of collimator lens, 722.4 mm; three prisms; focal length of camera lens, 405.5 mm. Collimator and camera corrected for $H\gamma$.

b. A single light flint prism instrument into which the Mills spectrograph is convertible, and which has been described in *Bulletin* No. 8 as "Spectrograph No. I." The lenses are the same as those mentioned above.

c. A spectrograph containing three dense flint prisms, similar to spectrographs III and IV described in *Bulletin* No. 8, except that the prisms were set at minimum deviation for λ 4950, which ray was brought to the center of the field. Plate No. 2268 was secured with the regular camera lens referred to above, and Nos. 2315E and 2316A with a lens of the same focal length corrected for λ 5900. In using the regular camera lens the plate-holder had to be tilted, but the combination of photographically corrected collimator and visually corrected camera gives in this region a field at right angles to the line of collimation.

The range of spectrum for the one prism instrument is normally from above λ 3700 to λ 5000. By slightly tilting the plate-holder equally good definition is afforded between λ 4000 and λ 6000. Two

exposures were made with the latter range for the purpose of photographing the nebular lines below λ 5007. They were unsuccessful, as no trace of radiation below the chief nebular line was secured. On account of the unfavorable positions of the comparison lines recorded upon them, neither of these plates was used to determine the wave-lengths of the lines actually photographed. The one spectrum of the Ring nebula secured (No. 2234C) was very weak, only the two nebular lines at λ 4959 and λ 5007 being recorded. It was accordingly not used. One of the plates of *G. C.* 4964 was also underexposed and rather poor. Values of wave-lengths determined from this nebula were therefore given weight one in making up the final list. In Table II will be found the results of measurements of the other plates secured with this instrument.

TABLE II.

Orion nebula 2 plates	*i*	*G. C.* 4390 2 plates	*i*	*G. C.* 4964 2 plates	*i*	*N. G. C.* 7027 1 plate	*i*
3726.35[1]	4						
3728.98[1]	3						
3835.8	1—						
3868.87	4	3868.94	5	3868.88	4+	3868.97	7
3889.13	4	3889.16	2				
3965.1	1—						
3967.69	3	3967.64	5	3967.57	4	3967.61	7
3970.24	5	3970.25	4	3970.08	2	3970.33	3
4026.7	1	4026.8	1—				
4068.7	1					4068.81	2
4101.95	6	4101.90	6	4101.88	4	4101.88	7
4340.60	10	4340.64	7	4340.67	6	4340.60	9
		4363.30	2	4363.5	2	4363.37	8
4471.77	2	4471.66	2			4471.7	1—
				4685.72	6	4685.75	9
						4740.0	1
4861.57	7	4861.47	6	4861.7	4	4861.44	8
4959.01	5	4959.13	8	4959.16	8	4958.97	20
5006.82	7	5006.94	25	5007.06	20	5006.83	80

The results given above are corrected for velocity in the line of sight. In making this correction the velocity for which the wave-lengths of the lines on any plate were corrected was determined from the hydrogen lines of that plate. This method of procedure was

[1] These wave-lengths depend upon the measurements of only one plate. In the first plate secured there were no comparison lines sufficiently close to them to give accurate determinations.

adopted for the purpose of eliminating any *error of the plate*, or relative shift of nebular and comparison lines which might result from a number of causes, such, for instance, as temperature changes and differential flexure during the exposures, which were necessarily long. In order to reduce any such effects to a minimum the comparison spectra were introduced at frequent intervals during the exposure. These precautions were taken even though with the longest exposures the definition was practically perfect. The correction for velocity was also applied to the wave-lengths of the hydrogen lines themselves. Had the negatives been somewhat stronger, velocity determinations could also have been made from a number of the helium lines.

The relative brightnesses of the lines as photographed are indicated in a way by the numbers opposite their wave-lengths. About all that can be said of the system is that for any one object, the larger the number, the brighter the line. Such estimates must of course be used with caution, as the apparent brightness of a line is greatly influenced by photographic and instrumental conditions.

Special interest attaches to the doublet at λ 3969. This has heretofore been observed as a single line and identified as $H\epsilon$, which is in fact its least refrangible component. The two components vary much in relative intensity in different nebulæ. In *N. G. C.* 7027, λ 3967.65 is much the stronger of the two, while in the case of the *Orion* nebula, in which the hydrogen series is relatively bright as compared with the chief nebular lines, it is much the fainter. In the *Orion* nebula there is an additional line at λ 3965 (helium).

The doublet at λ 3727 has heretofore always been observed as single.

The velocities in the line of sight determined from the low dispersion spectrograms are liable to an error of perhaps five kilometers a second. They are as follows:

Object	Velocity in line of sight
G. C. 4390	— 11 km per sec.
G. C. 4964	— 7
N. G. C. 7027	+ 6
Orion nebula	+ 17

The substances whose spectra were used for comparison purposes in these low dispersion spectrograms are iron (spark), helium, and hydrogen. The iron wave-lengths have been taken from Kayser's

paper[1] whenever possible, and otherwise from Rowland's solar spectrum wave-lengths. In the case of the iron line near λ 4958 a special investigation was necessitated by the fact that the line is double and too close to be resolved by the instruments used. The position of the line was measured on two plates secured with instrument *c* as used during the observations of nebulæ, employing as reference lines those at λλ 4941.77, 4948.40, 4964.90, 4968.75, and 4973.25 in the titanium spectrum. The values determined from the plate are:

λ 4957.77
4957.80

Mean 4957.78

This result confirms that of Dr. Hartmann,[2] who found the doublet to consist (in the spark spectrum) almost entirely of the least refrangible component, the wave-length of which is λ 4957.78. In this connection it may be stated that the coil used for our comparison spectra gives a spark of about an inch without a condenser in the secondary circuit. When a metallic spark is employed, a condenser of about 0.003 microfarads is placed in the secondary circuit. The wave-lengths of the helium lines are those determined by Runge and Paschen.[3] The hydrogen wave-lengths (see Table III) are, with two exceptions, taken from Ames' paper.[4] In one of the high dispersion observations to be described later, the lead line at λ 5005.63 was used.

The positions given in Table II for the two lines below $H\beta$ were not regarded as very trustworthy. Among the difficulties encountered in this region were the relatively low dispersion and the fact that the distance from the ray of minimum deviation was such as to make the determination of wave-lengths somewhat uncertain. It was for the purpose of securing more accurate determinations of the wave-lengths of the lines in question that the high dispersion spectrograms were made. The radial velocities determined from them are as follows:

[1] *Annalen der Physik* (4) **3**, 1900; also ASTROPHYSICAL JOURNAL, **13**, 329, 1901.

[2] *Sitz. der K. Akad.*, Berlin, **12**, 237, 1902; ASTROPHYSICAL JOURNAL, **15**, 292, 1902.

[3] *Sitz. der K. Akad.*, Berlin, July, 1895, 639 and 759.

[4] *Phil. Mag.*, July, 1890.

ORION NEBULA

Date			Line	Velocity[1]	Wt
1901	Dec.	8	$H\gamma$	+17.1 km	3
		16	$H\beta$	+16.1	2
		17	$H\beta$	+17.0	2
		18	$H\gamma$	+14.8	3

Mean +16.2 km ± 0.4 km per sec.

N. G. C. 7027.

Date	Line	Velocity
1901 Sept. 15	$H\beta$	+12.3 km per sec.

In the case of the *Orion* nebula the wave-lengths for the two chief nebular lines were corrected for the mean velocity given above. The resulting wave-lengths are:

ORION NEBULA λ	weight	*N. G. C.* 7027 λ	weight
4959.07 ± 0.05	2	4958.95	$\frac{1}{2}$
5006.91 ± 0.05		5006.80	

In determining these wave-lengths an interpolation curve was passed through $\lambda\lambda$ 4861.50, 4922.10, and 5015.73. The iron line λ 4957.78 is recorded on both plates of the *Orion* nebula. In both cases the computed value of the wave-lengths is λ 4957.73. A compromise correction of +0.03 was therefore added to the value of the second line as determined from the interpolation curve. In the case of the nebula *N. G. C.* 7027 the iron spectrum was not used, and the value given is that determined by interpolation.

Column one of Table III gives the finally adopted values of the wave-lengths. The values of the last two depend only upon the high dispersion observations. The values determined with instrument *b* are λ 4959.07 and λ 5006.90.

TABLE III.

Nebulæ	Vacuum tubes	Nebulæ	Vacuum tubes
3726.4 ±0.2		4101.91	4101.89 $H\delta$
3729.0 ± .2		4340.62	4340.63 $H\gamma$
3835.8 ± .5	3835.6 $H\eta$	4363.37±0.10	
3868.88± .05		4471.71±0.10	4471.65 helium
3889.14	3889.15 $H\zeta$	4685.73±0.10	
3965.1 ± .4	3964.9 helium	4740.0 ±1.0	
3967.65± .05		4861.54	4861.50 $H\beta$
3970.23	3970.25 $H\epsilon$	4959.05±0.05	
4026.7 ± .5	4026.3 helium	5006.89±0.05	
4068.8 ± .5			

[1] These velocities refer to the region immediately preceding the Trapezium.

The probable errors attached to the wave-lengths have been assigned in an arbitrary manner, but in accordance with the judgment of the observer.

For purposes of comparison the results of Keeler[1] and Hartmann[2] for the wave-lengths of the two chief nebular lines are appended. Keeler's value for the second line has been increased by 0.15 t. m. to allow for the difference in the positions of the iron doublet assumed by himself and the writer, respectively.

Keeler	Hartmann	Wright
4959.17	4959.17	4959.05
5007.05	5007.04	5006.89

The writer wishes to express his obligation to Dr. Campbell and Dr. Reese; the former for placing at his disposal the requisite instrumental equipment for the foregoing work and for valuable assistance, and the latter for great assistance rendered in the observations.

W. H. Wright.

May 26, 1902.

EFFECT OF SELF-INDUCTION UPON SPARK-SPECTRA OF METALLOIDS.

The well-known researches of Schuster and Hemsalech (reviewed in this Journal, **14**, 370–371, 1901) have been recently extended by M. A. de Gramont (*Comptes Rendus*, **134**, 1048–1050, 1205–1207), who has studied the variation of intensities in spectral lines with change in amount of self-induction. The substances examined include a large number of metalloids, especially in combinations occurring in minerals.

Among the results obtained with the condensed spark we note here only three, namely:

1. In general, the lines due to metalloids disappear before the spectra of metals are sensibly affected by increasing self-induction.

2. The C line of hydrogen remains long after increasing self-induction has extinguished the air lines. The extinction of this particular line demands an inductance of at least 0.00023 henries.

3. The spectrum of carbon, even in the strongly condensed spark, is rapidly weakened by the introduction of inductance. Using 0.00002 henries all carbon lines except *Cα* (λ6579) are diminished in intensity,

[1] *Publications of the Lick Observatory*, III. [2] *Loc. cit.*

while with 0.00006 henries they disappear almost completely. The disappearance of *Ca* occurs only when the inductance reaches 0.00350 heuries.

The reader is referred to the reference above given for the effect upon the spectra of *P*, *Te*, *As*, *Sb*, *C*, *Si*, *Ge*, *Th*, and *Na*.

H. C.

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ON THE THERMAL DEVELOPMENT OF THE SPARK SPECTRUM OF CARBON.[1]

By HENRY CREW and JOHN C. BAKER.

DURING the course of some experiments which Professor Basquin was making on the production of arc and spark spectra from the same electrodes, it was observed by one of us, standing at the eyepiece of his spectroscope, that the lines of the spark spectrum made their appearance gradually, and not suddenly, beginning at the instant at which the direct current feeding the arc was cut off and the high-voltage current producing the spark was switched on.

It was evident at once that the appearance of these lines in deliberate succession was due, primarily at least, to the gradual cooling of the electrodes and of the region between them. But we were uncertain whether, after all, the effect was not merely a physiological one, the lines first observed being the stronger lines, and those observed later being the weaker lines. We accordingly set about making a series of photographs which

[1] The expenses of this experiment were met by the consideration of the committee in charge of the Rumford Fund.

should show the spark spectrum at each successive instant, beginning at the time at which the arc current is interrupted.

At first our attempt was to employ metallic spark-electrodes; and in order to retard the development of the spark as much as possible, the carbon electrodes were enclosed between two saucer-shaped clay scorifiers as shown in Fig. 1. The terminals of the spark circuit were introduced into this cell at right angles to those of the arc. Various other forms of cells, hollowed limes from the stereopticon, clay pipes, fire brick, etc., were tried. But in each case, as soon as the region inside got hot enough to affect the character of the spark and render it quiet, we found (as indeed ought to have been anticipated) that the walls of the vessel became conducting.

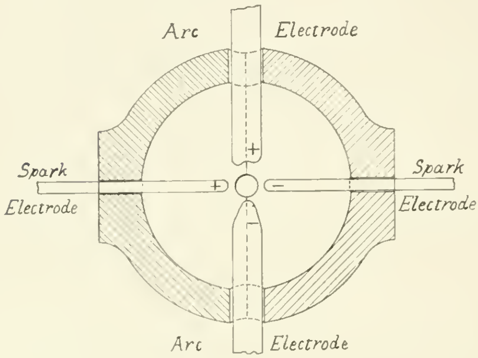


Fig. 1.

We tried next to get a gradual variation of temperature by moving the spark gap slowly from the center to the edge of an ordinary carbon arc, knowing that, at the center of the arc, the spark was quiet and non-luminous while just outside the arc it became noisy and brilliant. But in carrying the spark electrodes from one of these positions to the other, we encountered a peculiar discontinuity, *i. e.*, a position at which the spark *instantly* changed character.

When the spark was passed through the "horsetail" above the horizontal arc at a distance of from ½ to 2 centimeters from the arc, the quiet discharge mentioned above was still obtained and a spectrum of feeble intensity could be observed. When, however, the terminals were removed slightly further above the arc, a point was reached at which the discharge instantly assumed the ragged character of the ordinary cold

spark; and when the spark was then moved back toward the arc it did not resume its quiet character, but blew the "horsetail" away, and in most cases put out the arc. It did not seem possible to obtain any intermediate stages. The instability was very marked. The spark was liable at any time to break down into the ragged character and when it had once done so it retained that character until the circuit was broken.

APPARATUS AND METHOD.

Accordingly we had recourse to soft-cored carbons worked in air, using the same electrodes for both arc and spark; in other words, we used the hot region between the poles of an ordinary carbon arc as the heated medium in which to study the slowly developing spark.

The next step consisted in isolating the particular phase of the development which we wished to examine.

This was accomplished by means of a device (designed with the generous aid of Professor Basquin), which performs automatically the following cycle of operations:

1. Closes the arc circuit and lights the arc, thus heating the carbon electrodes and the region between them to a very high temperature.

2. After an interval of a few seconds, sufficient for the carbons to become thoroughly heated, interrupts the arc circuit.

3. After an interval which is less than one-tenth of a second, closes the spark circuit.

4. After a variable (but definite and measurable) interval of time, opens a shutter in front of the slit of the spectroscope and exposes the plate during any desired length of time, generally between ½ second and 1 second.

5. Interrupts the spark circuit.

1. Again closes the arc circuit; and so on, as before.

The arc was operated with 15 amperes showing 40 volts between the electrodes; while the spark was produced by a large induction coil of the type devised by Rowland in 1887 and described in Kayser's *Handbuch der Spectroscopie* (p. 183). This

induction coil, or step-up transformer, was operated on a 104-volt alternating circuit, of frequency 120, with a primary current of 20 amperes. In parallel with the spark gap was placed a capacity of $\frac{1}{50}$ microfarad. The arrangement of the circuit is shown in Fig. 2, where S_1 and S_2 are each double-pole mercury switches so fixed that *one can be closed only after the other is opened.* S_1 is kept closed by a spring until an electromagnet begins to close S_2 by rocking a light beam of which its armature is a part. The question of changing from arc to spark circuit is then merely a question of closing the battery circuit which actuates this electromagnet. This battery circuit is closed and opened by a continuously rotating switch (shown at the left in Fig. 3) which is driven at the uniform rate of 10 revolutions per minute by a small electric motor. This rate of rotation is maintained constant by means of a pair of cone pulleys and a heavy flywheel.

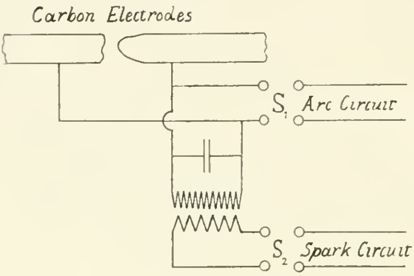


FIG. 2.

This same rotating switch, or commutator, by means of the sliding contact marked "2" in Figs. 3 and 5, opens the shutter in front of the slit of the spectroscope at any phase of the spark desired and holds the shutter open for a small but definite period of time varying usually from ½ second to 1 second. On this same rotating commutator shaft is a stud (D, Fig. 3) which, immediately after the arc circuit is closed, pushes a carbon rod into the arc gap for an instant and thus "lights" the arc.

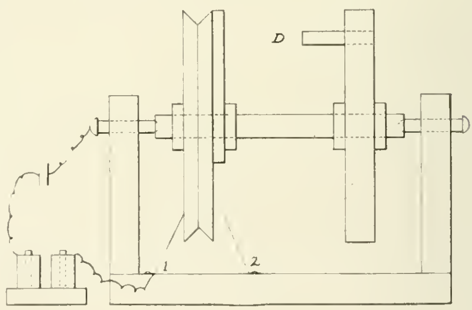


FIG. 3.

By clamping the sector *P* (Fig. 5) to the rotating commutator in successive angular positions about its axis one is enabled to open the slit for the successive phases of the spark which he may wish to photograph; not only so, but he can repeat any phase as many times as he likes and thus obtain a strong composite in cases where a single exposure would produce no visible effect.

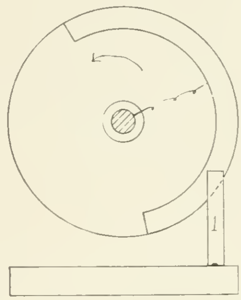


FIG. 4.

In this manner we have photographed the spark spectrum of carbon, with a ten-foot concave grating, in nine different phases[1] which may be roughly described as follows:

1. Exposure begins ⅙ second after breaking arc and lasts ½ second. Here the carbon poles are still white-hot and the spark is practically silent when compared with the noise which the cold spark makes. In this stage the luminosity is so exceedingly feeble that, with a slit of the same width as in the rest of the series, six to ten hours (*i. e.*, about 5,000 exposures) are required to get a fair negative.

2. Exposure begins ¼ second after breaking of arc and lasts for 1 second. The middle of the exposure, therefore, occurs ¾ second after the beginning of the spark. Here, again, the image of the spark on the slit of the spectroscope is quite invisible during the entire exposure.

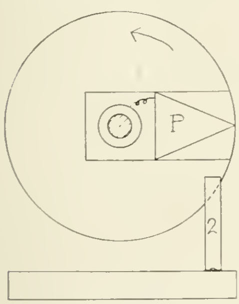


FIG. 5.

3. Exposure begins ½ second after breaking of arc, and lasts for 1 second; middle of exposure 1 second after beginning of spark. Here the image of the spark is barely visible just before the slit is covered. The spark is distinctly louder than in the preceding phases.

4. Middle of exposure 1¼ seconds after beginning of spark.

5. Middle of exposure 1¾ seconds after beginning of spark.

6. Middle of exposure 2⅝ seconds after starting spark.

7. Middle of exposure 5¼ seconds after starting spark. Here the electrodes begin to show merely red-hot instead of white-hot.

[1] The purpose of this experiment, it will be observed, is therefore fundamentally different from that in which Sir Norman Lockyer examined the spark spectra of salts volatilized in flames and which he described in *Proc. Roy. Soc.*, **30**, 22–31, 1879.

8. Middle of exposure 7⅞ seconds after beginning; spark distinctly noisy.

9. The last photograph in the series was taken at 12 seconds after the beginning of the spark, the duration of the exposure being, as in the preceding cases, 1 second. Even at this late stage a distinct *crescendo* is still noticeable in the noise of the spark.

The enormous increase of brilliancy from the hot spark to the cold may be judged from the fact that in order to make the cyanogen band at λ 3883 of uniform intensity the exposure time for the first of the series was 8 hours and for the last of the series 20 minutes.

RESULTS.

As in the case of the Swan spectrum and the carbon arc, so also in the case of the carbon spark, the flutings are, of course, the dominant features of the entire spectrum. The first question, therefore, which naturally arises, in the development of the spark, is concerning the order and the relative intensity in which these cyanogen bands make their appearance. A second question might be asked concerning the stage at which the air lines make their appearance. A third query is, when and how do the numerous metallic impurities present themselves? Our photographs permit at least partial answers to these three questions for the region lying between $\lambda\lambda$ 4500 and 3000. The phenomenon is one which cannot be accurately observed by the eye: and the exposure times are so long as to render photographing in the visible region well-nigh impracticable.

1. *Carbon flutings and lines.*—The cyanogen bands at $\lambda\lambda$ 4216, 3883, and 3590 all make their appearance on the first photograph of the series. Their relative intensity is practically the same as in the case of the spark between cold electrodes, which, for the sake of brevity, we shall hereafter call the "cold spark." In view of this fact we have employed these three bands as standards of intensity; and have called any two spectra of "equal intensity" when these three bands were of equal intensities on the respective negatives. Each member of the series was, in this way, made of practically the same intensity.

As to the carbon lines, very few appear in this region. The line at λ 4556.3 does not appear in the hot spark, *i. e.*, in the earliest phase of the series described above. The broad, hazy line at λ 4267.5 which Eder and Valenta[1] call the "chief carbon line" disappears completely on introducing inductance into the circuit of the cold spark. And it does not appear at all in the hot spark. These two facts raise the question as to whether this line[2] is due to carbon. The line at λ 3361 persists in the hot spark; but it also appears in the aluminium spark and, greatly enhanced, in the copper spark when there is no capacity in the circuit. As to the remaining lines which Eder and Valenta describe in this region λλ 3920.8, 3877.0 and 3848.0, they are weak, and we have not been able to identify them to our satisfaction.

2. *Air Lines and Flutings.*— Not one of the ordinary air lines appears on any photograph whose phase is earlier than ¾ second. On the plate whose phase is ¾ second appear only the very heaviest of the air lines, viz., λλ 4630, 4447, 3995, 3433, 3330. Indeed the elimination of air lines is so complete in these earlier phases that non-appearance in the hot spark might be used as one criterion for air lines, analogous to the inductance test discovered by Schuster and Hemsalech.

As to nitrogen *flutings* which appear in spark spectra, when the electrodes are close together or when inductance is placed in series with the condenser, the case is very different — quite reversed, indeed — from that of ordinary air lines. The nitrogen flutings, with heads at λλ 3371.1 and 3158.7 respectively, come out very strong in the earliest phase; at ¾ second they begin to weaken; after 3 seconds, only a trace of them is left.

The nitrogen flutings of wave-length longer than 3371 do not appear in the spark under the conditions in which we are

[1] EDER and VALENTA, *Denksch. K. Akad.*, Wien, 60, 249, 1893.

[2] The difference in behavior between this line at λ 4267.5 and the double red carbon line at λλ 6583 — 6577 leads Professor Schuster, on the basis of Herbert's experiments, to suggest that the red and blue lines may, at least, "belong to different spectra of carbon." *Note added Aug. 25, 1902.*

See *Phil. Mag.*, Aug. 1902, p. 207.

working, namely, a 3-millimeter spark gap in series with a condenser of $\frac{1}{50}$ microfarad capacity; no inductance.

We have not found any description of these nitrogen bands *as they appear in the spark spectra of elements in air at barometric pressure.* At first we took the band at λ 3371.1 to be a hitherto undescribed carbon band; and it was only through an excellent suggestion from Professor Hale that we discovered our mistake. He advised us to try the spark *without capacity.* On trying this experiment, we found the band at λ 3371.1 strongly present in spectra of aluminium, zinc, and other metals in air; but when the spark was worked in atmospheres of oxygen or coal gas, these flutings all disappeared save the merest trace of the strongest two.

The cold carbon spark (unlike that of metals) without capacity shows these bands only with extreme faintness; and the condensed carbon spark does not show them at all; the *carbon when white hot shows them strongly*, as indicated above.

In this connection, the question may be raised whether the band described by Professor Hutchins[1] does not belong to this nitrogen group. For we have found in the spark spectrum of aluminium a band, with its edge at λ 3914.41, which shows a weak line alternating with a strong one, exactly as in Hutchins' photograph. But on examining this spark in a current of oxygen, not the slightest trace of the band was found. Since it is found in metals, but not in the carbon spark, and since it disappears when nitrogen disappears, it seems to us more probably due to nitrogen[2] than to carbon.

[1]ASTROPHYSICAL JOURNAL, 15, 310, 1902.

[2]Mr. F. J. Truby has measured the first fourteen lines of this fluting, which form a group lying between the edge and the heavy impurity line at λ 3905.74. His values are as follows:

3914.41	head	3909.95	weak
3913.89		3909.30	strong
3913.35		3908.52	weak
3912.62	strong	3907.80	strong
3912.17	weak	3906.88	weak
3911.70	strong	3906.16	strong
3911.17	weak	3905.74	impurity
3910.61	strong		

There are possibly two other weak lines near the head which Mr. Truby's definition does not permit him to measure.

What is apparently the same band may be seen very distinctly on McClean's map of the spark spectrum of copper; and again a similar fluting has been found by Deslandres at the negative electrode of a spectrum tube filled with nitrogen. For Deslandres' drawing see *Comptes Rendus*, August 9, 1886. This is probably also the same band which is marked *very strong* at λ3914.4 in Hemsalech's[1] table of nitrogen bands. The fact that Hutchins is able to intensify the band he describes by making and breaking the arc circuit, would seem to indicate that it appears in the arc spectrum primarily in consequence of high electromotive force.

3. *Metallic impurities.*—The only electrodes which we have employed are the unplated, cored carbons, sold by A. T. Thompson, 25 Bromfield street, Boston, for use in projection lanterns. Their size is 7½ × ½ inches, and they are marked "imported." The metallic impurities which present themselves are practically only aluminium, calcium, copper, iron, and potassium. Possibly others might be detected by very long exposure or by study of portions of the spectrum other than that to which we have limited ourselves, namely, λ4500–λ3200. The strongest lines in this region of the hot-spark spectrum are two at λ4047.338 and λ4044.294 belonging to the principal series of potassium. They are faintly represented in the carbon arc; but *no trace of them can be found in the ordinary, or "cold," carbon spark.* Is it not rather surprising to find on a spark spectrum plate that the strongest lines are due not only to an impurity, but to an impurity which is introduced apparently by the condition of high temperature in the medium? For so far as the energy delivered by the spark itself is concerned, this would seem to be enormously greater in the loud and brilliant cold spark than in the quiet and invisible hot spark. We use the expression "high temperature" in this connection only with great hesitation, and then only with reference to the medium after the heating current has been cut off. *But this potassium pair persists very distinctly for five seconds after the heating* (*arc*)

[1] *Recherches Experimentales sur les Spectres*, etc., p. 126. (Paris, 1901.)

current has been interrupted. Accordingly, we find it difficult to imagine any electrical effect, other than heat, which would persist for this length of time, especially as the electrodes were placed always horizontally, so that strong convection currents were sweeping out anything in the nature of electrolytic products.

It seems not improbable that these effects of the hot spark are brought about through an increased conductivity—and, hence, a lowered electromotive force—between the poles of the spark gap, so that, in the series described above, the earlier phases partake of the character of the arc, while the later phases represent the spark. *If this be true, the nine members of this series of photographs constitute nine different steps between the arc spectrum and that of the spark.*

A similar diminution of E. M. F. between the hot poles is indicated by the work of Schenck,[1] who finds that, with hot poles, the "*Mg* spark line at λ4481 shrinks down close to the electrodes, while the arc triplet at λ5170 does not." And this view is rendered all the more probable by a fact noted by Professor Basquin,[2] viz., that an auxiliary cold spark gap in series with the hot spark gap suffices to render the spark lines immediately visible.

The general effect of the hot spark upon metallic impurities may perhaps be most clearly described in the following three statements:

1. Some *new* impurities are introduced, *e. g.*, potassium λλ4047.34, 4044.29, 3447.49, 3446.49. This is analogous to the introduction of the nitrogen fluting at λ3371 above mentioned.

2. Among lines *due to a single element* some may be diminished while others are enhanced in intensity. Thus the calcium pair at λλ3968.6 and 3933.8, and also the calcium pair at λλ3179.4 and 3159.0 are immensely diminished while the calcium line at λ4226.9 is so greatly enhanced by the hot spark that, after the potassium pair, it becomes the strongest in the

[1] Astrophysical Journal, 14, 131, 1901.

[2] Astrophysical Journal, 14, 15, 1901.

entire region studied. It is perhaps worth noting that all of the lines belonging to any one of Kayser and Runge's series are similarly affected. It would be interesting to know just how this behavior of potassium and calcium is explained in terms of the dissociation hypothesis.

3. The lines of some elements are affected either not at all, or very slightly, by the hot spark. This class is illustrated by the omnipresent copper pair at λλ 3274 and 3247; also by the aluminium pair between Fraunhofer's H and K; and by the great majority of the iron lines.

In general, it may be noted that there is nothing in the nature of a sudden change anywhere in the series. Indeed, the growth of the air lines and the diminution of certain impurity lines is so gradual and definite that one might use their relative intensities to determine the phase at which any particular photograph was taken. The triplet formed by the potassium pair at λλ 4047 and 4044, together with the strong iron line between them, serve to illustrate this principle and also to point out an exception

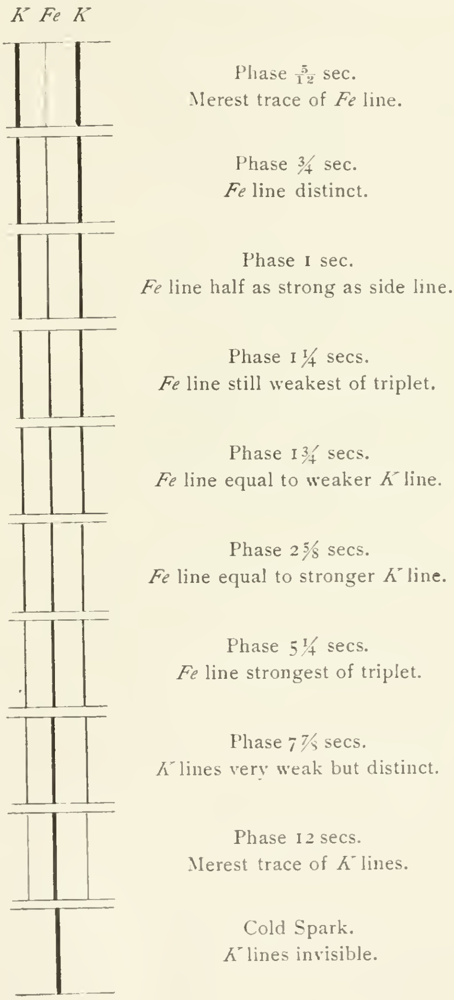


Fig. 6.

to the rule that the iron lines are generally unaffected by the hot spark. For, curiously enough, *this iron line increases* in intensity as the spark-gap (the medium) cools down, while, as noted above, *the potassium pair diminishes* with the temperature. In this comparison the three cyanogen bands are taken as the standard of intensity, and have essentially the same density on each plate. The triplet thus assumes the successive appearances shown in the accompanying figure. If we had measured the temperature of the region between the carbon poles at each of these nine phases, we could have identified with certainty each of these temperatures from the appearance (relative intensity) of the triplet. It is not to be forgotten that the temperature here referred to is *not* the much-talked-of and little-understood "temperature of the spark;" nor is it any temperature peculiar to certain "streaks," as perhaps is the case in the Geissler tube discharge. The temperature here referred to is that of the medium at the instant in which the shutter of the spectrograph is opened. The appearance of this triplet is then a criterion for a temperature, which may be measured directly with a thermo-electric couple of sufficiently fine wire; it is a function of the phase, and not of the duration, of the exposure.

NORTHWESTERN UNIVERSITY,
Evanston, Ill., July 19, 1902.

THE ABSORPTIVE POWER OF THE SOLAR ATMOSPHERE.[1]

By Frank W. Very.

The observations on which the present paper is founded were made at the Allegheny Observatory in the summer of 1882. Their original purpose was to furnish a test of Professor Langley's well-known theory of the "blue Sun." A concise account of the preliminary results appeared in a letter to *Nature* (**36**, 76, May 26, 1887), by Professor S. P. Langley, in which the bearing of the measurements of absorption by the atmospheres of the Sun and the Earth on this particular question were summed up in the general statement that, if we could view the naked photosphere of the Sun, it would appear of a violaceous or lavender tint.

The interpretation of the measures, as far as they affect our knowledge of the law of absorption by the solar atmosphere, still left much to be desired, and it is only recently that I have been able to get anything like a satisfactory conception of the absorptive process.

MEASUREMENTS OF RADIATION OF DIFFERENT WAVE-LENGTHS COMING FROM DEFINITE REGIONS OF THE SOLAR DISK.

The apparatus consisted of the achromatic glass lens of the great equatorial, 464 cm in focal length (aperture limited by the small size of the heliostat mirror to 16 cm), used in conjunction with a horizontal meridional heliostat, forming a solar image 4.27 cm in diameter upon the white-paper facing of the slit-plate of a spectro-bolometer. By pulling the guiding cords of the horizontal movement of the heliostat, the edge of the solar image could be easily and rapidly brought to tangency with fine lines, ruled on the white paper, and numbered, each position corresponding to a definite fraction of the radius of the solar disk falling upon the slit.

[1] Communicated by F. L. O. Wadsworth, Director Allegheny Observatory.

The spectroscopic outfit consisted of a glass prism, made by Hilger, with short focus glass lenses accurately adjusted for the particular point in the spectrum chosen for measurement. To get absolute values, corrections for absorption by the glass of four lenses and one prism, and for loss by reflection from silver would need to be applied. For these relative measurements, however, this is unnecessary, since each set of measures was made under constant instrumental conditions. The bolometer was 2 mm wide, and included successively seven groups of rays whose mean wave-lengths were: 0.416μ, 0.468μ, 0.550μ, 0.615μ, 0.781μ, 1.01μ, 1.50μ. The width of the slit varied between 0.2 mm and 0.4 mm, or from one-fifth to one-tenth the width of the bolometer, but in no case were any rays included whose source differed by more than one one-hundredth of the solar radius from the mean position chosen, except in so far as the occasional quivering of the image, produced by irregular refraction, may have momentarily thrown into the slit, rays from neighboring solar regions. Unfortunately, the accuracy which appertains to the verification of the position of the slit in the solar image, cannot be predicated in like degree of the thermal measures, which remain affected by all the fluctuations of our perpetually changing atmosphere, whose transmissive power varies from moment to moment. In order to secure the most reliable determinations of solar atmospheric transmission, and to eliminate, as far as possible, those errors produced by changes in the telluric absorption, it is desirable to make the measures rapidly and to secure a considerable number of comparisons. Accordingly, a particular fraction of the solar radius, and a particular wave-length having been selected, comparisons of heat at this point and at the center were made alternately, first on the east side, and immediately after on the west. Each marginal measure was then compared with the mean of the preceding and following ones on the center.

Two examples are given in full, one for minimum deviation $50° = \lambda 0.416\mu$ in the violet, and one for minimum deviation $48° = \lambda 0.550\mu$ in the citron. At each fraction of the radius are

given the galvanometer deflections observed, and the ratios of the deflections at stated points to those at the center found in juxtaposition.

TABLE I.

Violet, $\lambda=0.416\mu$	0.50 r		0.75 r		0.95 r		0.98 r	
	Defl.	Ratio	Defl.	Ratio	Defl.	Ratio	Defl.	Ratio
	div.		div.		div.		div.	
Center	26	....	23½	....	24	....	38	....
East	21¾	0.837	18	0.747	11½	0.500	9½	0.253
Center	26	....	24¾	....	22	....	....	....
West	21½	0.833	17½	0.726	10½	0.465	11½	0 307
Center	25½	....	23½	....	23¼	....	37	....
East	22½	0.882	17½	0.745	11½	0.502	11	0.297
Center	25½	....	23½	....	22½	....	....	....
West	21½	0.878	18	0.756	9½	0.417	11	0.297
Center	23½	....	24	....	23	....	37	....
Mean ratio, August 24	($3^h 35^m$)	0.858	($3^h 45^m$)	0.744	($3^h 55^m$)	0.471	($2^h 8^m$)	0.289
Calculated transmission by unit depth of atmosphere	....	0 372	....	0.561	....	0.710	....	0 735

Citron, $\lambda=0.550\mu$	0.50 r		0.75 r		0.95 r		0.98 r	
	div.		div.		div.		div.	[0.478]
Center	366½	....	345½	....	383	....	310	[0.536]
East	348	0.956	300	0.836	213	0.559	124	0.423
Center	361½	....	372	....	380	....	275½	....
West	324	0.903	302	0.822	223½	0.581	146	0.498
Center	357	....	363	....	389	....	310	....
East	347	0.940	....	....	....	....	....	....
	354	0.959	312	0.841	232	0.599	156	0.481
Center	381	....	378	....	385	....	338	....
West	332½	0.908	314	0.824	230	0.610	159	0.485
Center	352	....	383	....	368½	....	317½	....
Mean ratio, September 2	($1^h 20^m$)	0.933	($1^h 45^m$)	0.831	($1^h 55^m$)	0.587	($3^h 35^m$)	0.484
Calculated transmission by unit depth of atmosphere	......	0.640	....	0.697	....	0.785	....	0.835

The last line of Table I gives the transmission by a unit of the solar atmosphere (meaning by this the radial depth of the gaseous layers lying above the photosphere), computed by the ordinary secant formula. As in the case of the absorption of radiation by the Earth's atmosphere, it is obvious that there is considerable departure from the assumed law, and that with increasing depth of the absorptive layer the apparent transmission, by unit depth, derived in this manner, is not a constant, but becomes progressively greater in both cases, although possibly not from the same cause.

It will be seen that at every radial position without exception, the radiation falls off in the direction of the Sun's limb, and does so quite rapidly. No marked difference could be detected between the heat at similar radial positions on the east

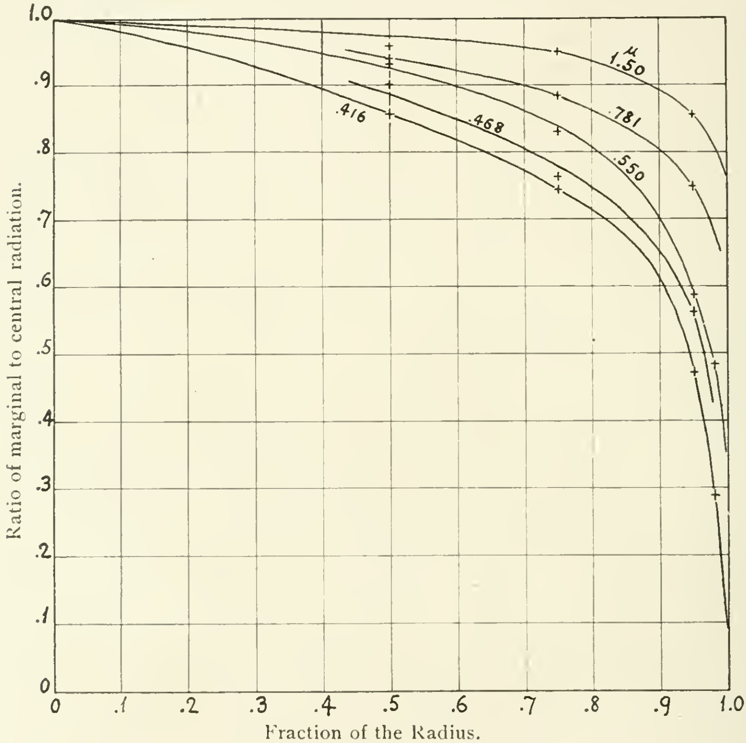


Fig. 1.

and west limbs; and Langley had previously proved that the thermal effect of the total or combined radiations from corresponding points on the north, south, east, and west limbs, as registered on a thermopile, is practically identical. See the paper "Sur la température relative des divers régions du soleil. 2me partie: Région équatoriale et régions polaires," *Comptes Rendus*, 80, 819, 1875. Also another paper: "Étude des radiations superficielles du soleil," *ibid.*, 81, 436, 1875, from which I

take the following results, stated as percentages of the total radiation from the center of the solar disk :

	N.	E.	S.	W.	Mean
.50 *r*	94.9	94.7	94.5	95.9	95.0
.75 *r*	86.5	85.6	85.5	84.0	85.9
.96 *r*	60.7	60.6	62.3	63.9	61.9
.98 *r*	50.8	47.7	49.5	52.2	50.1

The measures may therefore be presented in the form of a diagram in which the solar disk is divided into concentric annuli, to be regarded as contour lines exhibiting the varying thermal efficiency of different parts of the Sun's surface in the form of a symmetrical elevation ; or, better, the varying radiation may be displayed as a series of sections of such thermal diagrams for particular regions of the spectrum. Curves showing these relations are presented in Fig. 1, which shows the more rapid diminution of the radiations of shorter wave-length in the direction of the Sun's limb, in comparison with the falling off of the longer waves. A tabular view of the mean results of these observations follows :

TABLE II.

Ratios of Marginal to Central Radiation.

$\lambda =$	0.416 μ	0.468 μ	0.550 μ [1]	0.615 μ	0.781 μ	1.01 μ	1.50 μ
Deviation	50°	49	48°	47° 30′	46° 45′	46° 12	45° 28
0.98 *r*	0.289	.....	0.484	.....	.....	.....	.....
0.95 *r*	0.471	0.462	0.587	0.681	0.749	0.765	0.856
0.75 *r*	0.744	0.764	0.831	0.845	0.885	0.894	0.950
0.50 *r*	0.858	0.902	0.933	0.948	0.941	0.943	0.959

The only measures with which these can be compared are those of Dr. H. C. Vogel with a spectro-photometer. These give smaller ratios at the red end of the spectrum for the position 0.95 radius : violet 0.347, blue 0.456, green 0.440, yellow 0.460, red 0.580. The bolometer has an advantage over the eye in the red where the heat is great.

Repeated measures across the spectral regions where some of the great infra-red cold bands lie, proved that these absorption bands are not produced by the solar atmosphere, but since the bands broaden and deepen as the Sun goes down, they are unquestionably telluric. The absorption indicated by the marginal diminution of radiant intensity is of another order. I proceed to the further consideration and interpretation of these results.

THEORY OF THE ABSORPTION OF RADIATION BY THE SUN'S ATMOSPHERE.

If we compute coefficients of transmission, assuming uniform original radiation, and adopting the hypothesis that successive equivalent layers absorb equal fractions of the rays entering them, the striking result is obtained that the marginal measures appear to indicate a more highly transmissive atmosphere than those measures nearer the center, as shown in the final line of Table I. Asking now the meaning of this, let

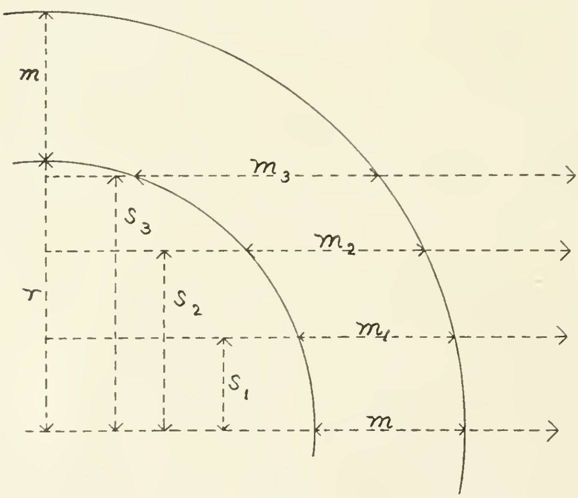


FIG. 2.

m (Fig. 2) be the thickness of the solar atmosphere, assumed to be homogeneous; $r =$ the Sun's radius; $p =$ the transmission through a radial section of this solar envelope; and let s_1, s_2, s_3, be distances from the center of the solar disk to the points on the

[1] Other series at wave-length 0.550 μ, omitting he readings at 0.98 of the radius, gave the following concordant results, scarcely differing except at 0.95 *r*: at 0.95 *r*, 0.618; at 0.75 *r*, 0.827; at 0.50 *r*, 0.936.

disk selected for measurement (expressed as fractions of the photospheric radius, r, which may be taken to be unity). Then $r + m = 1 + m$ will be the radius of the outer limit of the effective solar atmosphere. Let m_1, m_2, m_3, be the atmospheric paths of rays starting from the photosphere at distances s_1, s_2, s_3, from the center of the solar disk, and emitted in the direction of the Earth; and let R_1, R_2, R_3, be the ratios of the energy of these rays compared with central ones, or the values given in Table II. Then, other things being equal, if successive layers repeat the absorbent process according to the same law, and none of the rays vanish,

$$p = R_1^{\frac{1}{m_1 - m}} = R_2^{\frac{1}{m_2 - m}} = R_3^{\frac{1}{m_3 - m}} \; . \qquad (1)$$

The relative masses or depths of absorbent atmosphere in the several cases are to be computed by the equations:

$$m_1 = \cos\left(\sin^{-1}\frac{s_1}{1+m}\right) - \cos\,(\sin^{-1} s_1) \; ,$$

$$m_2 = \cos\left(\sin^{-1}\frac{s_2}{1+m}\right) - \cos\,(\sin^{-1} s_2) \; ,$$

$$m_3 = \cos\left(\sin^{-1}\frac{s_3}{1+m}\right) - \cos\,(\sin^{-1} s_3) \; .$$

In the first treatment of the problem, the unknown quantities being p and m, I shall assume the photospheric radiation to be equal for all parts of the disk. Since the values of p and m must then satisfy equation 1, it becomes possible, on this assumption, to infer the resulting depth of the efficient solar atmosphere. Omitting the observations at the violet end of the spectrum, which, on account of spectral non-homogeneity, may follow a different law, five wave-lengths remain for which transmissions are computed from the ratios of central, intermediate, and marginal radiations by the preceding formulæ: first (a) assuming an atmosphere of little depth; second (b) assuming an atmosphere whose depth is one half the radius of the sphere; and third (c) assuming an atmosphere whose depth is equal to the radius.

From measures at the center combined with those at the positions: 0.50 r 0.75 r 0.95 r

(*a*) For $m = \frac{r}{\infty}$, $m_1 - m = 0.155$, $m_2 - m = 0.512$, $m_3 - m = 2.202$.

(*b*) For $m = \frac{r}{2}$, $m_1 - m = 0.096$, $m_2 - m = 0.270$, $m_3 - m = 0.698$.

(*c*) For $m = r$, $m_1 - m = 0.070$, $m_2 - m = 0.192$, $m_3 - m = 0.446$.

TABLE III.

Position =	0.50 r			0.75 r			0.95 r		
λ	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*
0.550 μ	0.652	0.501	0.388	0.690	0.494	0.371	0.803	0.501	0.339
0.615	0.708	0.573	0.466	0.719	0.535	0.415	0.840	0.576	0.422
0.781	0.676	0.531	0.420	0.788	0.636	0.529	0.877	0.661	0.523
1.01	0.683	0.540	0.429	0.803	0.659	0.556	0.885	0.681	0.548
1.50	0.763	0.646	0.550	0.904	0.827	0.765	0.932	0.800	0.706
Mean p	0.696	0.558	0.451	0.781	0.630	0.527	0.867	0.644	0.508

Mean transmission - - - - - - - - - - (*a*) 0.781
" " - - - - - - - - - - - (*b*) 0.611
" " - - - - - - - - - - - (*c*) 0.495

The smaller transmissions of series (*b*) and (*c*), derived from measures at different radial positions, are more nearly accordant than those of series (*a*), and this, in the absence of any other explanation, would favor the doctrine of a solar atmosphere of great depth, substantially coinciding with the solar corona, rather than with the chromosphere. Nor need the hypothesis be dismissed as entirely impossible, since the peculiar law of solar atmospheric absorption coincides with that progressively increasing selective reflection which the floating particles and even the air molecules of our atmosphere exert on the shorter waves of light.

EFFECT OF DIFFRACTION BY FINE PARTICLES.

As the corona contains some scattered dust, a diffraction effect is possible, indeed it certainly exists, and the only question is as to its quantitative value. Professor C. D.

Perrine has found that the inner corona, to a height of 2′ to 8′, gives a continuous spectrum, such as might be produced by incandescent solid particles floating in the lower and denser layers of the solar atmosphere; but that above this zone, the coronal light is reflected sunlight showing the Fraunhofer lines. ("Preliminary Report of Observations of the Total Solar Eclipse of 1901, May 17–18," by C. D. Perrine; ASTROPHYSICAL JOURNAL, **14**, 349, December 1901.) Above this lower zone and to a height of half a degree, "comparisons with a sky spectrogram secured with the same instrument show that the coronal and sky spectra are sensibly identical in the blue and violet regions" (p. 354). Polariscopic observations also show that the light from the outer corona agrees with that of the sky in being polarized, and consequently we may assume that the coronal envelope, like the Earth's atmosphere, contains finely divided matter competent to either reflect or diffract sunlight, dispersing it in all directions, and acting upon the photospheric radiation selectively, that is, sending off a relatively larger percentage of the shorter rays.

It is somewhat difficult to judge by the eye in regard to the color of a faint light. Eclipse observers will probably agree that the coronal light appears almost white, or if it possesses any color this must be faint; but the spectrogram, it seems, decides the point in favor of a blue-tinted corona, with a light resembling that of the sky, and supplements the evidence derived from the colors of the solar image (reddish-brown near the limb and bluish at the center), as well as that from the spectro-bolometric comparisons along a radius of the solar disk, all declaring that the Sun is surrounded by a medium which excessively depletes the composite beam of its shorter waves and disseminates them in all directions as a feeble coronal illumination. If we accept Lord Rayleigh's theory that the blue light of the sky is diffracted by the air molecules, we must conclude that the corona is subdivided to molecular dimensions. Huggins, in his Bakerian lecture of 1885, arrived at an analogous conclusion, deciding that the coronal substance is so highly attenuated that

its particles, like those of comets' tails, are more strongly affected by electric repulsion than by gravitational attraction. Nevertheless, there must be a lower limit to the fineness of the coronal particles which give the polarized light, because a Bunsen flame does not reflect sunlight unless, by partial deprivation of air, the flame begins to assume an illuminating quality. If an image of the Sun be formed by a condensing mirror in the midst of a luminous or partially luminous flame, a brilliant blue light is emitted in all directions from the focal point, and is found to agree with sunlight in color, presence of Fraunhofer lines, and polarization. This light is sunlight which has been selectively reflected from solid particles of carbon, perhaps only a little larger than molecules, at any rate of excessively minute dimensions.

The corona appears to contain particles of some such magnitude, which probably diminish in size as the distance from the photospheric, or incandescent cloud surface, increases, until, at a considerable radial distance, they cease to diffract any but the shorter waves of sunlight. Dr. C. G. Abbot concludes from his bolometric measures of coronal radiation in the eclipse of May 28, 1900, that the corona seems "to be giving light in a manner not associated with a high temperature, or at least with the preponderance of infra-red rays usual in the spectra of hot bodies" (ASTROPHYSICAL JOURNAL, **12**, 73, July 1900). The preponderance of short waves and almost complete absence of infra-red rays in the coronal radiation is consistent with the supposition that the corona itself may be the medium which depletes the solar rays of their shorter waves. Coarser incandescent particles may diffuse as well as radiate the longer waves, but such particles probably only exist at low levels, where their presence has indeed been demonstrated by Campbell's spectrograms at the Indian eclipse of 1898, and inferentially by the continuation of selective depletion of radiation by the solar atmosphere as far as bolometric measures have been made in the infra-red spectrum.

We can subject the hypothesis of an extensive envelope,

depleting the rays by selective diffraction, to a further test by comparing the light diffused by the corona with direct sunlight. Suppose that three-fourths of the direct solar light is selectively reflected. An amount of light three times as great as that which reaches us directly from the Sun, is then diffused by some part of the corona lying between us and the photosphere, and is distributed over more than a hemisphere. Perhaps the fraction $\frac{3}{120000}$ might be taken as a rude approximation to the dilution of the light by this process. Now, during an eclipse of the Sun, the coronal light at 1′ from the Moon's limb has been found to have six times the intrinsic brightness of the Moon (*Washington Observations* for 1876, Appendix III, p. 214), and moonlight is about $\frac{1}{600000}$ of the intensity of sunlight. The luminosity of the corona also falls off very rapidly in a radial direction. At some point not much within the 1′ limit, the coronal luminosity is fifteen times brighter than moonlight, or in the ratio of the above fractions, and from a still narrower ring the proportion of light is much greater; but the light of the outer corona represents only a small fraction of the total loss of sunlight by selective reflection. The principal part of the diffusive action is due to the inner layers of coronal substance, and the efficient depleting atmosphere is at any rate confined practically to those regions which are sometimes called "the inner corona," constituting a very bright annulus 0.′5 or 1.′0 broad, as seen at the times of total solar eclipse, and even here the effect is greatest for the lowest levels.

EFFECT OF ATMOSPHERIC STRUCTURE.

Another possible explanation of the greater apparent transmission near the Sun's limb is that it results from the action of irregularly distributed ingredients in the solar atmosphere upon the transmitted sunbeam. Suppose that this atmosphere has a columnar structure, certain absorbent gases, or substances which deplete the rays, whether by absorption or by selective reflection, existing in isolated vertical currents in the midst of other inert

substances which may be more regularly distributed. Such a structure is indicated by some visible appearances, such as the serrations of the chromosphere and the penumbral filaments in Sun-spots; and as far as it exists, it must make itself felt in the nature of its absorptive effects. Let the height of these columns be ten times their diameter, and let us assume that they are radially distributed over the spherical surface, occupying one-fifth of its area (Fig. 3). Indicating by p_1 the coefficient of transmission through a layer of absorbent substance equal in depth to the average diameter of a column, if $p_1 = 0.9$, for central position and columnar structure, the photospheric radiant energy (P) escaping through the atmosphere is $0.8\,P + 0.2\,(p_1)^{10} \times P = 0.87\,P$, a larger quantity than would result if the same amount of absorbent material were spread horizontally in a layer of depth $= 2$, giving for central position and stratal structure, radiant energy transmitted $= (p_1)^2 \times P = 0.81\,P$.

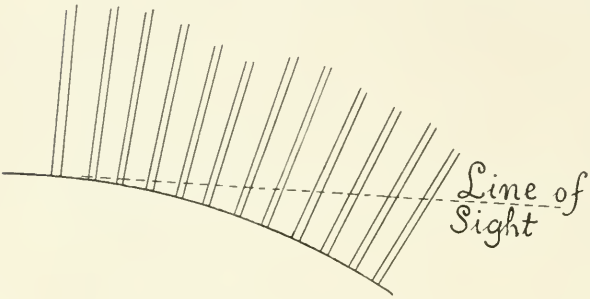


Fig. 3.

Consider now the effect on a ray from the edge of the visible solar disk, which has passed transversely through ten such columns. The intracolumnar path will differ but little from an intrastratal one, if the atmosphere be of slight depth relatively to the radius. Ruling out the hypothetical atmosphere of great depth, the marginal radiation transmitted is $(p_1)^{10} \times P = 0.35\,P$. Erroneously computed from the central value, supposed to be known, a smaller marginal value, $(0.87)^{10} \times P = 0.25\,P$, is obtained, or in appearance the marginal atmosphere is more transparent than the central. The effect of columnar structure in an atmosphere is therefore similar to that produced by greater depth, or by that selective absorption and extinction of individual rays which leaves the emergent beam more and more transmissible by succeeding layers of the same absorbent.

The Earth's atmosphere partakes of the same properties. Sudden variations in solar radiation measured by an actinometer of rapid registry, like Crova's, occur under a clear sky, and are of much wider range with high than with low Sun. In the middle of the day, the rays sometimes penetrate *between*, and sometimes *through* the absorbent columns for nearly their whole length, thus causing great and sudden variations; while, with a low Sun, the number of columns intersected does not vary greatly, and the solar radiation does not fluctuate so much. The effect is greatest in summer when ascensional movements of the air are most powerful, and it makes the summer air appear more transparent than that of winter, although the reverse is really the case, aqueous vapor, which is more prevalent in summer, being the chief absorbent among the constituents of the Earth's atmosphere.

Consider only the part of the general depletion of the sunbeam which is effected by selective reflection. If this exhibits any tendency to depart from a logarithmic law dependent on the depth of air penetrated by the rays, it must be on account of atmospheric columnar structure, and an irregular distribution of fine haze, perhaps constituted of incipient watery mist, which, in turn, owes its existence to the atmospheric movements which have produced the structure. But a large part of the discrepancy between transmissions computed from observations with different air-depths, results from a progressive disappearance of rays or particular wave-length in the upper air, mainly through absorption by aqueous vapor. In other words, the summer air, as well as the longer air-column traversed by the rays at low Sun, transmits the residual of a sifted solar radiation somewhat readily; but it does this because a larger number of the incoming homogeneous rays has been completely removed in the upper air. Locally selective absorption is of less importance in producing variation of apparent transmission by the solar atmosphere, because the Fraunhofer lines, especially in the infra-red, are nowhere so intense nor productive of such crowded arrangements into broad bands, as are the aqueous lines which chiefly concern telluric atmospheric absorption.

EFFECT OF PHOTOSPHERIC IRREGULARITY.

Quite apart from the distribution of the absorbent constituents of the outer solar envelopes is another factor which requires recognition, namely, the irregularity of the radiating photospheric surface. This irregularity will have but little influence on the apparent transmissive power of the solar atmosphere if the latter is exercised by an envelope of great depth ; but if all, or even a considerable part of the absorption is produced by the Sun's reversing layer, which is a stratum of comparatively slight depth, or even by an atmosphere as deep as the chromosphere, or the inner part of the inner corona, a marked effect must follow from the *form* of the radiating surface.

The photosphere is made up of brilliant "rice-grains" and their component "granules," separated by a relatively dark reticulation in which the light having come from greater depths, suffers larger absorption than where it proceeds from the summits of the photospheric clouds, or granules. Langley, who discovered the granules, estimated them to occupy one-fifth of the surface at the center of the Sun's disk ; but near the limb the light must come mainly from cloud summits, the bases being hid. Some of Janssen's photographs give even smaller ratios for the luminous dots. Assuming that the light at 0.95 of the radius comes entirely from granules which occupy but one-fifth of the surface at the center, the following estimate of intensities was made :

A drawing, which was considered fairly representative of the relative depth of light and shade in the rice-grain structure, was compared with a pair of black and white Maxwell's disks revolved with sufficient rapidity to produce a gray which could be matched with the shading on the drawing. The darker shading was equal to a gray of ⅛ white, ⅞ black ; the lighter was equivalent to a gray of ¼ white, ¾ black. Adopting an average gray of ⅙ white, we have for central light,

$$L_c = 0.2 + (0.8 \times 1/6) = 0.333 \ ,$$

and for light from the margin,

$$L_m = 1 \ .$$

Calling photospheric radiation from the center P, and taking the mean of the ratios at $0.95r$ for the five wave-lengths given in Table III with the exponent (m_3) appropriate to condition (a) we have:

$$\text{Central radiation} = P \times p = 1000 ,$$

$$\text{Marginal radiation} = 3P \times (p)^{3.2} = 734 ,$$

whence the coefficient of transmission is $p = 0.527$, which may be compared with the mean, $p = 0.495$ for an atmosphere of radial depth.

RESULTS OF A COMPARISON OF THE THEORY WITH THE OBSERVATIONS.

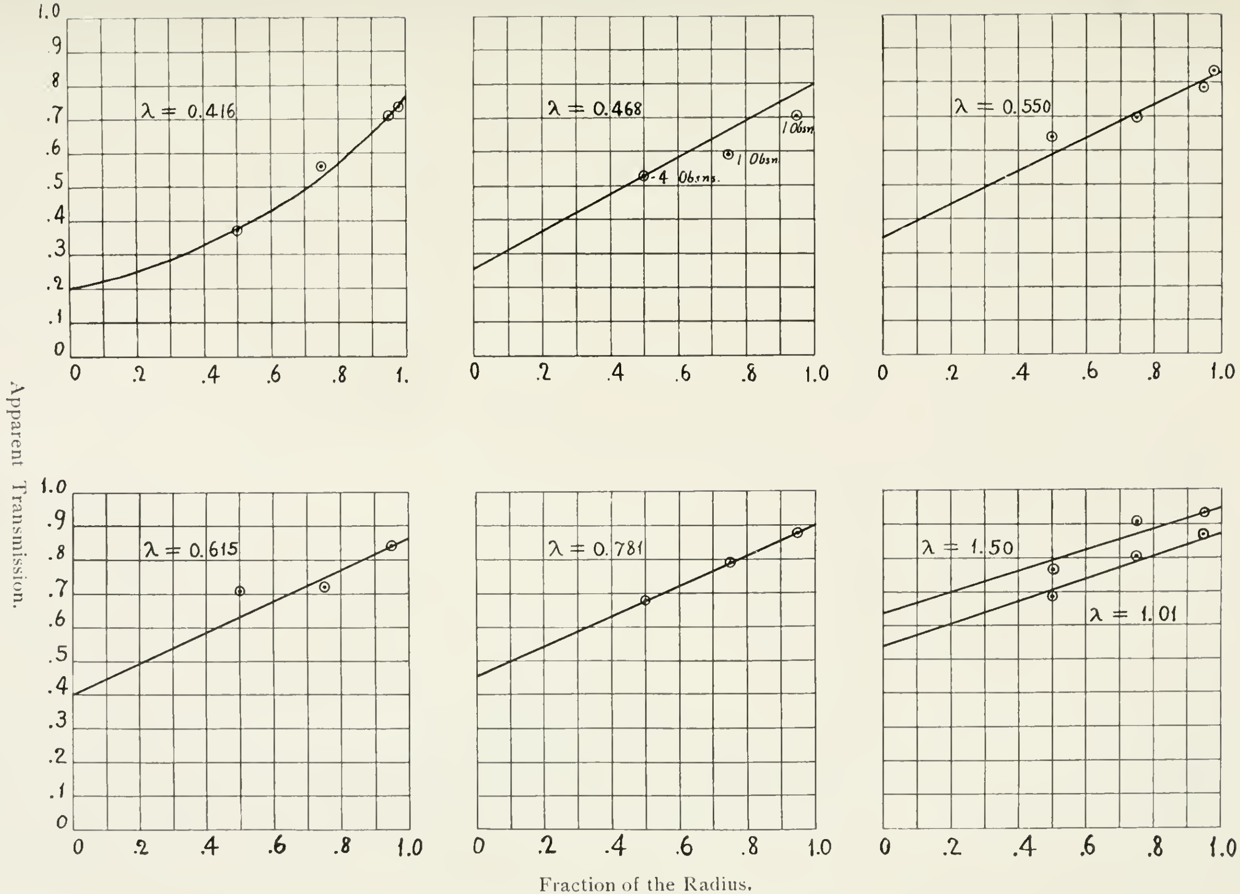
The result is that whether we attribute the discrepancies in the coefficients of solar transmission for radiation from different points of the disk to great depth of the solar atmosphere, or to increased photospheric brilliancy at the limb, the effect on the calculated transmission is similar. It is evidently necessary to diminish all of the computed coefficients of transmission, those from the marginal measures being reduced in greater proportion, and since the progressive extinction of particular spectral rays in the marginal radiation still further increases the effect, I think we may safely infer that not more than one-half of the red and infra-red radiation of the photosphere, is able to penetrate the solar atmosphere, while still less of the violet and ultra-violet gets through. It will be observed that the three causes of variation in the coefficients of transmission computed by the secant formula all act in the same direction, and all appear to be real.

Comparing the effects of increasing photospheric brilliancy towards the limb, and of great depth of solar absorbent envelopes, we find difficulty in separating them, because neglect of either action makes the calculated transmissions appear too large by similar amounts, the result being further complicated by our ignorance of the precise alteration effected by the peculiar structure of the Sun's outer envelopes. The form of the calculated transmission-curves from center to limb gives, however, a means of separating the influence exerted by the lack of homogeneity in the spectrum.

The exponential formula which is commonly used in the reduction of observations for atmospheric transmission has the form of the logarithmic curve whose equation is $y = a^x$; or if A is the radiant energy which passes through the solar atmosphere, P the original photospheric radiation, m the mass of the solar media through which the rays pass, and p the coefficient of transmission through the solar media, $A = P \times (p)^m$, where P and p are supposed to be constants. If the formula were followed, therefore, we should get the same value of p from observations at all points of the solar disk, or if we plotted them, letting abscissae be the fractions of the solar radius, and ordinates be the corresponding values of p derived by comparison of energies at these points with that at the center, the values of p should lie on a straight line parallel to the axis of X. If, however, some unknown action, increasing or decreasing from center to margin according to some regular law, is at work, our calculated values of p may follow a curve of some different shape. Still, if we could determine the form of this curve, the point at which it cut the axis of Y (ordinate at the Sun's center) would give us the value of p which we are seeking, namely, that for radial transmission through the solar atmospheric envelope. A considerable number of observations renders it nearly certain that the form of the above curve for p is a straight line inclined to the axis at such an angle that for the middle rays of the spectrum it would meet the axis of Y at about p=0.5, while for ultra-violet rays the intersections fall very near zero, and for extreme infra-red rays exceed 0.5, but fall below 0.9. These, therefore, are the real coefficients of transmission.

The curves of computed transmission from center to edge (Plate III) do not depart appreciably from inclined straight lines, except in the case of the violet rays. For these, however, the curvature is marked; that is, the apparent transmission of violet rays ($\lambda = 0.416\mu$) not only increases with the radius of the point of departure, but at an increasing rate. This suggests that the increment of locally selective absorption of violet light, indicated by the crowded Fraunhofer lines in this part of the spec-

PLATE III.



Computed Transmissions from Center to Edge.

trum, is accompanied by the successive vanishing of particular rays which drop out and leave a more transmissible beam of sifted marginal radiation. The percentage of vanished rays may possibly be estimated by the departure of the curve from a straight line (perhaps 10 per cent. for marginal violet rays). The general depletion by selective reflection or diffraction also increases in the direction of the shorter waves, and the combined effect produces complete extinction of ultra-violet rays by the solar atmosphere at about the same limit which the absorptive properties of the Earth's atmosphere place upon the spectra of extra-telluric sources. Theoretically, we might get an image of the Sun without any distinct edge by using only the extreme ultra-violet rays, and there is some advantage in studying them by the aid of an analyzing spectroscope, employing light from the center of the Sun's disk, provided the extra lens is sufficiently transparent to these rays.

The following table is a summary of the coefficients of transmission for the solar atmosphere, as calculated by the exponential formula, assuming uniform photospheric radiation, and perpetuity of all included rays. It is derived from observations on the points $S_4=0.98\ r$, $S_3=0.95\ r$, $S_2=0.75\ r$, $S_1=0.50\ r$, and the center. The computed coefficients being plotted with the radii from which they are derived as abscissae, the errors of observation have been eliminated by mean curves which are straight lines in every case but one. A slight adjustment has been made

TABLE IV.

Computed Transmissions of Solar Rays by the Solar Atmosphere. Neglecting the Influence of Structure.

$\lambda=$	$0.416\ \mu$	$0.468\ \mu$	$0.550\ \mu$	$0.615\ \mu$	$0.781\ \mu$	$1.01\ \mu$	$1.50\ \mu$
Deviation	50°	49°	48°	47° 30′	46° 45′	46° 12′	45° 28′
0.98 r	0.735	0.790	0.820	0.855	0.890	0.903	0.942
0.95 r	0.714	0.772	0.809	0.840	0.876	0.891	0.931
0.75 r	0.525	0.665	0.710	0.750	0.788	0.820	0.871
0.50 r	0.380	0.530	0.588	0.634	0.675	0.729	0.794
Central	0.200	0.258	0.345	0.400	0.450	0.540	0.638

for the series at deviation 46° 12′, but not beyond the limits of error of the observations. The last line gives the concluded vertical or central transmission.

The following final coefficients of transmission have been taken from a smooth curve through the positions obtained from the intersections of the secondary curves with the axis ($S_0 = 0$). The curves of apparent transmission of homogeneous radiations between wave lengths of 0.3μ and 2.6μ, deduced in the foregoing manner for different points of the solar disk, and exhibiting to the eye the numerical relations in Tables IV and V, are shown in Plate IV.

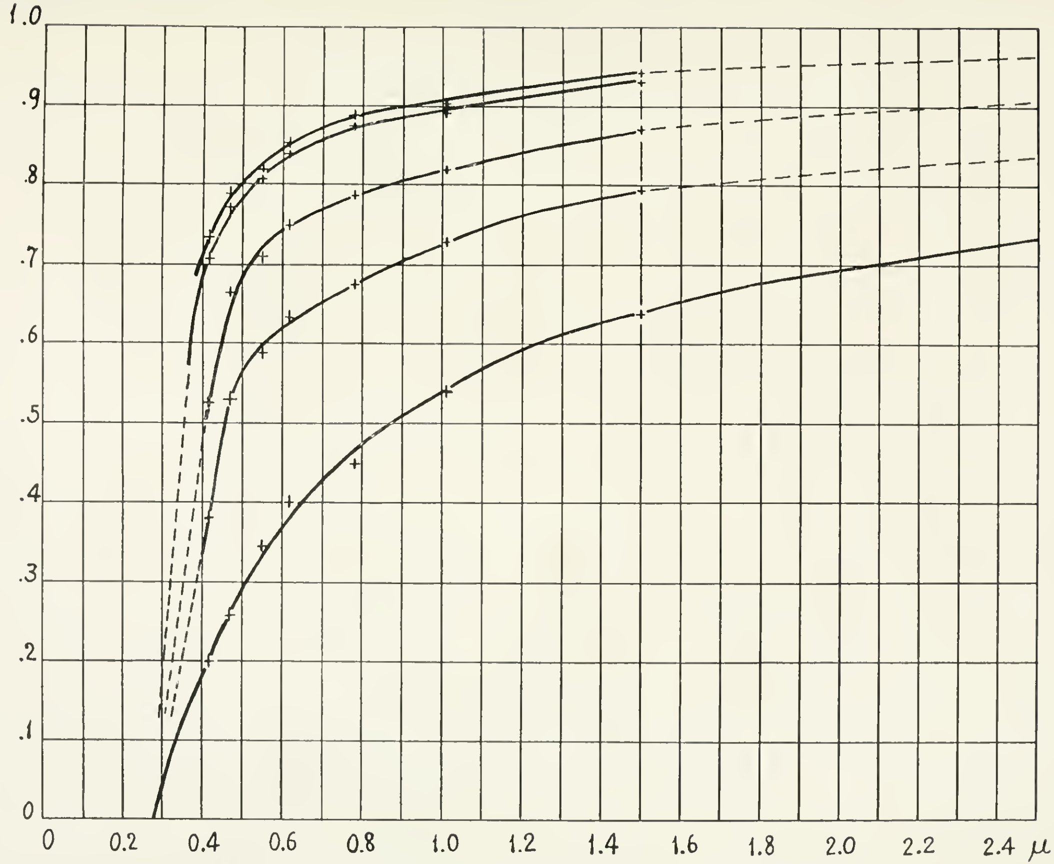
TABLE V.

Wave-length	$0.30\ \mu$	$0.40\ \mu$	$0.50\ \mu$	$0.60\ \mu$	$0.70\ \mu$	$0.80\ \mu$	$0.90\ \mu$	$1.00\ \mu$	$1.50\ \mu$	$2.00\ \mu$
Transmission.......	0.05	0.18	0.29	0.37	0.43	0.475	0.51	0.54	0.64	0.69

Some indefiniteness attaches to the figures given in Table V on account of the unsatisfactory definition of unit depth of the solar atmosphere. The values specified in this table relate to a mean depth for central regions where this depth is exceptionally great, because here we see down into the interior of the corrugated photosphere. The gases in the depressions are denser, and the equivalent atmospheric mass is greater than it is near the limb where the bottoms of the depressions are invisible. In any case the depth cannot increase as rapidly as the secant of the angle between the radius and the line of sight.

By comparison of these figures with those previously given, assuming an atmosphere of radial depth, it will be seen that the agreement between the results by the two methods is close. Since the various causes of deviation which have been suggested, all conspire to produce similar effects, it does not seem possible to discriminate further between them by mathematical treatment of the data. We have seen, however, that, while there can be no doubt that the principal cause of the depletion of the photospheric radiation is a selective diffuse scattering of the shorter ether-waves by particles of an excessive minuteness, constituting

PLATE IV.



Apparent Transmission Throughout the Spectrum for Each Radius.

an essential, if not the principal ingredient of the inner corona, there is good reason for rejecting the hypothesis of an atmosphere, or dust envelope, of great depth, at least as an efficient modifier of solar radiation; that the greater continuity and increased brilliancy of the naked photosphere at the Sun's limb is sufficient to account for a large part of the peculiarities, with possibly a small contribution from the columnar chromospheric structure; and that the successive extinction of special rays is a further concomitant in the violet and ultra-violet parts of the spectrum.

Assuming equal instrumental efficiency in other respects, the employment of an analyzing, in place of an integrating spectroscope, may be expected to increase the ultra-violet rays between 40 and 50 per.cent., the violet rays about 36 per cent., blue rays 30 per cent., citron 22 per cent., and crimson light 14 per cent.

If any variation in the absolute radiation of the Sun is ever certainly detected, it may possibly be found to be associated with some change in the quality or depth of the solar atmosphere, which can best be determined by spectro-bolometric measurements. Whatever change there may be in the solar atmospheric absorption, progressing or fluctuating with the time, it is a quantity of a very small order; yet it seems possible that such an effect may be discriminated by the summation of extensive series of measures similar to the foregoing, but made at maximum and minimum Sun-spot epochs.

As the larger fields of fact become exploited, we must strive for more minute detail. The work outlined here requires persistent systematic cultivation for a long time, such as can scarcely be obtained by the desultory efforts of individuals unable to command their time from year to year. It can be accomplished best at our larger observatories, where it can be made a matter of routine.

SOME RESULTS OF THE TOTAL ECLIPSE IN SUMATRA, OF MAY 18, 1901, OBTAINED WITH THE PHOTOHELIOGRAPH, AT FORT DE KOCK.

By G. H. Peters.

One of the three parties comprising the United States Naval Observatory eclipse expedition to the island of Sumatra was located at Fort de Kock. This village was about ten miles inside the northern edge of the path of the Moon's shadow, which had a width in that longitude of about 150 miles. The duration of total phase was 176 seconds, while on the central line it was 386 seconds.

The long duration of totality, even close to the limit of the shadow, rendered practical the establishment, near the edge of this path, of a large instrument to photograph the solar surroundings close to the photosphere. Successful spectroscopic work had already been done in similar positions in former eclipses; but no attempt had been made to secure photographs of the region close to the solar limb, with an instrument of this description, and from a like location.

Two instruments were in operation at this station, for recording photographically the eclipse phenomena: a large grating spectrograph, in charge of Dr. W. J. Humphreys, of the University of Virginia, an associate member of the expedition, and a 39-foot photoheliograph of 5 inches aperture, in charge of the writer; the entire station being under the control of Professor W. S. Eichelberger, U. S. N. These instruments were both used horizontally, taking their light from 10-inch silvered glass plane mirrors, at the extremities of the polar axis of a new Brashear coelostat.

The prime object in locating the photoheliograph here was to obtain large scale records of the solar surroundings close to the northern limb. These would show the chromosphere and prominences well uncovered, and at the same time photograph the coro-

nal rays at their departure from the Sun, close to their origin. It was also decided to give longer exposures on more rapid plates, near mid-totality, in order to obtain a record of coronal extension.

Notwithstanding some instrumental difficulties with which we had to contend, the results secured were extremely satisfactory. Moreover the station at Fort de Kock was the only one on the island where absolutely clear weather prevailed during totality. At the other stations the observers were unfortunate in having the sky more or less obscured by haze or thin clouds. The elevation of 3000 feet at this point also added to the atmospheric transparency, while with the Sun just past the meridian, and at a zenith distance of less than 30°, the best astronomical conditions prevailed.

There was, however, one exception to these most favorable conditions. It was found that the "seeing" at Fort de Kock was very unsteady both by day and at night on this and nearly all other occasions when the sky was clear. From our observations while on the island this seems to be a general feature of the climate. The close proximity of two high mountain peaks, to the south of the site selected, one an active volcano, each mountain rising to an altitude of 6000 feet above the table-land on which we were located, probably increased these unfavorable conditions. This poor seeing incident to the climate undoubtedly interfered considerably with the sharp definition of the eclipse photographs.

In all, ten photographs of the eclipsed Sun were obtained with the photoheliograph during totality, and the conditions and results are summarized in the following schedule.

The plates used were Seed's "Gilt Edge," 23 and 27, with a non-halation backing.

The purposes of plates 1 and 9 were to secure records of as much of the projecting chromosphere and prominences as possible, and at the same time impressions of the polar rays. It is to these regions, on the northern limb of the Sun, that special attention is invited.[1]

[1] The Editors regret that it has been impossible to obtain a satisfactory engraving of the plate sent by the author.

No.	Sensi-tometer of plate	Length of exposure seconds	Approximate period of count. From	To	Results and Remarks
1	23	½	½	1	Shows prominences and chromosphere on northeast limb, and faint coronal rays both polar and equatorial, and coronal disturbance.
2	23	1	9	10	Shows prominences and chromosphere on northeast limb. Stronger coronal and polar rays than No. 1. Polar rays emerging from chromosphere.
3	23	2	18	20	Shows prominences and small amount of chromosphere in northeast limb, together with considerable corona. Some clock rate is perceptible and development was checked in consequence.
4	27	5	28	33	A fine photograph of inner corona, showing coronal disturbance well in outer parts, besides the coronal loops, or arches, also prominences faintly.
5	27	60	43	103	Shows coronal streamers extending, faintly, to the edge of the plate, clock rate fairly good. Some effect of atmospheric unsteadiness noted.
6	27	20	113	133	Considerable clock irregularity, owing to bent worm screw of coelostat. Image considerably blurred. Developing not carried very far in consequence.
7	27	3	141	144	A fine photograph. Shows chromosphere on the northwest limb, prominences and inner corona, together with coronal disturbance and arches.
8	23	1	152	153	Some clock motion. Shows extensive chromosphere on northwest limb; polar rays emerging from chromosphere, corona faint.
9	23	½	161	161½	Comparable with No. 2, but showing slightly less extent of corona. The Moon's limb has here covered the bases of the features on the northeast edge.
10	23	½	176	176½	Taken immediately after third contact, shows a small ray of direct sunlight, giving position of contact, and but little of the corona.

The chromospheric crescent on these plates is not quite even in surface outline, but is broken up into *billow-like* ridges, resembling, to some extent, the rough limb of the Moon. Some of these *billows* are slightly larger than others, but all have the same general form, and are quite evenly distributed. Wherever they appear, the coronal rays seem to emanate from them. This is especially noticeable with respect to the polar rays, whose

well-defined outlines enable the latter to be traced more clearly to their source. On the original negatives, 2 and 9, these polar rays extend to a distance of 6mm above the atmosphere, the Moon's diameter being 117mm. Some of the polar rays do not coincide very well with these elevations of the chromosphere. In this case it is likely they originate, either in front of or behind the chromospheric limb, and apparently emerge from it only by projection.

The larger irregularities, or prominences, seem to interrupt the flow of coronal matter, which, in several instances, arches over them, forming hoods or envelopes, often with a dark arch between. From a study of these negatives it seems likely that when there is an upheaval of the chromosphere into billows, or in the process of prominence formation, matter in a finely divided or nebulous state is projected outward from their crests. When the eruption has reached a certain development, which is, perhaps, the limit of progressive activity, coronal matter from this part ceases to be given off, or is greatly diminished. This is often the case at the prominences where the coronal matter arches over from the chromosphere on either side.

Another important feature shown on these negatives is in connection with a double prominence at position angle 19°, where a coronal ray apparently emerges from behind a dark arch. This prominence is surmounted by a well-defined arch, which is manifestly composed of less luminous matter than the corona in its immediate vicinity. It is evidently not due to the scarcity of coronal matter. One of the rays composing the great wing in the northeast quadrant apparently emerges from behind this dark arch, extending throughout its whole southern side, and nearly to its top, while the arch maintains its dark appearance uniformly throughout its entire extent.

The corona on the western side of the Sun presents no unusual features, especially in its inner parts. Its structure corresponds, essentially, to that of the preceding year. On the eastern side, however, numerous and remarkable forms are exhibited, thus making a contrast between the two sides which is quite con-

spicuous. Briefly, these features consist of a great wing in the northeast quadrant, composed of a number of well-defined streamers, several of which are in closely associated pairs, extending over a series of medium-sized prominences. Another remarkable object is a coronal disturbance, near the solar equator, apparently the seat of a violent disruptive force, the coronal details resembling, to some extent, the structure seen in photographs of the *Orion* nebula. Attention is also attracted by a series of coronal arches, in the southeast quadrant, separated by darker concentric spaces.

While the north polar rays are apparently symmetrical with respect to the pole of rotation, those at the south pole are excentrically located, the displacement amounting to about 8° toward the east. At this latter point the rays are, moreover, projected directly away from the Sun. Considering the greater coronal activity on the eastern side of the Sun than on the western, this displacement is a striking coincidence.

These and other interesting features of the corona will be treated more at length in the official report, shortly to be issued in the publications of the United States Naval Observatory.

U. S. NAVAL OBSERVATORY,
June 14, 1902.

NOTE ON THE CONCAVE GRATING.

By H. C. Plummer.

In the case of a grating ruled with equal spaces on any concave surface the simple theory gives the familiar formula

$$N\lambda = \sigma\,(\sin\phi - \sin\psi)\,,$$

where ϕ and ψ are the angles, measured in opposite directions, between the normal at the center of the grating and the directions of the source of light and the focus, λ is the wave-length, N the order of the spectrum, and σ the grating space.

There must always, however, be a certain amount of aberration, and this is most easily estimated by calculating the retardation of the light diffracted at the extreme line of the grating as compared with that diffracted at the center. For the spherical grating this has been done by Glazebrook (1) in the case where the spaces are equal along the arc;[1] (2) in the case where the spaces are equal along the chord.[2] The former investigation is interesting as showing a disadvantageous way of ruling a grating. The latter shows that the retardation depends on the fourth power of the width of the grating.

The same method can be applied without difficulty and with the same degree of approximation to a grating ruled on any surface. As this does not seem to have been done, and the results in regard to the general degree of approximation appear to be not without interest, the investigation is now given.

The discussion is confined to the plane perpendicular to the lines of the grating. The curve of the grating, referred to the normal and tangent at the center as axes, may be represented by

$$x = ay^2 + by^3 + cy^4\,,$$

higher powers of y being beyond the degree of approximation contemplated. Let the co-ordinates of Q, the source of light, be $u\cos\phi$, $u\sin\phi$, and of Q_1, the focus, be $u'\cos\psi$, $-u'\sin\psi$.

[1] *Phil. Mag.*, **15**, 414, 1883. [2] *Phil. Mag.*, **16**, 377, 1883.

Also let (x, y) be the point P on the extreme line of the grating. Then

$$PQ = \{(u\cos\phi - x)^2 + (u\sin\phi - y)^2\}^{\frac{1}{2}}$$

$$= \{u^2 - 2uy\sin\phi - 2ux\cos\phi + y^2 + x^2\}^{\frac{1}{2}}$$

$$= \{u^2 - 2uy\sin\phi + y^2(1 - 2ua\cos\phi) - 2ub\cos\phi\,.\,y^3 + (a^2 - 2uc\cos\phi)y^4\}^{\frac{1}{2}}$$

$$= u - y\sin\phi + y^2\frac{2}{2u}(1 - 2ua\cos\phi) - b\cos\phi\,.\,y^3 + \frac{1}{2u}(a^2 - 2uc\cos\phi)y^4 - \frac{1}{8u^3}\{4u^2y^2\sin^2\phi - 4u\sin\phi(1 - 2ua\cos\phi)y^3 + [(1 - 2ua\cos\phi)^2 + 8u^2b\sin\phi\cos\phi]y^4\} + \frac{1}{16u^5}\{-8u^3y^3\sin^3\phi + 12u^2\sin^2\phi(1 - 2ua\cos\phi)y^4\} - \frac{5}{8u^3}y^4\sin^4\phi\,.$$

Herein powers of y above the fourth have been consistently suppressed. The result, by collecting the several terms, may be written:

$$PQ = u - y\sin\phi + y^2\cos\phi\left(\frac{\cos\phi}{2u} - a\right) + y^3\cos\phi\left\{\frac{\sin\phi}{u}\left(\frac{\cos\phi}{2u} - a\right) - b\right\} + y^4\left\{\frac{a^2}{2u} - c\cos\phi - \frac{b}{u}\sin\phi\cos\phi - \frac{\cos\phi}{4u^2}\left(\frac{\cos\phi}{2u} - a\right)\left(1 - 2ua\cos\phi - 5\sin^2\phi\right)\right\}.$$

The corresponding expression for PQ_1 is obtained at once by substituting u' for u and $-\psi$ for ψ in the preceding expression.

Hence it is seen that if the fourth power of the width of the grating be neglected,

$$PQ + PQ_1 = u + u' - nN\lambda$$

and

$$nN\lambda = y(\sin\phi - \sin\psi)\,,$$

when

$$\frac{\cos\phi}{2u} = a\,,\quad \frac{\cos\psi}{2u'} = a \quad \text{and} \quad b = 0.$$

Now, if ρ is the radius of curvature of the grating at the origin, $a = \frac{1}{2\rho}$, so that these conditions may be written

$$u = \rho\cos\phi\,,\quad u' = \rho\cos\psi \quad \text{and} \quad b = 0\,.$$

The first two are of course satisfied in Rowland's arrangement, in which the source and spectrum are on the circle described on the radius of curvature. The third condition merely implies that the curve of the grating is symmetrical with respect to the axis.

The term neglected in $PQ + PQ_1$ is now considerably simplified by these conditions, and can now be written

$$\delta = y^4 \left(\frac{a^2}{2u} - c \cos \phi + \frac{a^2}{2u'} - c \cos \psi \right)$$

$$= y^4 \left\{ \frac{a^2}{2} \left(\frac{1}{u} + \frac{1}{u'} \right) - 2ac \, (u + u') \right\}$$

$$= y^4 \left\{ a^3 \, (\sec \phi + \sec \psi) - c \, (\cos \phi + \cos \psi) \right\} .$$

As an example, let the grating be ruled on a spherical surface of radius R. The equation of the grating curve is

$$x = R - (R^2 - y^2)^{\frac{1}{2}}$$

$$= \frac{y^2}{2R} + \frac{y^4}{8R^3} ,$$

so that

$$a = \frac{1}{2R} , \quad c = \frac{1}{8R^3} .$$

Hence

$$\delta = \frac{y^4}{8R^3} \left\{ (\sec \phi + \sec \psi) - (\cos \phi + \cos \psi) \right\}$$

$$= \frac{y^4}{8R^3} (\sin \phi \tan \phi + \sin \psi \tan \psi) ,$$

which is equivalent to the expression given by Glazebrook.

As a second example, let the curve of the grating be parabolic. In this case $c = 0$, and it follows immediately that

$$\delta = \frac{y^4}{8R^3} (\sec \phi + \sec \psi) ,$$

where R is the radius of curvature. It appears therefore that a parabolic surface would be inferior to a spherical surface.

University Observatory, Oxford,
July 4, 1902

WAVE-LENGTHS OF CERTAIN LINES OF THE SECOND SPECTRUM OF HYDROGEN.

By Edwin B. Frost.

The attachment for producing the comparison spectrum in the Bruce spectrograph permits the use of a small vacuum tube when desired, in addition to the spark between metallic terminals. Tubes of a special shape were ordered from F. Müller, successor to Gressler, of Bonn, to be filled with helium.

The tube first tested showed the principal lines of helium, and, on longer exposure, a number of other lines, which proved to be those of the so-called "compound" line or second spectrum of hydrogen. These lines were fine and well suited for service as comparison lines, provided their wave-lengths were known with sufficient accuracy. This, however, is not the case: Hasselberg's wave-lengths were determined before Rowland's work had begun, and when standard lines were lacking; and Ames' later determinations of wave-lengths in the spectrum of hydrogen include but very few of the lines of the second spectrum.

I accordingly took last year a few plates of this spectrum, with comparison spectra of titanium and iron, and have thought it might be of service to others if the wave-lengths measured in the range of spectrum covered by the shorter camera (A) of the Bruce spectrograph were published.

The focal length of the camera is 456 mm, and the dispersion of the three prisms is such that at the wave-length of minimum deviation, $\lambda = 4481$, 1 mm = 13.21 tenth meters (Jena prisms); or, with the Mantois prisms first used, 1 mm = 13.86 tenth-meters. The tube in question was last autumn in such a condition as to bring out the principal lines clearly with an exposure of 60 seconds. Unfortunately it is now extinct, and as none of the other helium tubes have yielded this spectrum, we can at present obtain no new plates. It would have been desirable to obtain

some plates with camera B, of 607 mm focus, which gives sharper definition at the center of the field than does camera A. I have measured four plates: A 84 and 85, taken with the Mantois prisms, and A 210 and 267, taken with the Jena prisms. At my request Mr. Adams was kind enough to measure independently plate A 210.

The measurements were made with three different comparators, using different threads in different cases, and with the plate under the microscope in the position of violet toward left in some instances and violet toward right in others. The reductions to wave-length were made by Hartmann's simple formula (without exponent), and depend upon the wave-lengths of the titanium lines as given in Rowland's solar spectrum tables. The fit of the formula is checked and corrected at numerous points by the comparison lines measured.

The data as to the plates are as follows; Titanium lines measured and used in the reductions: on A 210, by Frost, 30, by Adams, 28; on A 84, 20; on A 85, 8; on A 267, 19. The three helium lines at $\lambda\lambda$ 4388, 4471 and 4713 were also measured and used in the reductions. On A 210 Adams also employed 7 iron lines in the reductions. The number of formulæ used on each plate, each covering a small section of the spectrum, were: A 210 by Frost, 3; by Adams, 2; A 84, 3; A 85, 1; A 267, 2 formulæ.

A 210 was measured in each case with violet to right, and with a single thread; A 84 with violet to left, and with a double thread; A 85 with violet to right and single thread; A 267 with violet to left and double thread. The double thread doubtless largely eliminates the physiological effect, which should here be small in any event, as all the lines are emission lines and dark on the negative. If these plates had originally been taken and measured with a view to the most precise determinations of wave-length possible with the instruments used, a more uniform procedure would have been followed; but in view of the variety of conditions described above, the mean of the different results should probably be fairly freed from individual peculiarities of the

different plates. Four settings were regularly made on each line, but on A 85, for which the exposure was such as to make the stronger lines very sharp, only two settings were ordinarily made on the hydrogen lines.

Although the measures from the different plates are doubtless entitled to different weights, the assignment of weights would be quite arbitrary, and I accordingly give the simple mean from the five measures of four plates. This gives to plate A 210 a double weight, which is proper, as it is the best spectrum for measurement, although not yielding nearly as many lines as the longer exposed plate A 84.

The intensities of the lines were roughly estimated when the settings were made, on a scale in which a fairly distinct line is recorded as 10 and the very faintest lines visible as 2 or 3. Inasmuch as the different plates were of very different degrees of strength these estimated intensities have very different absolute values for the different plates. Moreover A 84 and A 267 were only recently measured, a year later than the other two plates, so that the scale would presumably change greatly. The estimates on A 84 would be the nearest to the truth, as this plate includes many faint lines invisible on the other plates.

In the following table *d* denotes double, *n* denotes nebulous; *cv* signifies that there is a *comes* on the violet side, either too close or too faint for measurement; *cr* that there is a *comes* on the side toward the red end of the spectrum.

A comparison of these results with those of other observers is not given above, as Ames in his paper on the hydrogen spectrum (*Phil. Mag.*, **30**, 33, 1890) measured only 9 lines in the range of spectrum included above. For the 5 lines whose wavelengths are given by Ames to the hundredth of the tenth-meter, the differences F.—A. are as follows: λ 4412.47, +0.12; 4447.77, −0.08; 4498.69, −0.06; 4634.21, +0.06; 4683.97, +0.22.

Hasselberg's values[1] depend on Ångström's scale, hence are

[1] *Bull. Acad. Imp.* St. Petersburg, **11**, 307, 1880; **12**, 203, 1884; *Mem. Acad. Imp.* St. Petersburg, **30**, 1882; **31**, 1883. I have not had access to these papers, but have only examined his values as given in the "Index of Spectra" of Watts.

WAVE-LENGTHS OF LINES IN THE "SECOND SPECTRUM" OF HYDROGEN.

A 210 (F.)		A 210 (A.)	A 84		A 85		A 267		Mean
4358.51	8	4358.52	4358.51	15					4358.51
			4386.41	5					4386.4
4391.14	6	4391.15	4391.09	4					4391.13
			4391.90	3					4391.9
			4393.02	5					4393.0
			4398.28	2					4398.3
			4400.94	2					4400.9
			4401.79	2					4401.8
4412.48	12	4412.50	4412.44	15	4412.45	12	4412.47	8	4412.47
4413.70	3	4413.75	4413.65	2					4413.70
4414.43	5	4414.49	4414.38	6	4414.48*cr*	6	4414.38	3	4414.43
			4415.16	4			4415.16	1	4415.16
			4416.43	2					4416.4
4417.49	8	4417.48*d* ?	4417.49	12	4417.51	8	4417.50	7	4417.49
			4422.86	3					4422.9
			4423.43	2					4423.4
			4426.13	5					4426.1
4437.77	6								4437.8
			4445.44	7					4445.4
4447.83*nr*	7	4447.80*d*	4447.74*d*	15	4447.74	7	4447.73*cr*	4	4447.77
			4450.11	3					4450.1
			4453.22	2					4453.2
			4456.91	8					4456.9
			4458.97	6					4459.0
4461.17	12	4461.17	4461.14	20	4461.13	15	4461.14	10	4461.15
			4464.19*d*	3					4464.2
4467.36	4	4467.33	4467.33	6			4467.31	3	4467.33
4474.42	4	4474.44	4474.41	8			4474.43	4	4474.42
			4477.24	3					4477.2
			4482.19	3					4482.2
			4486.21*cv*	8			4486.21	3	4486.21
4488.05	7	4488.02	4488.00	12	4487.98	6	4487.98	5	4488.01
4490.65	9	4490.59	4490.68	15	4490.66	7	4490.63	6	4490.65
4493.83	2		4493.87	8			4493.83	2	4493.84
			4495.57	2					4495.6
4498.18	3	4498.30	4498.25	12	4498.28	5	4498.48	5	4498.25
4498.70	5	4498.74	4498.66	12	4498.67	7		5	4498.69
			4502.17*cr*	6					4502.2
			4503.96	6					4504.0
			4505.16	5					4505.2
4505.80	5	4505.81	4505.86*cr*	7	4505.73	3	4505.78	3	4505.80
			4507.05	7					4507.0
			4509.12	4					4509.1
			4510.98	12					4511.0
4514.50	3	4514.49	4514.47	5			4514.44	2	4514.48
4515.70			4515.72	2					4515.71
			4520.17	5					4520.2
			4521.62	3					4521.6
		4524.27	4527.36	12			4524.23	4	4524.29
			4529.37	4					4529.4
			4532.18	2					4532.2
			4533.30	4					4533.3

WAVE-LENGTHS OF LINES IN THE "SECOND SPECTRUM" OF HYDROGEN (*continued*).

A 210 F		A 210 A	A 84		A 45		A 267		Mean
			4534.61*nd*	10/10					4534.6
4538.42	4	4538.42	4538.46	10	4538.49	4	4538.36	2	4538.43
			4539.34	3			4539.26	2	4539.30
4543.92	3	4543.92	4543.90	12	4543.87	2	4543.86	3	4543.89
			4550.31	3					4550.3
4551.15	6	4551.11	4551.14	18	4551.16	15	4551.18	6	4551.15
			4554.32	7			4554.25	2	4554.28
			4557.46	3					4557.46
			4558.79*n*	15	4558.78*d?*		4558.67*dn*		4558.75
			4562.45	3					4562.4
			4563.94	8	4563.97	3			4563.96
4568.38	13	4568.33	4568.37	25	4568.37	18	4568.28	10	4568.35
4572.93	15	4573.02	4572.96	35	4572.91	16	4572.87	8	4572.94
4576.09*n*	4	4576.12	4576.11	10	4576.08	8	4576.02	3	4576.08
4578.20	5	4578.30	4578.27	12	4578.19	6	4578.14	4	4578.22
4579.65	2	4579.76							4579.70
4580.20	15	4580.28	4580.24*cv*	20	4580.16	20	4580.15	12	4580.21
4581.77	6	4581.78	4581.74	9	4581.74	7	4581.71	3	4581.75
4582.81	8	4582.81	4582.80		4582.76	12	4582.74	7	4582.78
			4598.73	6					4598.7
4607.57	6	4607.58	4607.65	12	4607.61	4	4607.51	3	4607.58
4618.48	5	4618.46	4618.52	15	4618.47	5	4618.41	6	4618.47
4625.53*n*	7	4625.58	4625.54	15	4625.55*n*	6	4625.47	4	4625.53
4628.19	9	4628.15	4628.17	20	4628.22	12	4628.12	10	4628.17
4632.01*nv*	13	4632.10*cv*	4631.97	80	4632.09*d*	15	4631.91*n*	20	4632.02
4634.16	15	4634.23	4634.21	50	4634.24*d*	15	4634.19*cr*	15	4634.21
4634.78	8	4634.79	4634.74	20					4634.77
			4645.51	4					4645.5
		4653.07	4653.23	12	4653.25	4	4653.16	3	4653.18
			4660.57	3					4660.6
			4661.63	5	4661.66	2			4661.64
4662.97	8	4662.92	4663.01	15	4662.92	8	4662.92	2	4662.95
			4671.47	5					4671.5
			4679.29	2					4679.3
4683.96	7	4683.92	4683.95	12	4684.06				4683.97
			4690.29	8					4690.3
4719.17	12	4719.1	4719.09	8	4719.1		4719.1		4719.11
4723.13	15	4723.1	4725.15	15	4723.2		4723.2		4723.16

Additional lines, seen but not measured: Plate A 84. Six pairs of faint lines precede λ 4386; 2 lines between λ 4388 and λ 4391; faint pair between λ 4417 and λ 4422; faint pair near λ 4437.

not directly comparable; but after applying systematic corrections to his scale, 70 of the 90 lines given above can be certainly identified with 70 of the 107 lines he measured (to tenths of a tenth-meter) in this region. The difference between our values

is in most cases not over 0.3 tenth-meters, and only occasionally rises as high as 0.6 tenth-meters.

Of course no guaranty can be given that the lines yielded by our helium tube are all to be assigned to the second spectrum of hydrogen, but the comparison with the results of Hasselberg indicates that at least the great majority were due to the same substance in the two cases.

It seems singular that this spectrum should not be more in evidence in celestial spectra. There are a few possible coincidences with lines of the chromospheric spectrum, and with lines in spectra of temporary stars. Of the five principal coronal lines, at $\lambda\lambda$ 3987, 4231, 4359, 4568, and 5303, all but the second fall near to the positions of lines observed in the second spectrum of hydrogen, but at present it cannot be asserted that this is more that an accidental coincidence.

YERKES OBSERVATORY,
August 21, 1902.

OBSERVATIONS ON THE MAGNETIC ROTATION OF THE PLANE OF POLARIZATION IN THE INTERIOR OF AN ABSORPTION BAND.[1]

By P. Zeeman.

1. The difficulties of a complete theory of emission are partly avoided in a treatment beginning with the absorption, and this may have been the reason why Voigt[2] has followed this procedure, though it must be granted that in his method an explanation of the mechanism of the phenomena as in Lorentz's theory cannot be given.[3] In Voigt's theory the separation of a spectral line by the action of a magnetic field is found as the separation of an absorption line.

Some particulars in this separation were anticipated by this theory[4] and confirmed by experiment.[5]

The long known phenomenon of the rotation of the plane of polarization and the magnetic separation of the spectral lines were closely connected.[6]

One result, however, of Voigt's[7] theory relating to the rotation of the plane of polarization in the interior of an absorption band seemed to be in contradiction with the results of Corbino[8] or at least were not confirmed by the experiments of Schmauss.[9]

[1] From the Proceedings of the Royal Academy of Sciences of Amsterdam, meeting of Saturday, May 31, 1902.

[2] *Wied. Ann.*, **67**, 345, 1899.

[3] For a comparison of the advantages of the theories of Lorentz and of Voigt, see Lorentz, Rapports, congrès, Paris, **3**, 16, 33, 1900; *Phys. Zeitschr.*, **1**, 39, 1899; *cf.* also Planck, *Sitz.ber. Ak. Berlin*, p. 470, 1902.

[4] Voigt, *Annalen der Physik*, **1**, 376, 1900.

[5] Zeeman, *Versl. Akad. Amsterdam*, December. 1899; *Archiv. Néerl.* (2). **5**. 237.

[6] *Cf.* also Larmor, "Æther and Matter," p. 203.

[7] *Ann. der Physik*, (4), **6**, 784, 1901.

[8] *Atti R. Acc. dei Lincei.* **10**, 137, 1901: *Nuovo Cimento*, February, 1902.

[9] *Ann. der Physik*, **2**. 280. 1900.

The theory of Voigt requires a negative [1] rotation of the plane of polarization in the interior of an absorption band; Corbino, however, only succeeded in observing a very small positive rotation.

It would be very remarkable, however, if there existed a disagreement between theory and observation in this special field so closely connected with other well-understood phenomena.

I have been experimenting for some time on this subject. In performing these experiments I have been aided in an excellent manner by Mr. Hallo.

I have succeeded in observing a negative rotation in the interior of an absorption band, the results of my observations being in perfect *qualitative* agreement with Voigt's theory.

2. The method used in the following observations on the rotation in sodium vapor is in principle the same as that which has been used by Voigt [2] in his demonstration of the double refraction of sodium vapor placed in a magnetic field. Hussel [3] had already used it in a determination of the natural rotation of the plane of polarization in quartz, and also Corbino in his first experiments on sodium.

By means of a system of quartz prisms (as has been used by Fresnel in his experiment on the division of a plane-polarized ray into two circularly polarized rays) a number of horizontal interference fringes are formed in a spectrum. The light traverses the prism in the direction of the axis, and the edges are horizontal and perpendicular to the slit of the spectroscope. The prism system (length 50 mm) was placed in my experiments as near as possible before the slit of spectral apparatus and a small Nicol, used as analyzer, behind the slit. The polarizing Nicol was placed, of course, before the electro-magnet (of the Ruhmkorff type). The spectroscope was a Rowland grating, for which I am indebted to the kindness of the directors of the Dutch Society of Sciences at Harlem. It has a radius of 6.5 meters, 10,000 lines to the inch, and a divided surface of nearly 14 cm.

[1] The magnetic rotation in the vicinity of the band is positive in sodium vapor.

[2] *Wied. Ann.*, 67, 360, 1899. [3] *Wied. Ann.*, 43, 498, 1891.

The grating was mounted for parallel light in the manner indicated by Runge and Paschen.[1] The source of light was in most cases the electric arc, in some the Sun.

Using this arrangement of the experiment we can deduce immediately from the deformation of the interference fringes in the neighborhood of the absorption bands, when the sodium vapor is under the action of the magnetic field, the value of the rotation of the plane of polarization for different wave-lengths. Fig. 1 of the plate gives an idea of the aspect of the fringes in absence of the field in the neighborhood of the sodium lines, considerable sodium being present in the flame between the poles. The observations were made in the second order.

3. In the experiment first to be described, the distance between the perforated poles was about 4 mm and the intensity of the field about 15,000 c. g. s. units. In this field was placed a gas flame fed with oxygen, and a small quantity of sodium was introduced in it by means of a glass rod. After removal of the polarizer and of the Fresnel prism the two doublets, in which the sodium lines are separated, in the inverse magnetic spectral effect were observed. Between the components of the doublet were seen the very narrow reversed sodium lines due to the arc light itself.

The polarizer and the prism were now introduced in their proper places. The field of view was then crossed by the above-mentioned (2) dark, nearly horizontal interference fringes.

I now wished to ascertain the deformation of the fringes by increasing continuously the quantity of sodium vapor, the field remaining constant. This method must be preferred for obvious reasons to the other which might have been followed also, viz., the examination of a flame with constant percentage of sodium under varying magnetic intensities.

The following observations refer to D_1:

If the quantity of sodium in the magnetic field was only extremely small, the interference fringe exhibited at the place of the reversed sodium line a protuberance — let us say *down-*

[1] KAYSER, *Handbuch*, 1, 482.

ward—the lines of the doublet being somewhat stronger just above the interference fringe. In Fig. 1 this behavior is represented schematically.

Increasing now the quantity of sodium (always remaining very small, however, absolutely) the interference fringes moved upward along the components of the doublet, whereas the part of the fringe between the components seemed no longer connected to the exterior fringes and assumed the shape figured schematically in Fig. 2.

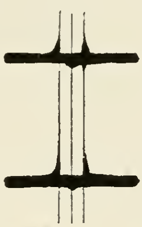


FIG. 1.

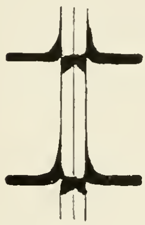


FIG. 2.

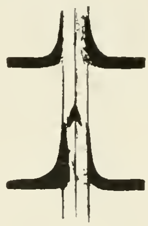


FIG. 3.

Increasing still further the density of the vapor the interior part of the fringe slid downward with increasing velocity and then resembled an arrow with point directed upward, the parts more removed from the medium line fading away and disappearing (see the schematic Fig. 3). At last the arrow entirely disappeared by the increase of the density of the vapor. It then became impossible to distinguish the fringes or any trace of structure in the field between the components. Considerable light was transmitted. The entire width of the components of the doublet was now about of the same order as the distance of their central lines.

A further increase of the quantity of sodium obscured the central part more and more (see below (8)).

The exterior fringes moved continuously upward while the density was being increased.

In a field of about 20,000 units the downward displacement could be followed over a distance of more than the double of the distance between two fringes, corresponding to a *negative* rotation of over $2 \times 180^\circ$, say 400°. The distance between the poles was 4 mm.

Some more accurate data will be given on another occasion.

In the case of D_2 the phenomena were in the main of the same character.

For D_2 it was, however, characteristic that the stage of the nearly or entirely vanishing of the interior fringes was reached with smaller field, whereas also the shape of the interior fringe differed from the one observed in the case of D_1. Hence there exists also in this case a difference between D_1 and D_2, a difference already known to exist in the phenomena of reversal, of the separation by a magnetic field, and of the rotation of the plane of polarization in the vicinity of the absorption band.

4. It appeared possible to keep each of the stages described in (3) stationary during a considerable time. Excellent photographs could be secured with plates which were sensitized for yellow light with erythrosine silver. Instead of the gas flame fed with oxygen it was easier, in the case of greater distances between the poles, to use a Bunsen burner wherein common salt was introduced.

5. If the density of the vapor was maintained as constant as possible and if it and the field intensity corresponded to the circumstances represented in Fig. 3 (3) then an *increase* of the field gave a motion of the arrow (Fig. 3) (3) upwards, corresponding to a *decrease* of the negative rotation and reciprocally. It was possible to observe by eye observation very clearly this decrease when the field was changed, *e.g.*, from 18,000 to 25,000. If the circumstances were more in accordance with Fig. 2 (3) then the same change of field produced a change only just perceptible of the negative rotation but in the same sense as mentioned in the case of Fig. 3.

An enlarged reproduction of one of the photographs is shown in Fig. 2 of the plate. The distance between the poles in this experiment was 6.3 mm, the field intensity about 14,000.[1] The negative rotation in the case of D_1 is somewhat less than 90°. In the case of D_2 only some traces of the interior fringes can yet be seen (3). The negative rotation is about 180°. In the photograph are seen also the reversed very narrow D_1 line and

[1] The intensities of the field were measured by means of a bismuth spiral in the center of the field. Probably the values given are somewhat too high. Measurements of the magnetic change of the spectral lines give lower values.

the broader D_2 line, which are due to the arc itself and have nothing to do with our subject.

6. The observations (3, 4, 5) agree qualitatively in an excellent manner with the conclusions from Voigt's theory. According to it, the negative rotation must be of the same order of magnitude as the positive one. This last was known from Macaluso's and Corbino's experiments to be very great. The enormous value and the sign of the negative rotation given in (3) may thus be regarded as a beautiful confirmation of the theory.

This is equally the case with the direction (5) of the change of the negative rotation with increasing field. In order to see this we must know the value of the quantity occurring in the theory $P=\frac{cR}{\theta}$ ($R=$ field intensity, c and θ parameters of the absorption band), for which the comparison must take place. It was possible to assign a value to P by comparison of the phenomenon with Voigt's Fig. 1.[1] This figure gives $n\chi_0$ (χ_0 angle of rotation, n a mean value of the index of refraction) as function of a certain variable Δ, whereas our phenomenon is a represention of χ_0 as a function of λ. Reducing the abscissa of the mentioned Fig. 1 to $\frac{1}{20}$ or $\frac{1}{25}$, we obtain diagrams resembling in the main features Fig. 2 of the plate. To the greater observed negative rotation (3) correspond values of P, which can be estimated at 5 or 8. The smallest easily observed rotations in the strong field used are probably in the vicinity of the critical value $P=1.73$.

7. The slope of the exterior interference fringes is greater toward the side of the greater wave-lengths than toward the violet, at least so far as the rotation due to one band does not influence visibly the rotation due to the other. At the same distances, if not very small, of each of the two D lines the rotation at the side of the violet is greatest. The interior fringes also show a slight asymmetry, so, *e.g.*, the point of the arrow in Fig.

[1] *Annalen der Physik*, 6, 789, 1901.

3 (3) ought to be asymmetrical. The part at the side of the violet is predominating.

It is clear that these phenomena depend upon an asymmetry of the dispersion curve.

8. With very dense sodium vapor, hence under circumstances which are beyond the last stage of (3), I observed phenomena very probably identical with those observed by Corbino. In my first experiments with those dense vapors I thought it absolutely necessary for securing sufficient intensity to widen the slit beyond the width used in the experiments already given. I now see, however, that this is unnecessary.

Using these very dense vapors one sees in the absorption band a horizontal part of an interference fringe, which seems to have undergone a very small displacement *upwards* by the action of the field. These horizontal parts are more ill-defined and broader and the whole phenomenon in the bands is darker than under the circumstances described in (3), (4), (5).

Figs. 3 and 4 of the plate will give a clearer impression of the change in the phenomenon than a long description.

Fig. 3 was obtained with a field of 4500 units and much sodium. I have made some measurements, according to a method not to be given here, concerning the displacement of the central (in horizontal and vertical direction) part of the interference fringe, and I have found a displacement which would correspond to a *positive* rotation of about 8° with both D lines. Fig. 4 was taken with a field of 10,700 and much sodium. The exterior interference fringes are very clear and much deformed; the rotation in the parts adjacent to the absorption band exceed 180°. The interior interference fringes are very indistinct. Their appearance would suggest that in the case of D_1 in Fig. 4 the stage reached for D_2 in Fig. 2 has been scarcely surpassed.

This, however, cannot be the case because there was too much sodium in the flame. A comparison with Fig. 2 will show that the lines are much broader in Fig. 4. Measurements taken on other negatives gave me for fields of 11,000, displacements of about $\frac{1}{16}$ of the distance between two fringes corresponding to a

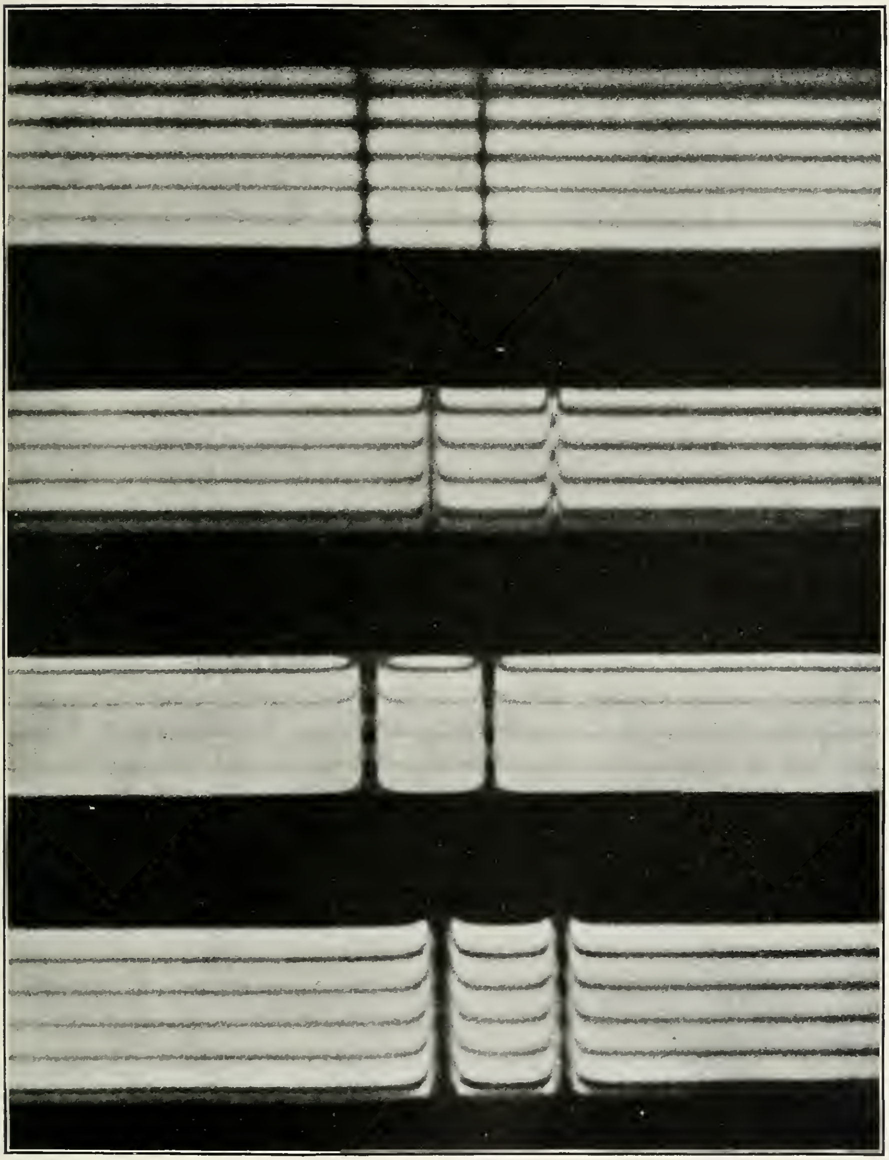
PLATE V

1

2

3

4



D_2 D_1

Magnetic Rotation in the Interior of an Absorption Band (P. Zeeman).

positive rotation of 11°. Hence the displacements in these cases are precisely of the same order of magnitude as in Corbino's experiments. The paleness of the borders of the band is easily accounted for by the remark that there the intensity of one of the circularly polarized rays largely exceeds the other.

I do not believe that these facts are in contradiction with theory. It is true that it requires for very high values of P a value zero for $(n\chi_0)_1$. If we must take as the locus of the fringe the mean vertical height, then really the rotation would be positive. It seems possible that with those broad fringes the case is different. It is also possible that the circumstances assumed in the theory are not wholly realized in the experiments with dense vapors. I am making some new experiments on this subject and therefore shall not discuss further the different possibilities.

EXPLANATION OF THE PLATE.

The plate gives about fourfold enlargements of the photographs.

Fig. 1. Interference fringes and absorption lines in absence of the field and with considerable sodium (2).

Fig. 2. Same lines. Field intensity about 14,000, little sodium (3) (5).

Fig. 3. Same lines. Field intensity about 4,500, much sodium (8).

Fig. 4. Same lines. Field intensity about 10,700, much sodium (8).

Minor Contributions and Notes.

SIX STARS WHOSE VELOCITIES IN THE LINE OF SIGHT ARE VARIABLE.[1]

The following six spectroscopic binaries, discovered with the Mills spectrograph, are additional to the thirty-two binaries already announced:

ϕ *Persei* ($\alpha = 1^h\ 37^m$; $\delta = +\ 50°\ 11'$).

The variable velocity of this star was discovered from the second plate. The observations are:

Date			Velocity	Measured by
1898	September	5	— 2 km	Reese
1900	December	16	+24	Reese
		16	+23	Campbell
1901	October	15	—10	Reese
	November	11	—12	Reese

This star has bright hydrogen lines, its bright $H\beta$ having been discovered by Espin (*Astr. Nach.*, No. 2963). The $H\gamma$ line may perhaps be best described as a comparatively narrow absorption line with very bright borders. The measures refer to the middle of the dark line. No other lines are apparent in the $H\gamma$ region.

η *Geminorum* ($\alpha = 6^h\ 09^m$; $\delta = +\ 22°\ 33'$).

The observations of this star thus far secured are as follows, the variable velocity having been discovered from the third plate:

Date			Velocity	Measured by
1900	January	15	+15.8 km	Reese
		15	+14	Stebbins
	January	21	+15.0	Reese
1901	October	13	+22.1	Reese
	November	6	+20.3	Reese
	December	4	+22.8	Reese
1902	February	2	+25	Reese
		2	+23	Stebbins

γ *Canis Minoris* ($\alpha = 7^h\ 23^m$; $\delta = +\ 9°\ 08'$).

[1] From *Bulletin* No. 20, *Lick Observatory, University of California.*

The observations of this star are:

	Date		Velocity	Measured by
1900	October	29	+44 km	Reese
		29	+44	Stebbins
1901	November	6	+41	Reese
		6	+40	Stebbins
	December	22	+54	Reese
		22	+53	Stebbins
	December	30	+50	Reese

The fourth plate is underexposed.

ζ *Herculis* ($\alpha = 16^h\ 38^m$; $\delta = +\ 31^\circ\ 47'$).

Early observations of the radial velocity of this star were obtained by Bélopolsky at Pulkowa, Campbell at Mount Hamilton, and Newall at Cambridge, as follows:

	Date		Velocity	Measured by
1893	May	18	—68 km	Bélopolsky
		22	—84	Bélopolsky
	June	2	—75	Bélopolsky
		3	—67	Bélopolsky
		4	—66	Bélopolsky
		14	—64	Bélopolsky
		16	—69	Bélopolsky
	Mean		—70.4	
1897	April	29	—69.1	Campbell
1898	May	11	—70.4	Campbell
		23	—70.0	Campbell
	August	19	—70.9	Campbell
	Mean		—70.1	
1897	June	14	—71.4	Newall
1898	May	16	—68.4	Newall
1899	April	29	—74.3	Newall
	Mean		—71.4	

The above observations afforded no evidence whatever of variable velocity.

Recent observations at the Lick Observatory are as follows:

Date			Velocity	Measured by
1901	July	1	—74 km	Wright
		1	—73.9	Reese
	August	6	—75.8	Reese
1902	April	13	—74.2	Reese
	Mean		—74.6	

The velocity has therefore changed about 4^{km} since 1898.

This star is a well-known visual binary, period about thirty-three years.

α Equulei ($\alpha = 21^h\ 11^m$; $\delta = +4°\ 50'$).

The variable velocity of this star was detected from the third of the following observations:

Date			Velocity	Measured by
1900	June	25	—26 km	Wright
	July	18	—22	Wright
1901	June	25	— 2	Wright
	September	1	—14	Wright
	October	15	—12	Reese
1902	June	2	—26	Stebbins

This star has a composite spectrum, discovered by Miss Maury, of Harvard College Observatory.

o Andromedae ($\alpha = 22^h\ 57^m$; $\delta = +41°\ 47'$).

The following observations of this star have been secured:

Date			Velocicy	Measured by
1900	October	9	—11 km	Wright
	December	17	—15	Campbell
		17	—17	Wright
1901	June	25	—20	Wright
	August	12	—12	Reese

These measures depend entirely upon the excellent $H\gamma$ line.

This star has a composite spectrum, discovered likewise by Miss Maury.

Before the discovery of the thirty-eight spectroscopic binaries with the Mills spectrograph, three had been discovered in the same list of stars by Bélopolsky, making forty-one binaries in about 350 stars observed. The proportion is therefore one binary star for every eight

observed, not taking into account a considerable list of suspected cases awaiting confirmation. The variable velocity of our Sun, due to its revolving planets, has a double amplitude of only a few hundredths of a kilometer. As the work progresses, and the degree of accuracy attainable increases, we shall probably find that there is a regular gradation of double amplitudes from that of the Sun up to those of the spectroscopic binaries already discovered, and it is possible that the star which is not a spectroscopic binary will prove to be the rare exception.

Acknowledgments are due to Messrs. Wright and Reese for continued efficient assistance in the line of sight work.

W. W. Campbell.

July 1, 1902.

OBSERVATIONS ON ζ *GEMINORUM.*

From March 10 to May 23, 1902, forty-two observations were made by Argelander's method on ζ *Geminorum.* Only a preliminary reduction is here attempted. The following light scale was deduced from the observations and used in the reduction. $\delta = 10.0$, $\lambda = 10.1$, $\iota = 11.7$, $\nu = 13.7$, $\sigma = 14.7$. The observations make it appear probable that a secondary maximum occurs $3^{d}.0$ before the principal maximum and that it attains the brightness of 3.88 mag. A secondary minimum is also indicated $1^{d}.6$ before the principal maximum. It has a magnitude of about 3.93. The following observed minima are compared with an ephemeris computed from the elements given in Chandler's *Third Catalogue of Variable Stars.* The minima are given in the Julian Day and G. M. T. The weights are proportional to the number of observations.

Obs. Min.	Wt.	C.-O.
2415823.18	2	+ 0.89
32.89	3	+ 1.33
53.83	2	+ 0.70
63.66	2	+ 1.02
73.81	2	+ 1.02
83.81	1	+ 1.18

The mean by weights gives a correction to the ephemeris of $+1^{d}.04 \pm 0^{d}.06$.

F. P. McDermott, Jr.

Princeton University,
June 1902.

ON THE RADIATION OF MERCURY IN THE MAGNETIC FIELD.

In the list of mercury lines whose separation in the magnetic field we measured,[1] the lines λλ 3655.00, 3650.31, 3027.66, 3025.79 belonging to the first secondary series were by accident not printed, and we did not notice the omission when reading the proof sheets. We ask permission herewith to supplement the omitted measurements:

Wave-lengths	Oscillations parallel to the lines of force	Oscillations vertical to the lines of force	Mean error	Intensity	Δλ	$-\Delta\lambda\ \lambda^2$	
		5.265		3	+265	−1.98	
		5.208		4	+203	−1.56	
	5.181			2	+181	−1.35	
		5.159		4	+159	−1.19	
	5.108			5	+108	−0.81	Possibly the same wave-length.
		5.102		3	+102	−0.76	
	5.049			3	+ 49	−0.37	
3655.00			0.0019				
satellite	4.949			3	− 51	+0.38	
		4.897		3	−103	+0.77	Possibly the same wave-length.
	4.891			5	−109	+0.82	
		4.846		4	−154	+1.15	
	4.822			2	−178	+1.33	
		4.789		4	−211	+1.58	
		4.734		3	−266	+1.99	
		0.480	0.0056	5	+170	−1.28	On some of the plates the components oscillating parallel to the lines of force are not separated. It may be that the separation consists only in an absorption of the middle line. All four components are much thicker than those of the other lines.
3650.31	0.353		0.0071	5	+ 43	−0.32	
principal line	0.262		0.0071	5	− 48	+0.36	
		0.141	0.0056	5	−169	+1.27	
3027.66 satellite	7.563	7.563	0.007	1	+ 97	−1.06	
	7.757	7.757		1	− 97	+1.06	
3025.79 satellite	.....	.....	.....	..	.....	......	Not measured because the components were too weak.

C. Runge and F. Paschen.

[1] See Astrophysical Journal, 15, 243, May 1902.

WAVE-LENGTHS OF CERTAIN OXYGEN LINES.

In determining the radial velocities of certain stars having spectra of the *Orion* type we have found that some of the best measurable stellar lines are those due to oxygen. The presence of oxygen lines in the spectra of *β Crucis* and several other southern stars has been demonstrated by McClean[1] and by Gill.[2] The wave-lengths of the oxygen lines in the spark spectrum of air, by Neovius, and by Trowbridge and Hutchins, are given only to the tenth of the tenth-meter, as the hazy character of the lines rendered their precise measurement very difficult on the scale presumably employed. As an uncertainty of 0.1 tenth-meter in the wave-length of the stellar line corresponds to about 6.6 km in the velocity deduced from that line, more accurate values of the wave-lengths of these lines were essential before we could use the corresponding stellar lines. The arrangement of the apparatus for the study of the spectrum of the spark under water and under various other conditions in the spectroscopic laboratory of the Observatory, made it an easy matter to procure plates with the concave grating showing the spark spectrum of different metals with their arc spectrum in juxtaposition. Messrs. Ellerman and Kent were kind enough to take for us such plates of the spectra of iron, titanium, nickel and cobalt. One of the advantages of the use of different elements is the avoidance of the disturbance of the measurements of the air lines by nearby metallic spark lines. Thus the air line at λ 4415.08 cannot be well measured on an *Fe* plate because of the nearness of the strong *Fe* line at λ 4415.293 ; and the case is similar for the oxygen (air) line at λ 4417.12 with the *Ti* spark. The 6.5 meter concave grating gives a scale of about 2.6 tenth-meters per millimeter on the negative, and we were surprised to find how accordantly the settings could be made on the air lines despite their breadth. The measures were made with three different measuring machines by the two observers, working entirely independently and using different comparison lines in many instances. Measures were generally made with both violet to left and violet to right, but they are treated below as separate determinations, no systematic differences being clearly evident between the measures in the two positions.

We did not attempt to measure all the air lines in the range of spectrum we included, but took only those lines which have consider-

[1] *Proc. R. S.*, 62, 418, 1898; "Spectra of Southern Stars," London, 1898.

[2] *Proc. R. S.*, 55, 196, 1899; ASTROPHYSICAL JOURNAL, 10, 272, 1899.

able strength in our stellar spectra and can be used in determining radial velocities.

In the following list the results of the two observers are given separately, with the number of measures made, generally on several different plates. The mean values are weighted according to the number of observations.

WAVE-LENGTHS OF CERTAIN OXYGEN LINES
(SPARK SPECTRUM OF AIR).

Frost		Adams		Weighted mean
λ	No.	λ	No.	λ
4317.27	3	Not measured	..	4317.27
4319.76	3	" "	..	4319.76
4345.67	4	4345.68	8	4345.68
4347.62	4	Not measured	..	4347.62
4348.12	4	4348.14	5	4348.13
4349.53	5	4349.55	8	4349.54
4351.51	3	4351.49	6	4351.50
4366.99	6	4367.03	8	4367.01
4415.08	6	4415.07	6	4415.08
4417.11	6	4417.13	6	4417.12
4447.16	6	4447.17	8	4447.16
4591.08	6	4591.06	6	4591.07
4596.30	6	4596.28	6	4596.29
4601.63	7	4601.63	8	4601.63
4607.32	5	4607.30	8	4607.31
4614.03	4	4614.04	6	4614.03
4621.56	4	4621.54	8	4621.55
4630.71	4	4630.70	8	4630.70
4638.95	4	4638.93	4	4638.94
4641.90	4	4641.87	4	4641.89
4643.25	4	4643.24	4	4643.24
4649.26	4	4649.24	4	4649.25
Not measured	..	4650.93	4	4650.93
4661.72	2	4661.73	4	4661.73

An estimate of the accuracy of these wave-lengths is somewhat difficult, and it did not seem worth while to separately determine the probable errors. For the line at λ 4367, for which the fourteen separate determinations are not in as good accordance as for the average air line, the mean error is ± 0.026; for the average line it would probably not exceed ± 0.02 tenth-meters.

EDWIN B. FROST and WALTER S. ADAMS.

YERKES OBSERVATORY,
August 25, 1902.

THE MIRROR OF THE CROSSLEY REFLECTOR—A CORRECTION.

Professor Keeler's most useful article describing the Crossley Reflector of the Lick Observatory, in the Astrophysical Journal of June 1900, contains the following paragraph :

> The large mirror, the most important part of the telescope, has an aperture of 3 feet, and a focal length of 17 feet 6.1 inches. It was made by Mr. Calver. Its figure is excellent. On cutting off the cone of rays from a star with a knife-edge at focus, according to the method of Foucault, the illumination of the mirror is very uniform. While the star disks, as seen in an ordinary eyepiece, are small and almost perfectly round, they are not, I think, quite so good as the images seen with a large refractor; still, they are very good indeed, as the following observations of double stars made recently for this purpose will show.

A correspondence with Sir Howard Grubb and Mr. J. Gledhill has proven to me that the figuring of this excellent mirror is the work of Sir Howard Grubb. Mr. Crossley's gift to the Lick Observatory included two mirrors essentially of the same diameter and focal length. These were distinguished by the letters A and B. An extract from Mr. Gledhill's letter is as follows :

> When we found that the A mirror was not good, and that the figure of the B was a little worse, we asked Dr. Common and Mr. Wassell (an expert in mirror testing by the Foucault method) to come and test them. They did so. Sir Howard Grubb was then called in, and he examined both mirrors. All the results of the tests agreed with ours. It was then decided to send the B mirror to be re-figured by Sir Howard Grubb ; this was done, and you have it as it came from his workshop.

It is the B mirror which has been used in all the work with the Crossley Reflector at the Lick Observatory.

In justice to Sir Howard Grubb I herewith make the correction which of course the late Professor Keeler would very gladly have made.

W. W. Campbell.

Mt. Hamilton, California,
August 21, 1902.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to uniformly employ the metric units of length and mass; the English equivalents may be added if desired.

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THE

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ON THE SEPARATION OF CORRESPONDING SERIES LINES IN THE MAGNETIC FIELD.[1]

SECOND PAPER.

By C. RUNGE and F. PASCHEN.

In our paper on the radiation of mercury in the magnetic field,[2] and in the first paper on the separation of corresponding series lines in the magnetic field,[3] we have discussed the so-called triplet series, which were discovered by Kayser and Runge, and by Rydberg in the spectra of *Mg*, *Ca*, *Sr*, *Zn*, *Cd*, and *Hg*. In addition to these series there have been found in the spectra of the alkalies as well as of *Cu*, *Ag*, *Al*, *In*, and *Tl* so-called doublet series.[4] It became interesting to investigate whether the same relations as to the separation in the magnetic field held good for these in the manner we have observed in the case of the triplet series.

As regards the alkalies, we did not, indeed, succeed in obtaining a separation of any lines whatever, with the exception of the

[1] Translated from *Sitzungsberichte der K. Akad.*, Berlin; Session of June 26, 1902.

[2] ASTROPHYSICAL JOURNAL, 15, 235, 1902.

[3] *Ibid.*, 15, 333, 1902. [4] See Kayser's *Handbuch der Spectroscopie*, II, 503-573.

D line, which had already been investigated by Cornu in the magnetic field. The other sodium lines were so lacking in sharpness in the spark spectrum that we could not recognize the type of separation in the magnetic field. No better result was found in case of lithium. We confined our observations to the D lines, and to those of *Cu*, *Ag*, *Al*, and *Tl*.

No principal series was found for *Al* and *Tl*, though for *Cu* and *Ag* the principal series is known, although represented by only one member. It appeared that the principal series of *Cu* and *Ag* exhibited precisely the same types as sodium in the magnetic field. And not only is the character of the separation the same, but when represented on the scale of vibration numbers the separation of the components is exactly the same with the same field-strength as in the case of the components of the D lines. We give in the following table the measurements of the distances of the components for the principal series of *Na*, *Cu*, and *Ag*. The measurements are reduced to the same field-strength, as the photographic plates were not made with exactly the same field-strength. We were careful to arrange so that on every plate, lines whose separation in the magnetic field had already been observed by us were also photographed. The lines thus simultaneously photographed were preferably the lines of the second subordinate series of triplets in the spectrum of *Zn*, *Cd*, or *Mg*, the separation of which we discussed in our first paper. These lines are admirably adapted for fixing the field-strength, as their separation is strong, and as the lines can be photographed very sharply with suitable self-induction in the secondary circuit. The two *Zn* and *Cd* lines, for which Mr. Faerber has investigated at the physical laboratory of the University of Tübingen the dependence of the separation upon the field-strength, belong also with these lines. If we make use of his measurements, which he kindly placed at our disposal, the field-strength to which all the measurements in this paper are reduced, is found to be equal to 31000 c. g. s. units. This should deviate from the truth by less than 1 per cent. according to Mr. Faerber's data. But if we depend upon the measurements of field-strength made by

Michelson, by Marchand and Blythswood, and by Reese, upon which we based our figures as to the field-strength in our paper on the radiation of mercury in the magnetic field, and to which our mercury measurements are referred, then we obtain a field-strength of 32000 units for the measurements in the present paper. The value 24600, which we then gave for the field-strength for the mercury lines, would be only 23850 units according to Faerber's measurements. We believe his determinations to be the more reliable.

The numbers in the following table, which contains the separation of the pairs of the principal series in the spectra of *Na*, *Cu*, and *Ag*, for a field-strength of 21000 units, indicate the separations of the components measured from their center of gravity on the scale of vibration numbers (number of vibrations in a path of one centimeter). Positive numbers signify distances in the direction of larger vibration numbers.

PRINCIPAL SERIES.

Na λ5896	*Na*[1] λ5896	*Na*[2] λ5896	*Cu*[2] λ3274	*Cu*[2] λ3274	*Cu*[1] λ3274	*Ag*[2] λ3383	*Ag*[1] λ3383	Mean	Remarks
−1.88	......	−1.84	−1.85	−1.88	......	−1.86	......	−1.86*s*	*s* denotes that vibrations were perpendicular to lines of force.
−0.90	−0.94	......	......	.....	−0.97	....	−0.97	−0.93*p*	*p* that vibrations were parallel to lines of force.
+0.93	+0.94	......	......	......	+0.97	......	+0.97	+0.94*p*	
+1.85	.....	+1.84	+1.85	+1.88	......	+1.86	......	+1.85*s*	
Na λ5890	*Na*[1] λ5890	*Na*[2] λ5890	*Cu*[2] λ3248		*Cu*[1] λ3248	*Ag*[2] λ3281	*Ag*[1] λ3281		
−2.28	......	−2.17	−2.34	......	......	−2.34	.....	−2.25*s*	
−1.43	.....	−1.38	−1.40	......	......	−1.39	......	−1.40*s*	
−0.47	−0.45		......		−0.40	......	−0.45	−0.45*p*	
+0.46	+0.45	......	.....	......	+0.40	......	+0.45	+0.45*p*	
+1.35	......	+1.33	+1.38	......	......	+1.39	......	+1.35*s*	
+2.36	......	+2.22	+2.36	......	......	+2.34	......	+2.30*s*	

For the plates designated by [1] the vibrations perpendicular to the lines of force, for those designated by [2] the vibrations parallel to the lines of force were cut out by a calc-spar. It is difficult to separate all of the components for the small wave-lengths of the *Cu* and *Ag* lines without this procedure.

The components (*p*) vibrating parallel to the lines of force are the strongest. The neighboring perpendicular vibrations

next follow. The extreme components, occurring for the smaller wave-lengths, are decidedly fainter.

The table shows that the separation of the three pairs of lines is identical within the accuracy of the observations. In taking the means triple weight has been given to the distances of the components of the sodium lines. The mean error of a measure of weight 1 is found from the deviations from the mean to be 0.056. On the scale of wave-lengths this corresponds to about 0.006 tenth-meters for the copper and silver lines. This can very well be ascribed to the inaccuracy of the observations.

The distances here exhibit a sort of law of multiple proportions similarly to the case of the components in the second subordinate triplet series. They are nearly equal to the even and odd multiples of a certain number.

		Distances of Components		Differences	
2×0.459=0.918	- -	−0.93	+0.94	0.01	0.02
4×0.459=1.836	- -	−1.86	+1.85	0.02	0.01
1×0.459=0.459	- -	−0.45	+0.45	0.01	0.01
3×0.459=1.377	- -	−1.40	+1.35	0.02	0.03
5×0.459=2.295	- -	−2.25	+2.30	0.04	0.01

The means of the above table have a mean error of 0.02, according to the deviations of the observations from the means. A mean error of 0.02 is also obtained from the differences of this table. Within the accuracy of the measurements, the distances therefore agree with the multiples of 0.459.

The distances of the components of the second subordinate triplet series are similarly multiples of a certain number, as we showed in our investigations of the radiation of mercury in the magnetic field and in our first paper on the separation of the series lines. For the same field-strength as that of the present measures that number would be 0.702. It is perhaps not due to chance but ultimately to a constant charge of the ions that these two numbers also bear a simple ratio to each other, for we have, nearly, 0.459:0.702 = 2 : 3.

As was found by Rydberg, the principal series stands in a close relation to the second subordinate series, which among

other things necessitates that the lines of a pair of the second subordinate series correspond in inverse order to the lines of a pair of the principal series. It could therefore be foreseen that the separation in the magnetic field would bring to light the same relation, and this has been confirmed by the fact. The two lines of the pairs of the second subordinate series are separated in exactly the same way as the two lines of the pairs of the principal series, both in respect to the relative intensities and to the distances of the components; but the smaller wave-length for the one is separated just like the larger wave-length for the other, and *vice versa*. The same thing holds good for the second subordinate series of *Al* and *Tl*, although no principal series has been observed for them. The following table gives a summary of all the measurements for the second subordinate series.

SECOND SUBORDINATE SERIES.

Cu λ 4531	*Ag*[1] λ 4669	*Al* λ 3962	*Al* λ 3962	*Al*[1] λ 3962	*Al*[2] λ 3962	*Tl* λ 5351	*Tl*[1] λ 5351	*Tl*[2] λ 5351	Mean
−2.20	......	−2.34	−2.31	−2.33	......	−2.33	−2.25	......	−2.293*s*
−1.37	−1.38	−1.39	−1.39	−1.39	......	−1.40	−1.39	......	−1.387*s*
−0.45	......	−0.45	−0.53	......	−0.47	−0.48	......	−0.47	−0.475*p*
+0.39	......	+0.50	+0.51	......	+0.47	+0.48	......	+0.47	+0.470*p*
+1.33	+1.38	+1.38	+1.40	+1.41	......	+1.39	+1.36	......	+1.379*s*
+2.30	......	+2.30	+2.31	+2.31	......	+2.35	+2.27	......	+2.307*s*

Cu λ 4481	*Ag* λ 4476	*Al* λ 3944	*Al* λ 3944	*Al*[1] λ 3944	*Al*[2] λ 3944	*Tl* λ 3776	*Tl*[2] λ 3776		
−1.93	−1.78	−1.86	−1.86	−1.84	.....	−1.95	......		−1.870*s*
−0.94	−0.93	−0.92	−0.89	......	−0.93	−0.87	−0.85		−0.904*p*
+0.98	+0.92	+0.94	+0.88	......	+0.93	+0.89	+0.86		+0.914*p*
+1.89	+1.80	+1.84	+1.87	+1.84	......	+1.93	... ..		+1.862*s*

For the measurements designated by [1] the vibrations parallel to the lines of force, for those designated by [2] the vibrations perpendicular to the lines of force, were cut out by a calc-spar. It was not possible to measure all of the components of the silver line at λ 4669. We were able to perceive two components, but only the two stronger ones, only when the vibrations parallel to the lines of force were cut out. Equal weights were given to all the measures in forming the means. It would not be correct to give a lower weight to the aluminium line than to the lines of

copper and silver, for although the wave-length is less, the measurement is nevertheless equally accurate, since the line is stronger and the components are consequently more readily seen. The mean would be only slightly changed, if different weights were given to the measures. The mean error of a single measure is computed from the deviation from the mean to be 0.039, which can doubtless be attributed to the uncertainty of the observations. The means therefore have a mean error of 0.016 or 0.015.

A comparison of the means with the values computed for the principal series gives convincing evidence that within the limits of accuracy of the observations, the larger wave-length in case of the pairs of the subordinate series is separated in just the same way as the smaller wave-length in the pairs of the principal series, and *vice versa*. The means are collected in the following table:

SEPARATION OF THE PRINCIPAL SERIES, AND OF THE SECOND SUBORDINATE SERIES.

Second Subordinate Series. Greater Wave-Length	Principal Series. Smaller Wave-Length	Differences
—2.29 *s*	—2.25 *s*	—0.04
—1.39 *s*	—1.40 *s*	+0.01
—0.48 *p*	—0.45 *p*	—0.03
+0.47 *p*	+0.45 *p*	+0.02
+1.38 *s*	+1.35 *s*	+0.03
+2.31 *s*	+2.30 *s*	+0.01
Smaller Wave-length	Greater Wave-length	
—1.87 *s*	—1.86 *s*	—0.01
—0.90 *p*	—0.93 *p*	+0.03
+0.91 *p*	+0.94 *p*	—0.03
+1.86 *s*	+1.85 *s*	+0.01

The difference has a mean error of between 0.024 and 0.026, according to the mean error of the means as above computed. The square root of the arithmetical mean of the squares of the differences is equal to 0.025. This agreement shows that as far as we are permitted by the accuracy of the measurements to draw conclusions, the separation is the same for the principal series and for the second subordinate series.

As to the first subordinate series, the two principal lines are split up into three lines each by the magnetic field. But the satellite, which accompanies the principal line of smaller vibration number on the side toward smaller vibration numbers, consists in the magnetic field of eight components, of which six vibrate perpendicularly and two parallel to the lines of force. The measurements for the four elements are collected in the following table:

FIRST SUBORDINATE SERIES.

Cu λ 5220.2	*Ag* λ 5471.7	*Ag* λ 5471.7	*Tl*[1] λ 3529.6	*Tl* λ 3529.6	*Tl*[2] λ 3529.6				Mean
−2.26	......	......	−2.09 *s*	−2.20	......				−2.18 *s*
−1.07	−1.15	−1.12	−1.40 *s* −0.84 *s*	−1.11	−1.07 *p*				−1.10 *p*
+1.12	+1.15	+1.12	+0.83 *s* +1.49 *s*	+1.11	+1.07 *p*				+1.11 *p*
+2.20	+2.22	......	+2.01 *s*	+2.20	......				+2.16 *s*
Cu λ 5218.3	*Ag* λ 5465.7	*Ag* λ 5465.7	*Ag* λ 5465.7	*Al*[1] λ 3092.8	*Al* λ 3092.8	*Tl*[1] λ 3519	*Ti* λ 3519	*Cu*[1] λ 4062.9	
−1.52	−1.52	−1.48	−1.54	−1.42	−1.39	−1.52	−1.50	−1.40	−1.48 *s*
+0.02	0.00	0.00	+0.03	......	−0.03	......	0.00	.. ...	0.00 *p*
+1.50	+1.52	+1.48	+1.51	+1 42	+1.39	+1.52	−1.50	+1.40	+1.47 *s*
Cu λ 5153.3	*Ag* λ 5209.2	*Ag* λ 5209.2	*Al* λ 3082.3	*Al*[1] λ 3082.3	*Tl*[1] λ 2768	*Cu*[1] λ 4022.8	*Cu* λ 4022.8		
−1.12	−1.19	−1.14	−1.21	−1.26	−0.96	−1.20	−1.10		−1.15 *s*
−0.04	+0.02	−0.01	−0.03	0.00	......	......	−0.03		−0.01 *p*
−1.16	+1.18	+1.15	+1.24	+1.26	+0.96	+1.20	+1.12		+1.16 *s*

In the case of the plates designated with[1] the vibrations parallel to the lines of force, in the case of those designated with[2] the vibrations perpendicular to the lines of force, are cut out. Only in the case of thallium did we succeed in completely separating the satellites. In the case of the other elements the components vibrating parallel to the lines of force run together with those adjacent vibrating perpendicularly, so that we can only observe the center, which coincides with the components vibrating parallel to the lines of forces. The separation of the satellite could not be observed at all in the case of aluminium. Two other faint components could be observed near the aluminium lines at

λ3082.3, presumably belonging to another line lying near to λ3082.3. Equal weights are given to all measures in taking the means. The means are computed in case of the satellite only for the two outside components vibrating perpendicularly to the lines of force, and for the two components vibrating parallel to the lines of force.

The mean error of a single measure from the deviations from the mean is computed to be 0.065, which is explained by observation errors, and would not justify any conclusion that the separation of corresponding lines is a little different from the different elements. The greatest deviations from the mean occur in the case of the thallium line at λ2768, and correspond here to a difference of wave-length of about 0.015 tenth-meters. The components lie very close together on account of the short wave-lengths and are very difficult to measure separately.

Kayser and Runge have observed in the spectrum of copper still another pair of lines, with wave-lengths λ5782.30 and λ5700.39, which does not belong to the series, although the vibration numbers have the same difference as in case of the pairs. It is interesting to note that the separation of the two lines by the magnetic field conforms to the separations occurring in the first subordinate series. The smaller vibration number has the separation of the larger vibration number in the pairs of the first subordinate series, and the larger vibration number has conversely the separation of the smaller vibration number in the pairs of the first subordinate series, namely, the separation of the satellites. The agreement is shown in the following table.

The reversal in the succession for the lines of the two pairs recalls the relationship of the second subordinate series to the principal series. We should be led to the suspicion that there are two principal series, of which one bears a relation to the first, the other to the second subordinate series, but for the present this must remain only as a suspicion.

Kayser and Runge also observed in the spectrum of silver a pair of lines not belonging to the series and corresponding to the pair in the spectrum of copper just mentioned. We have unfor-

Cu λ5782.3 Greater Wave-length of Pair.		First Subordinate Series Smaller Wave-length Mean.		Remarks
− 1.15		− 1.15		
0.00		− 0.01		
+ 1.15		+ 1.16		
Cu λ5700.4 Smaller Wave-length of Pair.		First Subordinate Series. Greatest Wave-length. Satellite. *Tl.*		In the measures under *b*, the vibrations parallel to the lines of force were cut out by a calc-spar. In the measures under *a*, the two vibrations parallel to the lines of force so fall upon those vibrating perpendicularly that the neighboring perpendicular vibrations cannot be separated from them.
a	*b.*	*a.*	*b.*	
− 2.20	− 2.32	− 2.20	− 2.09	
− 1.18	− 1.52 − 0.68	− 1.11	− 1.40 − 0.84	
+ 1.12	+ 0.72 + 1.51	+ 1.11	+ 0.83 + 1.49	
+ 2.26	+ 2.30	+ 2.20	+ 2.01	

tunately not succeeded in observing this pair in the magnetic field, as it cannot be seen on our plates.

All these types observed in the series of pairs in the spectrum of *Na*, *Cu*, *Ag*, *Al*, and *Tl* also occur in a number of pairs found in the spectra of *Mg*, *Ca*, *Sr* and *Ba*, and from this we should be inclined to regard them also as series lines, although they cannot be resolved into series, but always give only a single representative of a series. In respect to the number of their components as well as to their distances and their ratios of intensity, the separation is the same in all details. We should, therefore, not hesitate to apply to these lines the same designation as principal series, first and second subordinate series. Some of the strongest lines in these spectra are included among these lines; for instance, the calcium lines H and K without doubt constitute a pair of a principal series, and therefore correspond to the two D lines in the spectrum of sodium. The following table contains the measurements, together with the means of the distances of the corresponding lines of *Na*, *Cu*, *Ag*, *Al*, and *Tl*.

The vibrations parallel to the lines of force were cut out in case of the measurements designated by [1], those perpendicular to the lines of force where designated by [2]. The principal series and the second subordinate series are difficult to observe in case of

Principal Series Greater Wave-length				Second Subdivision Series Smaller Wave-length				Mean		Mean of Corresponding lines in *Na, Cu, Ag, Al, Tl*	
Mg λ2803	*Ca* λ3969	*Sr* λ4215.7	*Ba* λ4934	*Ma* λ2939	*Ca* λ3706	*Sr* λ4162	*Ba* λ4525	Principal.	Subordinate.	Principal	Subordinate
−1.75	−1.82	−1.84	−1.83	−1.79	−1.83	−1 85	−1.80	−1.81*s*	−1.82*s*	−1.86*s*	−1.87*s*
−0.98	−0.91	−0.98	−0.94	−0.99	−0.89	−0 93	−0.96	−0.95*p*	−0.94*p*	−0.93*p*	−0.90*p*
+0.92	+0.94	+0.97	+0.93	+0.99	+0.89	+0.93	+0.96	+0.94*p*	+0.94*p*	+0.94*p*	+0.91*p*
+1.79	+1.80	+1.84	+1.84	+1.79	+1.83	+1.84	+1.84	+1.82*s*	+1.82*s*	+1.85*s*	+1.86*s*
Principal Series Smaller Wave-length				Second Subordinate Series Greater Wave-length							
Mg λ2796	*Ca* λ3933.8	*Sr* λ4078	*Ba* λ4554	*Mg* λ2937	*Ca* λ3737	*Sr* λ4306	*Ba* λ4900				
−2.22	−2.25	−2.32	−2.26	......	−2.27	−2.27	−2.22	−2.26*s*	−2.25*s*	−2.25*s*	−2.29*s*
−1.37	−1.36	−1.39	−1.37	−1.44	−1.41	−1.37	−1.38	−1.37*s*	−1.40*s*	−1.40*s*	−1.39*s*
−0.41	−0.45	−0.49	−0.48	−0.33	−0.43	−0.47	−0.46	−0.46*p*	−0.42*p*	−0.45*p*	−0.48*p*
+0.41	+0.48	+0.45	+0.46	+0.33	+0.43	+0.45	+0.45	+0.42*p*	+0.42*p*	+0.45*p*	+0.47*p*
+1.37	+1.33	+1.38	+1.36	+1.44	+1.37	+1.37	+1.33	+1.36*s*	+1.38*s*	+1.35*s*	+1.38*s*
+2.22	+2.27	+2.36	+2.28	......	+2.32	+2.28	+2.27	+2.28*s*	+2.29*s*	+2.30*s*	+2 31*s*

FIRST SUBORDINATE SERIES.

	Ca[1] λ3181 4	*Ca*[2] λ3181.4	*Sr*[1] λ3475	*Sr*[2] λ3475	*Sr* λ3475	*Ba*[1] λ4166.2	*Ba*[2] λ4166.2	Mean of the corresponding lines of *Cu, Ag, Al, Tl*
					−2.28	−2.06		−2.18*s*
	−1.45		−1.41			−1.46		−1.40*s*
		−1.09		−1.09	−1.14		−1.11	−1.10*p*
Satellite						−0.78		−0.84*s*
		+1.09				+0.70		+0.83*s*
				+1.09	+1.10		+1.11	+1.11*p*
	+1.45		+1.41			+1.44		+1.49*s*
					+2.32	+2.16		+2.16*s*
	Mg λ2798	*Ca* λ3179.4	*Sr* λ3465	*Ba* λ4130.9				
	1.38	−1.52	−1.53	−1.49				−1.48*s*
	0.00	0.00	0.00	−0.02				0.00*p*
	+1.38	+1 52	+1.53	+1.51				+1.47*s*
	Mg λ2791	*Ca* λ3159 0	*Sr* λ3381	*Ba* λ3892.0				
	−1 04	−1 12	−1.17	−1.12				−1.15*s*
	0.00	0.00	+0.03	+0.01				−0.01*p*
	+1.04	+1.12	+1.14	+1.11				+1.16*s*

Mg, since these lines reverse easily. The appearance of the line on the photographic plate may thus turn out very differently, according as the line was reversed during the whole or during a part of the exposure. We were only able to measure those plates on which no reversal was to be seen. In addition the wave-lengths of the line are small and consequently the differences of wave-lengths of the components are slight. The

separation of the satellite is also difficult to observe in the first subordinate series, and we only succeeded in obtaining the complete separation in the case of barium. Nevertheless, it seems to us as not open to doubt that the separation of the satellite is the same for the other elements. On cutting off the vibrations parallel to the lines of force, the components vibrating perpendicularly to the lines of force run together into two diffuse lines. If none of the components are cut out, the stronger components vibrating parallel to the lines of force are superposed upon the others. We cannot then distinguish from them the neighboring components vibrating perpendicularly, and we also perceive in addition to them the outside components vibrating perpendicularly. No satellite was observed in the case of *Mg*, either because none exists, or, as is more likely, because it falls so near to the principal line that it is concealed by it.

In the spectrum of barium, as of copper, still another pair was found, yielding the same difference in the vibration numbers, although not belonging to the three series. The wave-lengths are λ 6497.07 and λ 5853.91. We measured the separation in the magnetic field of the latter and found that it agreed with the separation of the corresponding copper lines, and had the type of satellite of the first subordinate series. The distances of the components are given in the following table :

Ba [1] λ 5853.9	*Ba* [2] λ 5853.9	*Cu* λ 5700.4	Remarks
$-2.19s$		$-2.26s$	[1] Vibrations parallel to the lines of force are cut out.
$-1.47s$		$-1.52s$	
	$-1.14p$	$-1.18p$	[2] Vibrations perpendicular to the lines of force are cut out.
$-0.73s$		$-0.68s$	
$+0.72s$		$+0.72s$	s Vibrations perpendicular to the lines of force.
	$+1.14p$	$+1.12p$	
$+1.47s$		$+1.51s$	p Vibrations parallel to the lines of force.
$+2.19s$		$+2.28s$	

We did not measure the other line at λ 6497.007, but we were able at least to satisfy ourselves that it is resolved into a triplet by the magnetic field, like the corresponding copper line.

The principal results of this paper are these:

1. The pairs of lines observed in the spectra of the elements *Na*, *Cu*, *Ag*, *Al*, *Tl*, *Mg*, *Ca*, *Sr*, and *Ba*, when separated by the magnetic field, exhibit a number of types which repeat themselves from element to element. Here the separations of the same type agree to the smallest detail for the different elements; that is, not only is the number of components the same, but also their ratios of intensity, and their distances when we consider the lines as represented on a scale of vibration numbers.

2. The pairs of lines may be arranged according to types, as those of the principal series, those of the first subordinate series and those of the second subordinate series. The types of the principal series and of the second subordinate series are the same, but with the succession inverted, as was to be suspected *a priori*, according to the relation found by Rydberg between the principal and the second subordinate series.

3. The distances of the components of the principal series and of the second subordinate series from the unaffected line are multiples of the same number on the scale of vibration numbers. They are equal to the even multiples for the one line and to the odd multiples for the other line. We have previously communicated a similar law in case of the second subordinate series of triplets in the spectrum of *Mg*, *Ca*, *Sr*, *Zn*, *Cd* and *Hg*. The number which in that case represented by its multiples the distances of the component stands very nearly in the ratio of 3:2 to the number found here.

The experimental part of this investigation was carried on by the authors in common, but on account of his removal to Tübingen, F. Paschen was unable to participate in the measurement of the photographic plates or in the discussion of the measures.

OBSERVATIONS OF THE AURORA MADE AT THE YERKES OBSERVATORY, 1897–1902.

By E. E. Barnard.

Williams Bay seems to be well situated for observations of the aurora. When we first came here in 1896–7, there were frequent and brilliant auroras. In the succeeding years they became less so, and with the exception of what seemed to be special outbursts they appeared to die out altogether.

A few of the first were not recorded, but when the record began a strict account was kept of all auroras seen, and frequently their absence was noted at times near when they had been seen, or for other reasons.

Some of the phenomena of these displays were wholly new to me. My previous experiences had been with very inconspicuous auroras, with the exception of the magnificent display of April 16, 1882, which I saw at Nashville, Tenn., and which I have not seen equaled since.

Following is a list of the records made here, which may be important, as they cover a Sun-spot minimum, and will doubtless bear on the connection between Sun-spots and auroras.

Ninetieth meridian time is used in these notes.

1897.

March 27. $8^h\ 10^m$. A faint aurora has appeared 10° east of the north point. It is decided but not bright, and stretches along some 30° of the horizon.

April 23. Fine aurora from 8^h. Finest at 9^h, when extraordinarily sharp arch formed. Fine streamers.

July 30. Fine aurora in early part of night, but no streamers had appeared when it clouded up. Cleared at $14^h\ 30^m$. From this time on till daylight the aurora was very strong, with a well-formed arch, very dark beneath— streamers not mentioned and probably not present.

August 19. Magnificent aurora all night. Very active at $12^h\ 30^m$ to $13^h\ 0^m$.

August 29. Auroral arch at midnight; died out before dawn. Did not notice any streamers.

October 29. Fine aurora from 9^h to 10^h.

December 20. $9^h\ 15^m$. Through breaks in the clouds in the low north there is a brilliant aurora with streamers moving to the *left*. It is evidently a very bright affair; cleared at $9^h\ 30^m$; the aurora kept up until about 10^h, with brilliant streamers, then gradually died down. At $13^h\ 5^m$ there were no streamers, but a diffused auroral glow with rapid pulsations — horizontal strips of flashing light.

$15^h\ 50^m$. The aurora seems to have died out.

$16^h\ 0^m$. The aurora is not dead. The same strange rapid flares of light are going on.

$16^h\ 30^m$. The aurora is still fluttering. At a small altitude a bright strip of horizontal light will appear and rapidly spread to the *left*, and then back again and out. The motion is extremely rapid.

December 21. $14^h\ 10^m$. I see the glow and a smoky arch for the first time tonight. It did not make any display up till daylight.

December 22. Up to $7^h\ 30^m$ no aurora had appeared.

December 23. No aurora as late as 10^h.

December 24. No aurora — clear till $16^h\ 30^m$.

1898.

January 15. Cleared at 12^h; from midnight until $14^h\ 30^m$, when it clouded again, a strong auroral glow.

January 16. At $9^h\ 30^m$ there was a bright aurora with streamers which moved slowly to the *left*. Heavy arch. This died out in half an hour. As late as $14^h\ 30^m$ it had not again appeared.

Observer absent from January 29 to February 16.

March 14. $7^h\ 20^m$. There is a fine long auroral arch, no streamers. A vertical line from β *Ursae Minoris* will cut the exact summit of the arch. From this on until $8^h\ 30^m$ the aurora was superb — a great double arch and magnificent streamers of a crimson color — the aurora filled all the north up to and above the pole. At about $8^h\ 10^m$ there were persistent white masses in the northwest, low near the horizon. At $8^h\ 40^m$ the aurora is quieter, with large white glow from north horizon and whitish masses in the low northwest. This is by far the finest aurora I have seen here. $10^h\ 0^m$. Fine aurora again. The arch remained as late as 11^h, when it clouded up, though the activity had ceased. At 10^h *Vega* could be seen shining brilliantly through the dark space below the brilliant arch.

March 15. $7^h\ 10^m$. Sky covered with a magnificent aurora clear to the zenith; at $7^h\ 15^m$ great streamers running south of zenith to the southeast. Rapid pulsations in the light in the north all the time The streamers below pole moved to the *left*. The pulsations seemed to be waves of light concentric with the arch, ascending with great velocity and following each

other rapidly at intervals of one or two seconds. Twice a brilliant and enormously long irregular ray of light about 1° or 2° broad stretched across the sky south of the zenith and perpendicular to the meridian. This had a slow motion to the south and was sinuous. A white, comet-like ray — perfectly resembling a large comet — extending from near the east horizon through *Jupiter*, remained stationary for upwards of an hour. Patches and wisps of nebulous light appeared in all parts of the sky.

$9^h\ 50^m$. The sky has been luminous all over the north for some time, though quiet, but the arch is forming again. It is a very long, low arch whose center is in the same vertical with β *Ursae Minoris*, and whose altitude is one-fourth of that of the star.

$12^h\ 0^m$. The aurora has been active for the past half hour — splendid streamers shooting up from the arch. These streamers all move very slowly to the left. In the beginning, before the arch broke, bluish white masses of intense light appeared on the arch and moved very rapidly to the *right*.

$12^h\ 50^m$. The aurora is still very active, sending up streamers as high as the pole.

$14^h\ 0^m$. The aurora is dead except for occasional flashings up.

$15^h\ 0^m$. Very quiet but occasional flashings up.

March 16. No trace of aurora as late as $11^h\ 30^m$.

March 19. $10^h\ 35^m$. There is an auroral glow in the north which has appeared within the hour. $10^h\ 55^m$. The aurora is getting brighter, with arch and faint streamers. $12^h\ 10^m$. The aurora is moderately bright and slightly active. The streamers are ordinary and move to the *left*.

The aurora died down after midnight, the arch and glow disappearing. Two brilliant fluctuating clouds, however, remained, or rather appeared, after the arch had gone; one of these was almost due east and the other was west of the pole. These would brighten up for a few seconds very brilliantly and then die down again. The one to the west remained thus until early dawn.

March 20. From $15^h\ 30^m$ to $17^h\ 20^m$ no aurora.

March 23. No aurora during the night.

March 24. No aurora until midnight, when streamers shot up for half an hour and then died out.

$13^h\ 30^m$. The aurora seems to have entirely died out.

March 28. $14^h\ 10^m$. An auroral glow under pole has started up. No other remarks, though observing all night until 18^h.

March 29. $14^h\ 0^m$. There is a slight aurora.

April 20. No aurora during the night. Clear all night.

May 3. $13^h\ 0^m$. There is a strong auroral arch in the north which, in spite of Moon and haze, is conspicuous and shoots up streamers. This is the first aurora since the record of March 29. I have kept a close lookout for the aurora on every clear night.

Observer absent from May 24 to June 8.

August 12. Aurora began faintly at $8^h\ 30^m$. Brightest at $10^h\ 30^m$, when there was a strong arch and feeble effort at streamers. After this it died out, and at $13^h\ 30^m$ there was only a feeble glow.

September 2. $10^h\ 15^m$. There is a strong double (concentric) aurora arch. At $10^h\ 25^m$ the aurora is extremely brilliant and active, though the Moon is nearly full. It makes a brilliant display of streamers and breaks up into great masses of intense light. The display is unusually fine.

$11^h\ 50^m$. The aurora has been dead for an hour or so. No further record of it as late as $16^h\ 15^m$.

September 10. $9^h\ 0^m$. There is a very faint auroral glow in the north. $11^h\ 30^m$. There is a strong aurora with arch. $12^h\ 30^m$. The aurora is active, it (the arch) breaks up into masses of light but no streamers.

$13^h\ 0^m$. A magnificent and superb display of aurora.

The most striking feature of this was a great comet-like mass of intense light with head to the southwest of *Orion*, and stretching across the sky slightly south of the zenith, to the west horizon. It was some 20° wide and very much resembled some of the photographs of Brooks' comet of 1893. The stars shone through the brightest part of it. It moved slowly to the southeast, and faded out about $13^h\ 10^m$, but would then brighten up near *Orion*. So bright was the display on this date that at times the light in the north cast a distinct shadow of a person across the ground.

September 11. No aurora at night.

September 12. $8^h\ 10^m$. There is an intense auroral light shining through the clouds in the northwest in the region of α *Canum*. It is very brilliant and makes a glow through the rather thin clouds. It seemed to be a large roundish mass of light.

September 15. 14^h. There is an auroral spot 10° above the horizon in the northwest and a similar one in the northeast 10° high. These faded and brightened rapidly. By $16^h\ 30^m$ the one in the northwest had extended in a fragment of arch nearly under the pole, and was very bright as daylight killed it out. The one in the northeast had disappeared.

September 19. Dense smoky yellow haze all day. Could not see across the lake (1 ½ miles). Though the night was more or less broken with clouds, observed all night, and there is no record of aurora. This haze doubtless had nothing of an auroral nature.

October 14. Auroral glow from 11^h until 12^h; no arch. Feeble streamers for a while.

November 5. No aurora.

November 7. No aurora.

November 10. No aurora.

November 11. $7^h\ 30^m$. During the past ten minutes a feeble aurora has started up in the north.

$9^h\ 35^m$. The aurora is quite strong, with feeble efforts at streamers.

$10^h\ 15^m$. The aurora is brighter.

$12^h\ 40^m$. The aurora has dimmed down.

November 22. $15^h\ 0^m$. Faint aurora, which did not last long.

November 23. No aurora.

December 13. $9^h\ 50^m$. An aurora with a low arch has started in the north. It is moderately bright. There has been no aurora for a long time. I have looked out for auroras every clear night. $13^h\ 23^m$. The aurora is very strong — like a strong dawn, but very low. It extends pretty far east. There is no definite arch or streamers. $15^h\ 30^m$. The auroral glow is still strong among the clouds.

1899.

January 28. $10^h\ 0^m$. Aurora before Moon-rise and strong arch after Moon-rise.

February 11. $8^h\ 0^m$. Strong auroral arch — the first seen since January 28. $11^h\ 50^m$. The arch is very bright, but there are no streamers. $12^h\ 5^m$. The auroral arch is now breaking up. *Vega* is shining through the darkest part under the arch. Temperature 23° (F.) below zero. $14^h\ 50^m$. The aurora is very active. It is splendid. There are no streamers above the arch, but there are brilliant masses of bluish-green light, like cometary tails, projecting up in the arch and moving to the right. There are quick waves of ascending light, and the arch is broken, and double in places. This is the finest display I have seen in a long time. $15^h\ 20^m$. The auroral arch is broken up into cloud masses — no definite arch, but dark below. The ascending waves of light are very rapid all along the line. Temperature, 24° (F.) below zero. $17^h\ 30^m$. The aurora is very active. Great streamers. No pulsating waves. $17^h\ 15^m$. The aurora is bright and sending up streamers to the east. Temperature, 25°.5 (F.) below zero.

February 12. No aurora during the night.

February 15. $12^h\ 5^m$. There is a fragment of a bright arch in the northeast over *Vega*. It is about one-fourth of an entire arch, beginning at the northeast horizon and ending abruptly. There is no glow or indication of auroral light anywhere under the pole or near it. *Vega* shines through the dark part under the fragment of arch.

May 1. $8^h\ 35^m$. There is a pretty strong auroral arch. $9^h\ 25^m$. The arch is quite strong — no streamers. $14^h\ 30^m$. The aurora has nearly died out. It was very active about 11^h. The arch was low, $= \frac{1}{10}$ altitude of *Polaris*. There was a thin inner arch part of the time, and some bright spots forming and a few streamers.

May 2. No aurora during the night.

May 4. $11^h\ 0^m$. There is a rather noticeable aurora that started up about 10^h. By midnight the aurora was very bright.

May 18. A faint aurora was visible from 13^h to 14^h.

June 28. Cleared at midnight, showing a strong and active aurora. This was bright in spite of the Moon at $13^h\ 0^m$, but died out shortly after $14^h\ 0^m$.

June 29. $9^h\ 0^m$. The sky is partly covered with thin hazy clouds. There is the eastern part of a decided auroral arch seen under the clouds. To the left of the pole the arch cannot be seen on account of twilight and clouds. There is a brightish streamer in the east near the horizon, inclined 10° or 15° to the south at upper end. $9^h\ 30^m$. The full arch is very strong. It seems to me that the center of the arch is further to the right than usual.

$10^h\ 5^m$. The aurora is brighter. There are a few faint streamers to the left of the pole. The arch is not black beneath — it is a light gray; there is little contrast with the bright arch. The sky is darker above the arch, but is covered with a diffused glow. The aurora became very active about 11^h and threw up streamers. By $11^h\ 30^m$ it seemed to have nearly died away. There were bright greenish-blue-white masses in the lower part of the arch that moved rapidly to the *right* as they faded.

June 30. Up to $13^h\ 30^m$ there had been no aurora, but at $14^h\ 15^m$ a feeble aurora started up. $15^h\ 30^m$. The aurora did not get fairly started before dawn killed it out.

July 1. No aurora during the night.

July 2. Clouded at 11^h. Before this there was no aurora.

July 5. $10^h\ 10^m$. There is a distinct but feeble auroral spot 20° to the left of north at 10° or 15° altitude, but no other indications of aurora. $10^h\ 40^m$. The luminous appearance now seems to be almost under the pole and to slant to the east at its upper end.

July 8. No aurora during the night.

August 5. $9^h\ 10^m$. There is a patch of auroral light to the right of the pole at an altitude of some 10° and several faint streamers at that point. There has been no aurora during any of the nights of observation lately. A close watch has been kept.

September 12. $12^h\ 0^m$. There is a slight aurora very low on the north-northeast horizon. $14^h\ 35^m$. The auroral light has all gone. There was no display.

September 15. $15^h\ 35^m$. For the past half hour a strong auroral glow has been visible low in the north. It has now become quite bright.

September 25. $10^h\ 0^m$. Strong auroral glow — the first in a long time.

September 26. $7^h\ 40^m$. Slight auroral glow in the north. $10^h\ 50^m$. The aurora has been very feeble, but it is now strong, with a heavy arch. $12^h\ 0^m$. There are some streamers in the north.

September 30, October 1, October 2. No aurora during the night.

October 9. No aurora during the night.

October 12. No aurora during the night.

October 28. No aurora.

November 3. $13^h 0^m$. There was a strong auroral glow in the north horizon, but it did not last long.

November 4. No aurora.

Observer absent from December 8 to 1900, March 8.

1900.

April 30. $9^h 40^m$. Strong aurora in north, with streamers one half way to pole. Arch low and irregular. $10^h 40^m$. The aurora seems to have died out, except a faint glow.

May 4. $12^h 0^m$. There is an auroral glow, with dark clouds below it in the north. $13^h 30^m$. There is a dark bank of clouds in the north with the aurora. The auroral light is pretty strong, but irregular. No streamers.

Observer absent from May 8 to June 1.

September 27. $14^h 5^m$. There is a slight auroral glow under the clouds in the north. No arch.

1901.

Observer absent from February 6 to July 26.

November 18. $13^h 30^m$. A slight auroral display. A few streamers shot up from the north horizon under the pole.

1902.

April 10. Cleared at midnight, exposing a rather bright auroral glow in the north horizon. This increased and finally formed a distinct arch — dark below; the lower part, or base of the arch, being only 5° or 6° high. The top of the brightness just reached to *α Cassiopeiae*. It was most distinct at $13^h 0^m$. By $14^h 0^m$ it had quite faded out. There were no streamers while I watched it. The arch was quite bright, and was very low and long. This is the first aurora I have seen since the last record some months ago (November 18).

May 8. $13^h 0^m$. There is a very low auroral arch in the north. It is dark beneath and was not there before midnight. $14^h 30^m$. The aurora is very strong. The space beneath it is very dark. The arch is bent down in the middle as if it were made of two arches touching each other. There are faint streamers to the right of north. $14^h 45^m$. It is intensely bright, casting a shadow. There are outrushes of bright matter from the dark part, to the left of north, that form very faint, but not decided streamers. These bases of streamers, as it were, eat into the dark arch and break its symmetry badly. $15^h 10^m$. It has all died out, and has left only a faint glow. The aurora seemed to be most active to the left of the pole. The top of the dark arch — base of bright arch — was only 5° high — the top of bright arch about 10° high.

In connection with the magnificent aurora visible here on September 10, 1898, it is well to call attention to the fact that this

disturbance seems to have been general, as great displays were visible in England and on the continent about this time, mainly on September 9.

In speaking of the display of September 9, in Finland, Baron Kaulbars says the aurora was "not only one of the most splendid seen, but also that has appeared in our latitude for a long series of years." It ceased about midnight in Finland.

In *Nature* for 1898, September 8, Mr. C. E. Stromeyer, of Whitby, England, says that on September 2, from $7^h\ 45^m$ to $8^h\ 15^m$ there was a display visible with the center of rays apparently resting on the horizon, about north 25° east. "The rays revolved from west to east at the rate of about 20° in 10^m."

In an interesting account of the aurora of September 9, given in *Nature* for September 15, 1898, pp. 490–491, it is stated that there were bright auroras visible in England also on September 2, 7, and 10.

In connection with the recent display it is interesting to note that the unusually large spot which came over the eastern limb of the Sun on Saturday, September 3, was on the central meridian of the Sun's disk on Friday, the 9th, at about the time the aurora was at its maximum.

And still further, the automatic recording instrument for magnetic declination in the physics department at South Kensington showed a large disturbance the same evening. From the photographic record it appears that the disturbance began about 7:30 P. M., and in fifteen minutes reached a value of 30′ of arc; by 8:00 P. M. the declination was normal again, but immediately afterward the needle traveled on in the opposite direction to the first displacement, and reached a second maximum eastward about 8:15. By 8:30 the needle had again assumed its normal position, and no further disturbance, other than the usual diurnal one, has yet been recorded. Thus the declination magnet was deflected over 1° in the hour from 7:30 to 8:30 P. M. This leaves little doubt as to the definite connection between the position of the spot on the solar disk, the magnetic variation, and the aurora.

A letter from Dr. Charles Chree in the same number of *Nature*, p. 468, shows that a conspicuous magnetic storm was recorded at the Kew Observatory while the aurora was in progress. In ending his letter Dr. Chree says:

The curves had become fairly quiet by midnight of the 9th, but there was a recrudescence of the disturbance between 8:00 A. M. and midnight of the 10th, and subsequent smaller movements occurred on the 11th.

There are several features in connection with these auroral displays that will bear special notice.

When the arch is strongly formed, the space beneath it to the horizon has usually a dark, smoky look as if there were a bank of dark, smoky clouds filling all beneath the arch. This gives the impression of being densely opaque. But it is really transparent, as shown by several observations of Vega through its apparently densest portion. It would appear, therefore, that this smoky blackness is merely due to contrast with the bright arch above. This is very surprising, for if there is any material substance that fills this space it is certainly transparent. The motion of the streamers is moderate. They move slowly, say a degree in a couple of minutes, towards the left. (I use the terms right and left to avoid ambiguity.) Mr. Stromeyer in England (*Nature*, September 8, 1898) saw them moving from west to east, *i. e.*, in the reverse direction to the motion they have here. It seems to me that this is an important feature.

The intensely bright bluish or greenish-white balls that appear in the base of the arch have a very rapid motion from the left to the right, contrary to the motion of the streamers, and the velocity is very much greater than that of the streamers. They burst out very bright and move rapidly to the right along the arch and fade out as they go, usually existing for only a portion of a minute.

The pulsating clouds of light are remarkable features. They are usually 5° or 10° in diameter and very bright—for a short interval they will almost fade out and then quickly become very bright again—as if some one were capriciously turning on and off their light.

The comet-like objects are as nearly like a great naked-eye comet as possible. Their light is of the same color as that of a comet and their form almost that of a comet with a straight tail. The one seen on March 15, 1898, in the east near Jupiter, could easily have been mistaken for a real comet.

The arches are very various in their heights. Sometimes the arch is very low, being only a few degrees above the north

horizon. At other times it reaches nearly half-way to the pole. The center of the arch is usually 20° or 25° east of north.

It seems to me that a faithful record of the altitude of the arch and the azimuth of its summit must be important. Just why the arch should at one time be close to the horizon and at another time many degrees higher, will doubtless be an important feature when the nature of the aurora is better known. To stop and locate these features with reference to the stars when one is busy with other work, and especially to reduce such observations, requires considerable time from one who is busy enough with other work in which he is more especially interested. I have therefore thought that a crude instrument made of wood, consisting of a horizontal and a vertical circle roughly graduated to degrees, with a wooden rod as pointer, might be constructed and used for quickly observing the altitude and azimuth of the arch. Such an instrument we hope soon to have placed on the roof of this Observatory for quickly locating important features of the aurora.

The incomplete arch of February 15, 1899, was a singular feature and unique so far as my observations go.

The appearance of an aurora here is a capricious affair; there seems to be no means of telling when one will occur.

A very active aurora will appear one night, and the next night there will be no trace of anything of the kind, nor perhaps, had there been any trace of one the night before.

The bright auroras that we shall doubtless have in the next ten years will perhaps give important information from a spectroscopic standpoint. Some of the principal features should also be easily photographed with short-focus, quick-acting lenses, but the exposures must be very short, because of the motion of the streamers.

These observations of aurora have been made in the hope that they might some day be valuable. It seemed a pity that no record should be kept of their appearance, though my interest has lain in other directions altogether.

Yerkes Observatory,
September 2, 1902.

NOTES ON SPECTRO-PHOTOMETRIC ADJUSTMENTS.

By L. B. Tuckerman.

About two years ago[1] Professor D. B. Brace described a new and simple form of spectro-photometer. Several of these instruments have been since in almost continuous successful use in the laboratory of the University of Nebraska. In this time several alterations have been made in the design of the instrument which render its adjustment much simpler and easier. In view of the growing importance of spectro-photometric measurements in various physical and chemical-physical investigations it has been thought worth while to describe the modified instrument in connection with a discussion of the conditions which afford the greatest accuracy with ease of observation.

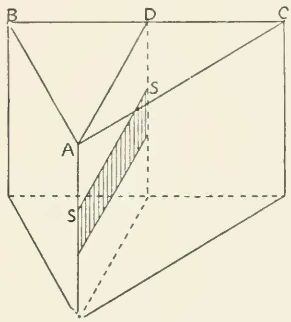


Fig. 1.

The instrument consists essentially of a double prism P (Figs. 1 and 2) with a narrow silvered strip SS on the face AD of the right half. Light of the same wavelength is thus brought by direct transmission from the collimator T, and after reflection at the silver strip from T', to the same focus in the observing telescope R. When the eyepiece is removed and the prism viewed through a slit in the focal plane of R, the eye sees three fields (Fig. 3), the central one $ABCD$ illuminated by light from the right collimator T', and the upper and lower ones, ACF and BDG, by light from the left collimator T. The fields meet in the sharp edge of the silver strip.

Lummer and Brodhun[2] have given the following three condi-

[1] *Phil. Mag.*, 48, 420–430, 1899, and Astrophysical Journal, 11, 6–24, 1900.

[2] *Zeitschrift für Instrumentenkunde*, 9, 42, 1889.

tions which should be met by an ideal photometer screen where the principle of equality is used.

1. Each of the comparison fields must receive light from only one light-source;

2. The bounding line between the two fields must be as sharp as possible; and

3. It must vanish at equal illumination of the two fields.

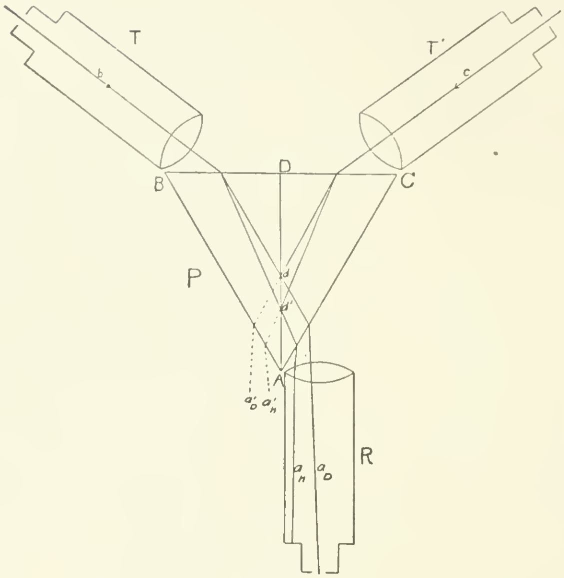


FIG. 2.

The third condition evidently includes uniform illumination of each field and for spectrophotometric work the added condition that the field be of uniform tint throughout. The first of these conditions is evidently reached if the cementing substance between the two half-prisms has a refractive index sufficiently near that of the glass. (For glasses of low refractive index (*circa* 1.5), balsam may be used, but for those of higher index α-monobromonaphthalene is more satisfactory.) The second condition is fulfilled even with non-homogeneous light when the edges of the silver strip are perpendicular to the refracting edge of the prism.[1] To secure a field of homogeneous tints when the slits are illuminated with white light, the light must pass in parallel rays through the prism, to retain its homocentricity, and the ocular slit must lie in the same plane as the image of the collimator slit and be somewhat narrower than the pupil of the

[1] O. LUMMER and E. BRODHUN, *Zeitschrift für Instrumentenkunde*, 12, 137–138 1892.

eye.[1] With these conditions fulfilled, the tint of the field appears uniform throughout, while with imperfect focusing the tint varies from one side of the field to the other.

In adjusting the instrument for use, more trouble was found in meeting the remaining requirements. The illumination would in general vary in intensity across the field, causing the appearance shown in Fig. 4. Because of the sensibility of the instrument a variation of one or two tenths of 1 per cent. across the field was noticeable and troublesome in careful work. This uneveness might be due: (1) To an uneven silver deposit. Care in the silvering, especially in insuring that the temperature of the silvering bath and the prism should not be widely different, easily obviated this. (2) To an imperfect slit. If one of the jaws of the slit were slightly displaced along the axis of the collimator (Fig. 5), the effective width of the slit was greater on one side of the field than on the other. This trouble should not be present in any good slit. (3) To unsymmetrical path of the two collimators. Owing to the more oblique reflection at the edges of the lenses, parts of the field illuminated from them will appear slightly darker than those illuminated from the center. If as shown in the diagram (Fig. 6) light from the central part of the lens of one collimator is matched from that of the edge of the other and *vice versa*, it is evident that a match cannot be obtained across the whole field. To remove this the central part of the field must be illuminated from the central part of the lenses of both collimators. It is desirable also that the light should pass centrally through the observing telescope and that the image of the field should not shift laterally as the telescope is rotated to bring other colors into the field. These conditions determine completely the relative positions of the prism, the axis of the instru-

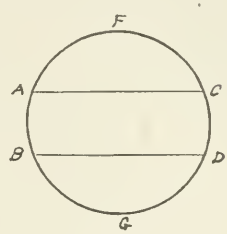


Fig. 3.

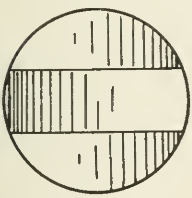


Fig. 4.

[1] H. v. Helmholtz, *Physiologischen Optik*, pp. 289 and 301, 1896.

ment and the axis of collimation of the collimators and telescope. The axis of the instrument must pass through the image of the center of the silver strip as seen through the telescope and the axes of collimation of the collimators must meet the prism one-fourth way from the edge. With the instrument as first constructed it was impossible to meet these conditions fully without considerable alteration, and even when the instrument was so altered, the adjustment was inconvenient.

FIG. 5.

The new instruments have been so planned as to make the adjustment rapid and easy. A convenient size of prism (6 cm) was decided upon, and the position of the collimators, telescope and prism calculated for the glass used ($n_D = 1.65$–1.66) and for minimum deviation for the sodium line. The collimators and telescope are mounted permanently on their arms in the required position. The prism is placed on a base having a projecting pin directly underneath the image of the center of the silver strip, which fits into a socket in the axis of the instrument. The details of the new design are given in Fig. 7. O is the vertical axis of the instrument, lying in the center of the image AD' of the silver strip AD. Or and Or' are the perpendicular distances of the axes of collimation of T and T', respectively, from O. O lies a distance AO from the prism angle A, Op below the surface of the prism and KO from the center K of the prism. The side of the prism $AB = 6$ cm. $Or = 1.8$ cm. $Or' = 2.6$ cm. $AO = 1.7$ cm. $Op = 0.1$ cm. $KO = 2.1$ cm. T is fixed, while T' and R rotate about the axis

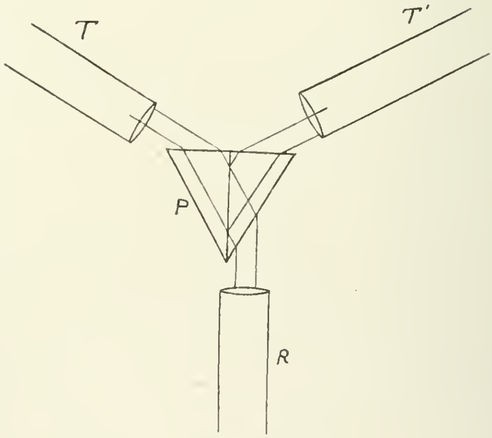


FIG. 6.

O, their positions being read on micrometer screws. With this arrangement all that is necessary for adjustment is to place the prism properly on the base, set it on the instrument, rotate it to minimum deviation for the image of the D line from the left hand collimator *T* and clamp. When the right hand collimator *T′* has been rotated about *O* till the two images of the sodium line coincide, the light passes symmetrically and centrally through the collimators and telescope. This arrangement obviously precludes the use of the instrument with prisms much different in size or varying much in refractive index from the glass chosen. The limit, however, is not narrow. Prisms as small as 5.7 or as large as 6.3 cm on a side could be used, though not so conveniently, and the refractive index may vary from 1.63 to 1.67 without seriously altering the adjustment. As the dispersion of the glass used is large enough for all ordinary work, the advantages of simplicity and ease of adjustment more than compensate for the limitations imposed by the design.

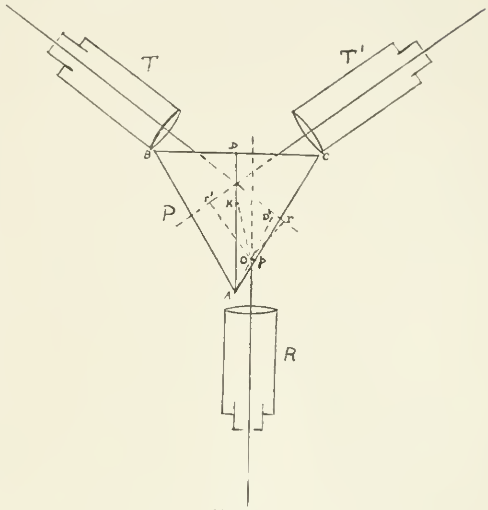


Fig. 7.

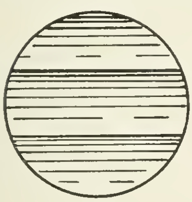


Fig. 8.

Another appearance of the field became noticeable when these adjustments had been made (Fig. 8), a match being obtainable only on one edge of the silver strip at a time. This might be due to unevenness in the silver strip, but the likelihood of that is small and the effect was never traced to that cause. A slight tilting of the prism on its base causes the light from the top of one slit to be matched with that from the bottom of the other and the same results as were found in lateral symmetry

were to be expected. When cross-hairs are placed across the center of the slits and the prism tilted till their two images coincide, the light comes from corresponding parts of the slits of both collimators, and the trouble disappears. Usually a strip of paper under one corner of the prism will correct this. With a uniform source of light the two fields will now appear evenly illuminated and a perfect match is obtainable, the dividing lines becoming almost invisible.

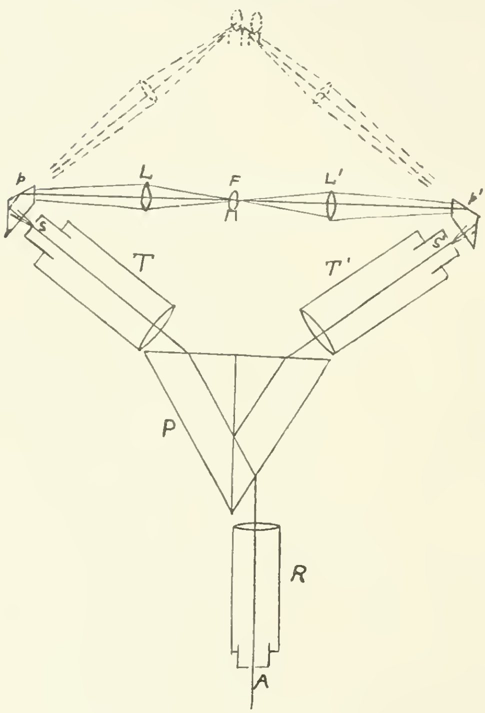


Fig. 9.

For the comparison of two sources of light, ground glass may be used, or if acetylene is made the standard of comparison the free flame is usually sufficiently uniform. For work on absorption the advantage of having both comparison fields lighted by the same source, and thereby avoiding errors due to variation in the sources, has long been recognized. It has, however, been difficult with mirrors and ground or opal glass to secure not only uniformity but even sufficient illumination to give the required sensibility. To avoid this loss of light the following arrangement (Fig. 9) was devised by Professor Brace. The light is taken from a flat flame F, and passes through the lenses L, L' by which after a double reflection at the prisms p, p' it is brought to a focus on the jaws of the slits S, S' of the collimators T, T', forming there images of the flame. The opposite sides of the same point of the flame serve as sources and if the light passes centrally through the lenses L, L' the field is evenly illuminated. The adjustment is

effected by placing a flame in front of the slit of the ocular telescope at A, then adjusting prisms and lenses until the light passes centrally through the lenses, and the images of the two slits S, S', thus illuminated, fall upon the same point of a screen which replaces the flame at F. As the light for both sides comes from the same point in the flame, comparisons are wholly free from disturbances caused by alterations in the intensity of the source. Flickerings which are readily seen in the flame produce no noticeable effect in the relative illumination of the fields. As there is usually an area of several square milimeters in a flame which has uniform intensity, a displacement of the flame from F towards L or L' has for quite a range of movement no effect upon the relative intensities of the two comparison fields, so that the variations which might be feared from that cause do not exist. The two prisms p, p' are exactly similar in shape, but p is silvered on one of its rear faces at the same time that the comparison prism is silvered. It is so cut that the light falls upon this silvered face at an angle of 60°, which is approximately the same angle as that at which the light is reflected from the comparison strip. This effectually compensates for the selective reflection of the silver strip, which would otherwise become noticeable in the blue by altering the reading for equal illumination of the two fields. The difference of setting for different colors sometimes runs as high as 2 or 3 per cent., but with a compensation prism p, a match for one color is, within one or two-tenths of 1 per cent., a match throughout the spectrum.

Several other arrangements were tried and abandoned, either because of lack of constancy or other faults. For instance the arrangement shown in the dotted lines (Fig. 9), in which the flame does not lie in the line joining the centers of the two lenses, was discarded because a slight lateral shifting would cause the light to be taken from different parts of the flame and produce marked variations in the relative intensity of illumination of the two fields. Both ground and opal glass were tried in the place of the lenses L, L', but uniform opal glass is very hard to obtain,

the surface of ground glass is easily ruined by handling, and, as mentioned above, both cut down the intensity so much (95 to 98 per cent.) that very intense sources are necessary to secure even moderate illumination of the field. The use of constant separate sources in absorption work was at first thought to offer some advantages, especially in greater intensity of illumination of the field, but although it was possible to prepare large and uniform incandescent filaments, which would not vary in intensity more than two or three-tenths of 1 per cent. in the course of an hour's work[1], they were expensive and fragile and gave no greater intensity than is now obtained by the system adopted.

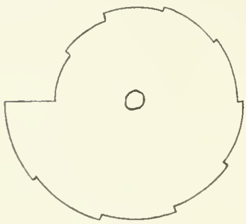


FIG. 10.

S' (Fig. 9) is a bilateral slit which is either used alone or in conjunction with a rotating sector for measuring the changes in intensity. If the slit is used alone it must first be calibrated optically[2] (preferably with a rotating sector) for various widths and different wave-lengths and for the source of light used, since owing to the curvature of the luminosity-curve the illumination is not, in general, proportional to the slit width.[2,3] The sector used is a cardboard or metal disk cut in a series of steps (Fig. 10) which divide the circumference equally, thus cutting out a definite fraction of the light at each revolution. By placing the different steps in succession before one slit, different fractions of the light are cut out, and by altering the width of the other (bilateral) slit to match, it can be completely calibrated.[2] When the instrument is in constant general use, the calibration will in the end save time, but where only a few observations are to be made, or where measurements are being made on strongly absorptive substances, the combination of sector and slit without calibration is often more advantageous, by saving time in the first case, and in the second by avoiding the large effect of zero

[1] E. V. CAPPS, ASTROPHYSICAL JOURNAL, 11, 25-35, 1900.

[2] *Phil. Mag.*, 48, 420-430, 1899, and ASTROPHYSICAL JOURNAL, 11, 6-24, 1900.

[3] D. W. MURPHY, ASTROPHYSICAL JOURNAL, 6, 1-10, 1897.

errors in reading the width of very narrow slits. The curves given by Mr. Capps[1] for flint glass show that if great accuracy is wanted, the proportionality of slit-width and intensity of field, especially in the red, is not to be relied upon for variations of intensity much exceeding 2 per cent. with a slit-width of 0.5 mm, so that in working with that width a sector of at least twenty steps would be necessary if the maximum error is to fall

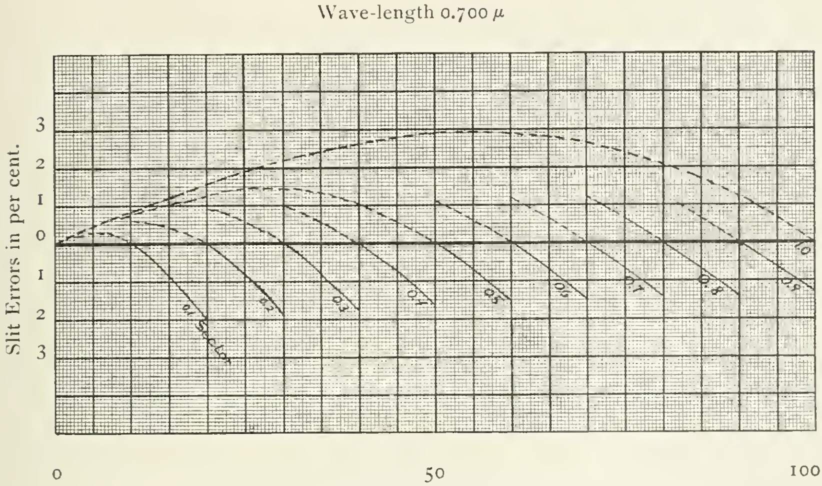


FIG. 11.

within 3 per cent. With the slit-width of 0.25 mm, a ten-step sector would give sufficiently accurate results. The variations of these errors due to lack of calibration for various intensity ratios is plotted from Mr. Capps' tables[1] for a ten-notch sector, a 0.5 mm slit and a wave-length of 0.700 μ, where he found the maximum error. The full lines give effects of decreasing, and the dotted lines of increasing the slit-width to obtain a match (Fig. 11). Of course in the measurement of strong absorption the sector can be used in conjunction with a calibrated slit, if high accuracy is needed. This method of simultaneous use of

[1] ASTROPHYSICAL JOURNAL, 11, 25–35, 1900.

sector and variable slit has been used by Professors Brace and Moore since 1899.[1] It has also been noted and its advantages pointed out by Brodhun[2] in a review of Professor Brace's article.[3]

UNIVERSITY OF NEBRASKA,
Lincoln, Sept. 23, 1902.

[1] D. B. BRACE, *Bull. Amer. Phys. Soc.*, **1**, No. 2, pp. 29–31. Abstract of article read December 28, 1899.

[2] *Zeitschrift für Instrumentenkunde*, **20**, 210–212, 1900.

[3] *Phil. Mag.*, **48**, 420–430, 1899, and ASTROPHYSICAL JOURNAL, **11**, 6–24, 1900.

THE PHYSICAL CAUSES OF THE DEVIATIONS FROM NEWTON'S LAW OF GRAVITATION.[1]

By PETER LEBEDEW.

THE question of the deviations from the general law of gravitation and their physical causes is one of the oldest questions of astrophysics — older, indeed, than Newton's law itself. It was propounded by Kepler three hundred years ago, and was answered by him in a way that we could not improve upon today. This deviation from Newton's law appears in its most striking and at the same time most simple form in the case of comets' tails, where it occurs as a pronounced repulsive force from the Sun. The development of the views as to the nature of this repulsive force and its basis in physics forms one of the most interesting chapters of astrophysics. The intimate association of an astrophysical theory with the current physical views and results can be followed in its course through three centuries.[2]

In 1608 Kepler expressed the opinion that comets' tails are the evaporizations of the heads, which move independently of the heads and are repelled by the Sun instead of being attracted. Kepler attributes the cause of this repulsive force to the solar radiation. The corpuscular theory of light was in his time the prevailing view in optics, and it is a necessary mechanical consequence of this theory that the rays of light should exert pressure upon the bodies they encounter, whence the repulsion of the very small particles of vapor by the solar rays could be explained. Newton[3] distinctly pointed out in 1687 that Kepler's assignment of the repulsion to the forces of pressure of the radiation might

[1] A paper presented at the Göttingen meeting of the *Astronomische Gesellschaft* on August 4, 1902, communicated by the author.

[2] An extended account of the history of this development may be found in DE MAIRAN'S *Traité physique et historique de l'Aurore Boréale* (Second Edition), Paris, 1754; and in Zöllner's *Wissenschaftliche Abhandlungen*, Zweiter Band, Zweiter Theil, Leipzig, 1878.

[3] *Principia*, Lib. III. London, 1687.

be assumed as a method of explanation, but he did not himself adopt Kepler's view, attempting to bring the repulsion of the comets' tails under his law of gravitation and considering it merely as an apparent repulsion. He made the hypothesis that the universe is filled with a gaseous medium denser than the gases of the comets' tails, which therefore are buoyed up (according to the law of Archimedes) and are therefore only apparently repelled by the Sun.

Euler[1] in 1744 clearly perceived the difficulties associated with Newton's hypothesis and returned to Kepler's view, similarly attempting to explain the repulsion by the pressure of light. But as an active opponent of the emission theory of light, Euler[2] had adopted the Huyghens theory and assumed that light is an undulatory motion with longitudinal displacements in the ether. He treated the matter as a series of mechanical impacts which the incident longitudinal waves exert upon the bodies affected, and in this way accounted for the existence of the pressure of light.

In the middle of the eighteenth century, some thirty-five years before Cavendish investigated in the laboratory the law of attraction, de Mairan and du Fay (1754) made the first attempt to test experimentally the pressure of radiation which should cause the deviation from Newton's law. De Mairan and du Fay[3] planned their research with admirable skill, but they encountered difficulties (air currents) which could not be overcome with the experimental means of the eighteenth century, and they were compelled to leave unanswered the question of the existence of the mechanical pressure of light.

Attention was directed in the nineteenth century to the motion of comets by the pioneer investigations of Olbers. He regarded the repulsive force on comets' tails as a fact firmly established by observations; as to its physical foundation, Olbers[4] in 1812

[1] See ZÖLLNER, *loc. cit.*, p. 525.

[2] *Histoire de l'Academie Royale de Berlin*, 2, 121, 1746.

[3] DE MAIRAN, *Traité*, *loc. cit.*, p. 371.

[4] *W. Olbers Leben und Werke*, 1, 331. Berlin, 1894.

discarded the views of Kepler as well as those of Newton as hypotheses which could not be based on experiment, and he expressed a new suspicion in a very cautious way: "One can scarcely refrain from thinking of something analogous to our electrical attractions and repulsions." If we reflect that Olbers expressed this suspicion as the theory of electricity was celebrating its first triumphs, we can comprehend his suggestion of electric forces, the laws of whose action were known from the direct experiments of Coulomb (1785).

The electrical theory of Olbers now became prevalent. The decrease of the electrical force with the square of the distance (which also applies to the pressure of light) sufficed for Bessel [1] to give in 1836 a simple theory of comets' tails, and to compute the absolute magnitude of the repulsive force from the curvature of the tail. From the measurement of many comets' tails, Bredichin[2] found the magnitude of the repulsive force to be characteristic for different component substances in the tail, and he determined its value to be 0.2, 1.1, and 17.5 times the force of attraction.

The electrical theory of Olbers depends upon two hypotheses —first, that the Sun has a constant electrical charge, and second that the separate molecules of the gases of the tail receive charges upon leaving the head, the sign of which agrees with that of the solar charge. No progress worthy of mention has been made in the course of years in establishing a physical basis for these two hypotheses. The assumption that the Sun is a charged body could be connected with the magnetic phenomena of the Earth only by introducing additional hypotheses; but the absolute magnitude of the solar charge could not be measured, indeed not even its sign could be determined. Phenomena of electrification of the separate molecules of gas under the conditions assumed by the second hypothesis have not been discovered by the physicist in the laboratory.

The similarity of the optical phenomena in comets' tails and in Geissler tubes is often invoked in support of the assumption

[1] *A. N.*, 13, 185, 1836. [2] *Annales de l'Observatoire de Moscou* (2), 1, 45, 1886.

of an electrification of the materials of the tail, but such an argument is unpermissible and contradicts the principle of the conservation of energy, according to which every phenomenon of luminosity is associated with the giving out of energy, which is not possible for molecules of gas having a constant *electrostatic* charge. The cause of the luminosity of comets' tails is probably to be attributed to the fluorescence of strongly illuminated gases, which has been demonstrated by the direct experiments of Lommel[1] and of Wiedemann and Schmidt[2].

The serious objections which may be raised against the electrical theory were pointed out by Zöllner[3] (1872), who most fully worked out the electrical theory, but declared himself ready to give up his theory and adopt that of Kepler if the existence of a pressure of solar radiation could be proven.

The question of the existence of such a pressure was solved independently of astronomical theories thirty years ago, by Maxwell[4] (1873) as a result of his electro-magnetic theory of light, and by Bartoli[5] (1876) as a consequence of the second law of thermodynamics. These theoretical investigations agreed in indicating that the pressure (p) of radiation must necessarily exist, and that it stands in a simple relation to the quantity of energy (E) falling upon the body per second, as a beam of parallel rays, and to the velocity of light (v). For an absorbing body this relation is

$$p = \frac{E}{v} .$$

For the solar radiation at the distance of the Earth this gives a pressure of 0.5 milligram per square meter. It has recently been possible for me[6], as well as for Nichols and Hull[7], to

[1] *Wied. Ann.*, **19**, 356, 1883.

[2] *Wied. Ann.*, **56**, 18, 1895; **57**, 447, 1896.

[3] *Ueber die Natur der Kometen*, p. 198. Leipzig, 1872.

[4] *Treatise on Electricity and Magnetism*, § 792.

[5] *Il Nuovo Cimento*, **15**, 195, 1883.

[6] *Ann. der Physik*, (4), **6**, 433, 1901; ASTROPHYSICAL JOURNAL, **15**, 60, 1902.

[7] *Physical Review*, **13**, 307, 1901; ASTROPHYSICAL JOURNAL, **15**, 62, 1902.

demonstrate the existence of this pressure of light by direct laboratory experiments, and the formula due to Maxwell and Bartoli was quantitatively confirmed.

Before these experimental researches had been made, Fitzgerald[1] in 1884 applied the theoretical results of Maxwell to the deviations from Newton's law in the case of the motion of comets, but he committed the error of extending his results to the gaseous molecules of the tail without considering that Maxwell's deduction only holds for bodies which are large in comparison to the wave-lengths of the incident radiation. This error was avoided in the considerations simultaneously advanced by Lodge[2] and by myself[3] as to the repulsive force of the Sun, and by myself[4] as to the deformation and disintegration of comets' heads. In his theory of the solar corona Arrhenius[5] has recently made the same mistake again.

For a spherical body of large dimensions as compared with the wave-lengths of the solar radiation, the resulting action (F) of the Sun is expressed in terms of the force of gravitation as

$$F = 1 - \frac{1}{10{,}000} \cdot \frac{1}{r\delta},$$

where r is the radius in centimeters and δ is the density of the body (referred to water as unity)[6]. It is evident from this that the deviations from Newton's law for a body whose dimensions exceed one meter are far less than the limits of error of the most delicate astronomical observations.

For the head of a comet consisting of a swarm of stones smaller than 1 cm, the deviation from the law of gravitation can just be proven under favorable conditions of observation; for still smaller stones the deviation will be correspondingly

[1] *Proc. Roy. Soc. Dublin*, 1884. [2] *Nature*, 1891.

[3] *Wied. Ann.*, 45, 292, 1892. At that time I had unfortunately overlooked Fitzgerald's paper.

[4] *Rapports présentés au Congres internationale de Physique à Paris*, 2, 133, 1900.

[5] *Physikalische Zeitschrift*, 2, 81, 1900.

[6] See *Wied. Ann.*, 45, 294, 1892. The Sun's rays exert a somewhat greater repulsion upon bodies which are not spherical, as for such the ratio of surface to volume is greater.

larger. The converse proposition may however be asserted — that if there is no appreciable deviation from Newton's law, and if the extent of the uncertainty of the observations is known, the lower limit can be assigned for the size of the stones of the comet's head.

If the head consists of a swarm of meteorites of unequal sizes, some sufficiently small, the swarm will be deformed and will disintegrate — as must be especially conspicuous in the case of comets which have become periodic. The prediction, in the ordinary manner, of the orbit of such a swarm will show greater departures from the subsequent observations. It is possible that the peculiar movements of the Bielids can be explained in this way.

The formula cited above fails for the case of dust particles with dimensions of the thousandths of a millimeter — which are therefore comparable with the wave-lengths of the incident solar radiation. Schwarzschild[1] has shown that in this case the repulsive force reaches a maximum at certain dimensions and then rapidly decreases for smaller dimensions.

Gaseous molecules which are exposed to the solar radiation develop resonance phenomena, which are accompanied by forces of pressure from the incident rays, as I have shown by computations[2]. But in this phase of the subject, which is of especial importance in astrophysics, we must still await the results of direct experimental researches.

As we review the historical development of our views as to the physical cause of the deviations from Newton's law of gravitation, we see that the view of Kepler, after being displaced first by Newton's pressure theory and then by Olbers' electrical hypothesis, again appears, now based upon the pressure of light theoretically deduced by Maxwell and Bartoli, and demonstrated by direct experiment. Kepler's theory has received a physical

[1] *Berichte der Münchener Akad.*, 1901.

[2] *Wied. Ann.*, **62**, 170–172, 1897. For gases the amount of the pressure is equal to the quantity of energy absorbed divided by the velocity of light. The ratio of absorption to mass is very different for different gases.

foundation, and we are now obliged to assert that the Sun does possess repulsive forces, the magnitude of which is obtained from the laboratory experiments. We can now quantitatively predict in advance the deviations from Newton's law which must necessarily occur, or we can discuss the consequences of them.

The question whether electrical actions may arise in one case or another, also carrying with them an appreciable departure from Newton's law, must at present be left as an open question. Not until we have taken into quantitative account the action of the pressure of light, which is unquestionably present, can we conclude as to the presence or absence of further forces, and assert whether additional assumptions are necessary or whether that of Kepler is alone sufficient.

MOSKAU,
August, 1902.

NITROGEN BANDS *VS.* "NEW HEADS TO CYANOGEN BANDS" IN ARC SPECTRA.

By PERCIVAL LEWIS and A. S. KING.

IN the June number of this JOURNAL Professor C. C. Hutchins[1] published a reproduction of a photograph of the spectrum of the arc between copper electrodes, in which appears a band with its head at λ 3914.47, which he assumes to be a new head to the group of cyanogen bands beginning at λ 3883.55. He found another head at λ 4278.5, and possibly one above λ 4606, which he thinks may bear a similar relation to the cyanogen bands at λ 4216 and λ 4606.

Deslandres[2] has shown that the most prominent of the negative-electrode bands of nitrogen has its head at λ 3914.6, and Hasselberg[3] places the head of another of these bands at λ 4278.6. Their general appearance is not unlike that of the cyanogen bands, except for the very characteristic property, pointed out by Deslandres,[4] that the lines composing these negative-electrode bands are alternately strong and weak in the neighborhood of the heads. This peculiarity is strikingly shown on Professor Hutchins' plate.

The writers have measured the wave-lengths of all the lines on the plates of Hutchins and Deslandres between the limits λ 3914 and λ 3883, with the exception of those crowded together in the head, and the results are given below. As these measurements are taken from prints on ordinary paper, the individual results are liable to considerable error, but the general agreement between the parallel columns is very striking. The results are not selected, but all the lines between the limits are given, and there is a line-for-line correspondence. The identity of the two

[1] ASTROPHYSICAL JOURNAL, **15**, 310, 1902.

[2] *C. R.* **103**, 375, 1886.

[3] *Mem. de l'Acad. St. Petersburg*, 1885 (7), **32**, No. 1.

[4] *Ann. Chim. et Phys.* (6), **15**, 57, 1888.

bands is shown very clearly by making a diagonal scale from one plate and comparing it directly with the other.

Hutchins Copper arc in air	Deslandres Nitrogen, negative pole	King Carbon arc, Nitrogen atmosphere	Hutchins Copper arc in air	Deslandres Nitrogen, negative pole	King Carbon arc, Nitrogen atmosphere
3914.47	3914.60	3914.55	3904.55	3904.45	3904.61
14.00	13.95	....	03.61	03.35	....
13.55	13.50	....	02.55	02.35	....
13.32	13.20	13.10	01.47	01.30	....
12.90	12.85	12.76	00.35(?)	00.05	....
12.62 }	12.40	....	3898.94	3898.85	....
12.17 }		....	97.69	97.60	....
11.75	11.8	....	96.30	96.35	....
11.13	11.20	....	95.18	95.00	....
10.85	10.60	....	93.70	93.65	....
10.06	10.00	....	92.30	92.20	....
09.45	09.40	09.43	90.80	90.80	....
08.65	08.60	....	89.45	89.30	....
08.00	07.85	....	87.75	87.70	....
07.14	07.10	06.80	86.20	86.10	....
06.40	06.25	06.46	3884.40	3884.45	....
3905.55	3905.35	....			

On Hutchins' plate there is the appearance of an overlapping band beginning at about λ 3889, and below that point several lines seem double. This may have slightly affected some of our measurements.

On some of the plates taken by one of us[1] while studying the cyanogen bands in the carbon arc, some peculiar shadings and grouping of lines attracted our attention. This was especially marked near the head of the λ 3883 band, particularly on some plates taken with the arc in a nitrogen atmosphere. Some faint lines were present on these plates and absent from others of equal exposure and development. It seems unlikely that they were due to metallic impurities; the carbons were prepared from calcined crystallized sugar, and showed scarcely a trace of any impurity. The wave-lengths of such of these lines as appeared particularly differentiated from the carbon lines in the neighborhood were measured, and the results are given in the third column of the above table. The carbon lines are crowded so closely in this region that it would not be difficult

[1] A. S. King, Astrophysical Journal, 14, 323, 1901.

to find almost any desired wave-length; but it is to be noted that these extra lines appearing on some plates only were first selected by inspection, and their wave-lengths afterward determined quite independently of the numbers in the first two columns; there was no selection or elimination. On some plates the line at λ 3914.6 is very sharply defined, on others entirely absent. The results seem to make it appear at least possible that this band of the negative-pole nitrogen spectrum may be found even in the carbon arc.

In the light of these measurements it seems probable that Professor Hutchins has made a mistake in his conclusions; but at the same time he has observed a very interesting and important fact, for, so far as known to the writers, no one has ever before found in the arc spectrum the bands or lines of any permanent elementary gas except those of hydrogen.

Several facts besides the identity of wave-lengths with those of the nitrogen band oppose the view that this band is due to cyanogen. It has never been observed before, notwithstanding all the varied conditions under which the cyanogen spectrum has been studied. Professor Hutchins found that moistening the copper electrodes with oil greatly intensified the band at λ 3883, but had no effect on that at λ 3914, as one might expect if this were a cyanogen band. The physical appearance of the lines is also different; in the photograph the cyanogen lines run across the field, while the lines of the other band are confined to the neighborhood of one pole — presumably the negative.

On the other hand, some analogies with spark spectra make it seem reasonable to assume the possibility of the appearance of the nitrogen spectrum under the special conditions of Professor Hutchins' experiment — namely, with a hissing or discontinuous arc, which is kept cool by periods of extinction and by the high thermal conductivity of copper. The fact that on starting the arc the negative pole was tipped with a bright glow would further lead us to look particularly for the negative-pole bands. We should certainly expect to see nitrogen bands or lines in the spectrum of the transformer discharge between

terminals of nickel or cobalt, in which cases Hutchins observed the band at λ 3914, while the cyanogen bands were absent.

The bands were most clearly brought out by intermittently starting and stopping the arc, thus keeping it cool and reducing the amount of metallic vapor present. It is to be noted that the copper lines in the plate are stronger near the hot positive electrode, and grow fainter as they approach the cooler negative pole; the air seems to pick up the current at the point where the metallic vapor fails, and to become luminous in consequence. In spark spectra we see similar effects; the metallic lines are stronger at the poles, where the vapor is denser, while the air bands or lines bridge the cooler gap. In the case of the spark discharge it has also been shown[1] that the first partial discharge passes through air; the succeeding sparks go by preference through the metallic vapor produced by the first spark. If the same be true of the arc we can see why the nitrogen bands are brought out more strongly by the intermittent arc.

The writers have endeavored to find air bands visually in the spectrum of the arc, between copper electrodes, but without success. The band at λ 3914 is too near the limits of the visible spectrum to expect to see it. Deslandres states that this band is the strongest in the negative-electrode spectrum of nitrogen, and its occurrence in the arc while all other parts of the nitrogen spectrum (except the fainter band at λ 4278) are absent, indicates that of all the spectra of nitrogen that of the negative pole appears under the lowest conditions of temperature or current intensity.

It seems as though this band should be found on some of the plates taken by Professor Crew with the rotating metallic arc, when some of the less volatile metals were used.

UNIVERSITY OF CALIFORNIA,
Berkeley, August 20, 1902.

[1] SCHUSTER and HEMSALECH, *Phil. Trans.*, 193, 189, 1899.

THE TOTAL LIGHT OF ALL THE STARS.

By Gavin J. Burns.

My attention has been called to the article in your issue of December last on the above subject. Professor Newcomb there observes that he cannot learn that any attempt to measure the amount of light received from various regions of the sky has ever been made. As I have made a number of observations on this subject during the last four years, in the course of which I have gone over nearly the same ground as Professor Newcomb, I venture to send a short account of them.

1. *Relative brightness of different portions of the sky.*— My attempts to measure the relative brightness of the sky have not been very successful. Like Professor Newcomb, I was much surprised at the smallness of the difference between the brightness of the galaxy and the rest of the sky. My first attempt was based on the principle of Abney's revolving sectors. The brighter part of the sky was viewed through an aperture in front of which a disk with openings in it was made to revolve rapidly, while the less bright part was viewed through a similar aperture but without the revolving disk. I obtained no results of any value by this method.

I next tried the following plan: I procured several pieces of ordinary clear glass about 10 cm by 7 cm. I then looked at the part of the Milky Way I wished to examine through one, two, three, or four thicknesses of glass, and compared the brightness of the Milky Way seen through the glass with some other portion of the sky. The result was that it required four thicknesses of glass to reduce the light of the Milky Way near *Orion* to the general luminosity of the extra-galactic sky, while it required seven thicknesses to produce the same results in the brightest part of the region between *Cassiopeia* and *Cygnus*. I have been unable to detect any difference in the luminosity of the sky outside the visible boundary of the Milky Way. I next

determined by the best means at my disposal the proportion of light transmitted by the glass. This I found to be 52 per cent. for four thicknesses and 36 per cent. for seven thicknesses. It follows that the luminosity of the Milky Way is from two to three times as great as that of the rest of the sky.

2. *Amount of sky light compared with star light.*—The method I adopted was to compare the brightness of the sky with the brightness of a star which was out of focus in a telescope. I used a 9.5 cm refractor with a power of 100. A suitable star was selected and put out of focus to such an extent as to make the brightness of the apparent disk equal to the brightness of the sky. The latter was viewed through a hole in a black screen which subtended an angle roughly equal to the angular diameter of the disk of the star. One eye was applied to the telescope and the sky viewed with the other. Each observation recorded is the mean of a right-eye and a left-eye observation. The following table gives the results:

Name of star	Magnitude H. P.	Angular diameter of disk
μ *Leonis*	4.14	12°
ζ *Ursae minoris*	4.49	11
δ *Ursae majoris*	3.41	17
β *Eridani*	2.87	17
κ *Geminorum*	3.61	11.5

The last column gives the apparent diameter of the star when put out of focus to such an extent as to equal the light of the sky. Let $D =$ this quantity.

Also, let $a =$ aperture of telescope,

$e =$ aperture of pupil of eye,

$b =$ fraction of light, transmitted by telescope,

$x =$ area of sky that gives an amount of light $=$ that of star of magnitude m;

$y =$ Ditto $=$ that of star of magnitude o.

Then

$$x = \left(\frac{De}{a}\right)^2 \frac{\pi}{4b}$$

and

$$y = x \times 10^{\frac{2m}{5}}.$$

In order to determine x, the values of e and b must first be found. I measured e by finding the diameter of an opaque disk which would stop all the light coming from a small distant hole. I found it to be between 6.6 mm and 7.1 mm (0.26 and 0.28 inch). I assumed $e/a = 0.07$. I determined b by turning the telescope to the Sun and allowing its image to be formed on a sheet of white paper, which was put at such a distance as to give an image of the same brightness as the part of sheet exposed to the direct rays of the Sun. The image was then 6.35 cm in diameter. Consequently

$$b = \left(\frac{6 \cdot 35}{9 \cdot 53}\right)^2 = \frac{4}{9} .$$

The results are as follows :

μ *Leonis* gives - - - - - - -	$y = 56.4$
ζ *Ursae minoris* gives - - - - -	$y = 65.4$
δ *Ursae majoris* gives - - - - -	$y = 57.8$
β *Eridani* gives - - - - - - -	$y = 35.1$
κ *Geminorum* gives - - - - - -	$y = 31.9$
Mean - - - -	$y = 49.3$

Or half a square degree of non-galactic sky gives as much light as a star of magnitude 5.0.

I believe the discordance in the above observation is largely due to the difference in color between the sky and the image of the star in the telescope, and the consequent difficulty in making a comparison. The object-glass of my telescope is of a perceptibly greenish tinge. I believe better results could be obtained with a smaller telescope having a thinner object-glass, or with a reflector.

3. *Faintest magnitude visible.*—In the course of the observations above recorded I discovered, somewhat to my surprise, that the faintest stars visible to the naked eye could be seen through a telescope with the aperture cut down to 3 mm (0.125 inch). Allowing for loss of light in the telescope itself, only about one-tenth of the light reaches the eye through this aperture which would enter the unaided eye. It appears, therefore that on a perfectly dark background stars of the eighth magnitude would be readily visible.

KENDAL HOUSE, HOLLAND ROAD,
WEYMOUTH, ENGLAND.
September 2, 1902.

COÖPERATION IN OBSERVING RADIAL VELOCITIES OF SELECTED STARS.

By Edwin B. Frost.

It is a somewhat peculiar fact that in the present state of spectrographic work it is not possible to find published determinations of the radial velocities of as many as a dozen stars (a few spectroscopic binaries excepted) which have been observed at three different observatories. This anomalous condition is probably in part due to the fact that in the past two or three years most of the large spectrographs designed for line of sight determinations have been in process of construction or reconstruction; and perhaps in part also to the disinclination of the observers to publish their results while subject to uncertainties as to the wave-lengths of comparison lines or stellar lines.

For those not engaged in work of this kind, probably the most satisfactory evidence of the accuracy of the determinations would be gained from a comparison of the results of different observers and instruments. And while each observer doubtless satisfies himself of the accuracy of the operation of his instrument by frequent measurements of the velocity of the Moon and planets, nevertheless there would be great satisfaction in finding that the different observers were obtaining substantially identical results, although using different comparison spectra, different stellar lines, and with generally different conditions of observation, measurement, and reduction. The history of progress in other fundamental measurements would suggest that systematic differences between the results at different observatories are likely to become more apparent as the accuracy of the determinations increases. The limit of accuracy ordinarily attainable at present in determining the velocity of approach or recession of a star of the solar type with sharp spectral lines is not known, and cannot be satisfactorily established by comparing

observations made with one instrument, in which there may be present sources of systematic error, however small.

For reasons like these and for others which have doubtless often occurred to spectroscopists, it would seem highly desirable to have some common action by different observers which would furnish a basis for an estimate of the accuracy of spectrographic work in general, and of that of each observer and instrument in particular.

It is therefore very satisfactory to be able to report that a beginning has been made with a simple plan of coöperation in the observation of the radial velocities of a few selected stars, in which it seems probable that the observers with the principal spectrographs now in use will take part as far as possible. The present status of the scheme can perhaps be best indicated by quoting here two circular letters recently sent out by the writer.

YERKES OBSERVATORY, FEBRUARY 20, 1902.

DEAR SIR: For reasons briefly stated in a paper in the January number of the ASTROPHYSICAL JOURNAL, a reprint of which accompanies this letter, it has seemed to the writer that the time has arrived for an attempt at coöperation in the observation of certain stars (which might be called "fundamental velocity stars") by the spectrographs now engaged in such research.

Opinions may differ as to the present desirability of such coöperation and as to its details; hence the principal object of this letter is to inquire whether, and under what circumstances, you would be prepared to coöperate in the observations.

In order to expedite arrangements, should it prove that several spectrographs will coöperate, a provisional list of stars is submitted for your consideration, in the hope that you will freely suggest any changes in the list, or propose still other stars.

In the preparation of the list attention has been given to the distribution of the stars in R. A. and Dec., to the character of their spectra, and to their magnitude. Spectra of nearly all of these stars have been obtained at the Yerkes Observatory, and it is believed that the lines will in all cases be found suitable for accurate measurement. Stars known to be visual[1] or spectroscopic binaries and those varying in brightness have been excluded from the list.

Although the time required for merely obtaining the plates of these spectra will not be significant, yet in view of the expenditure of time neces-

[1] β *Leporis* excepted.

PROVISIONAL LIST OF TWENTY "FUNDAMENTAL VELOCITY STARS."

Star	R. A.	Dec.	Mag.	Class
β Cassiopeiae	0^h 04^m	+58° 36′	2.4	XII *ab*
α Arietis	2 02	+23 0	2.0	XV *a*
α Persei	3 17	+49 30	1.9	XII *ac*
α Tauri	4 30	+16 19	1.0	XVI *a*
β Leporis	5 24	−20 51	3.0	XIV *a*
γ Geminorum	6 32	+16 29	2.0	VIII *a*
β Geminorum	7 39	+28 16	1.1	XV *a*
β Cancri	8 11	+9 30	3.8	XV *a*
α Hydrae	9 23	−8 13	2.2	XV *a*
α Crateris	10 55	−17 46	4.1	XV *a*
β Corvi	12 29	−22 50	2.8	XIV *a*
α Boötis	14 11	+19 44	0.0	XV *a*
α Serpentis	15 39	+6 44	2.7	XV *a*
β Ophiuchi	17 38	+4 36	2.9	XV *a*
η Serpentis	18 16	−2 56	3.4	XV *a*
γ Aquilae	19 42	+10 22	2.8	XV *a*
ε Cygni	20 42	+33 36	2.7	XV *a*
ε Pegasi	21 39	+9 25	2.4	XV *a*
α Aquarii	22 01	−0 49	3.2	XIV *ac*
γ Piscis	23 12	+2 44	3.8	XV *a*

The classification of Miss Maury has been used above.

sary for measurement, the list of twenty stars may seem too long, if each one is to be observed three times annually with each coöperating spectograph. To the writer it would seem sufficient, at the beginning, if the list should be limited to ten or twelve stars.

It is, of course, desirable that the same comparison spectrum should be employed as is regularly used with the given spectrograph ; variety between the different spectrographs being desirable. Agreement should be reached as to the number of star lines to be measured on a plate, and the suggestion is made that the number should not exceed twenty; but it would seem undesirable to make any effort that the same lines be used with the different instruments.

Awaiting with interest your reply, and hoping for your coöperation, and an expression of your views, I am, etc.

YERKES OBSERVATORY, JUNE 26, 1902.

DEAR SIR: The circular letter in reference to regular spectographic observations of certain selected stars, of date of February 20, was addressed to Messrs. Bélopolsky, of Pulkowa, Campbell, of Mt. Hamilton, Deslandres, of Meudon,Gill, of Capetown, Lord, of Columbus, O., Newall, of Cambridge, England, and Vogel, of Potsdam. Replies have not yet been received from Messrs. Gill and Newall, but from all the others letters have now arrived, in

general favorable to the suggestions of the circular letter, It therefore seems desirable to communicate without further delay the substance of the responses that have been received, so that some degree of coöperation may be begun as soon as possible.

Taking up the letters in order, a condensed statement may be made as follows:

I. A. BÉLOPOLSKY (dated March 18, 1902): "Ich nehme gewiss gerne Theil in der von Ihnen vorgeschlagenen Arbeit." It would be well (1) if a general scheme of the observations could be worked out, covering, for instance, points like these: Should each star be observed in each position of the instrument (camera above, camera below) or in only one? Should the comparison spectrum be photographed at the beginning and end or at middle of exposure to star? (2) While leaving the choice of a comparison spectrum open to each observer, it would be interesting to have an additional comparison spectrum, as hydrogen, common to all observers; making the exposure to the hydrogen tube, for instance, at the middle of the star's exposure. (3) The star list should be longer, and stars with large velocity are desirable. (Stars below the equator could hardly be observed at Pulkowa.) (4) Epochs of observation should be arranged to eliminate errors of reduction to Sun, making the reduction once positive, and once negative. (5) As possible substitutes for the southern stars the following are suggested: Beta *Andromedae*, Mag. 2.3; Iota *Aurigae*, 3.0; Epsilon *Leonis*, 3.2; Epsilon *Virginis*, 3.0; Beta *Boötis*, 3.0; Delta *Boötis*, 3.0; Zeta *Herculis*, 3.0; Gamma *Cephei*, 3.5.

II. W. W. CAMPBELL (June 14): "It gives me pleasure to say that I think the scheme is an excellent one in its general features, and the Lick Observatory will be glad to coöperate in carrying through a well-considered program. The subject is one whose importance impressed itself upon me a few years ago, and was the subject of numerous conversations between Professor Keeler, Mr. Wright and myself. . . . . It seems to me that ten stars are sufficient, and that twelve should be the maximum, three observations each per annum." He suggests that the following ten stars be omitted from the list: Beta *Cassiopeiae* (on account of its poor lines), Alpha *Tauri*, Gamma *Geminorum*, Beta *Cancri* Alpha *Hydrae*, Beta *Corvi*, Alpha *Serpentis*, Eta *Serpentis*, Epsilon *Cygni*, Alpha *Aquarii*. "In past years we have secured a considerable number of observations of all the twenty stars on your list, so that we have a good starting point for all of them. (June 19.) Further consideration of the matter convinces me that thirty of these standard plates per year as a maximum will answer all purposes fully as well as sixty."

III. H. DESLANDRES (March 14): "Votre idée d'une coöperation de tous les observatoires est excellent, et je l'accepte immediatement." Ten stars, suitably distributed, would be sufficient, at least for the beginning. To

render the observations fully comparable, it would be well to designate for each star a period of ten days within which the participators should make one or two spectrograms of that star.

IV. H. C. LORD (February 29): The list should be of ten or twenty stars, and each should be observed on five nights, then published. After this work had been completed each observer could use the list as he chose, preferably taking one of them on each observing night as a control star. Thirty spectrograms of one star would be more desirable than three spectrograms annually of ten stars.

V. H. C. VOGEL (March 1902): "In Beantwortung Ihres Schreibens vom Februar 1902, theile ich Ihnen mit dass ich Ihren Vorschlag dass eine Anzahl von Sternen von allen denjenigen Beobachtern, die sich mit spectrographischen Geschwindigkeitsbestimmungen beschäftigen, regelmässig jedes Jahr beobachtet werden soll, für ganz zeitgemäss halte, und dass ich schon früher den Gedanken gehabt habe einen ähnlichen Vorschlag zu machen." It is recommended that the number of fundamental stars be made as small as possible, but that they should be observed more than three times yearly. The stars Alpha *Persei* and Alpha *Boötis* are suggested, as they can be consecutively observed for a long time by northern observers. From its high declination Alpha *Persei* offers opportunity for measurement at very different hour angles. Ten star lines would be sufficient for a plate, and observers would not be expected to exceed twenty star lines. For the sake of uniformity in publication it is suggested that the following data should be given: (1) Place and date. (2) Greenwich Mean Time, for the middle of the exposure time. (3) Value of the linear dispersion in some clearly intelligible form, using either $\mu\mu$ per $\frac{\text{mm}}{4}$ or tenth-meters per mm. (4) Velocity of the star in kilometers referred to the Sun, reduced with Schlesinger's tables: (*a*) Plate so placed on microscope carriage that increasing readings correspond to increasing wave-lengths; (*b*) Plate in the opposite direction. (Note. It would be interesting to have the results of the two series of measures given separately.) (5) Velocity of the star in kilometers referred to the Sun, the mean of the results (*a*) and (*b*). (6) Mean error, $\epsilon = \pm\sqrt{\frac{\Sigma v^2}{n-1}}$, in kilometers, of the determination of the velocity from a single line. The mean of the separate settings of the micrometer thread should be taken as the value for the line, and in both positions of the plate under the microscope. (7) Number of lines (m) measured in the spectrum. (8) Mean error, $\epsilon = \pm\sqrt{\frac{\Sigma v^2}{m(n-1)}}$ in kilometers of the final velocity of the star deduced from one plate. It is desirable that the observers should give once for all the values employed for the wave-lengths in the comparison spectrum and in the stellar spectrum, as for

instance: comparison spectrum, iron, wave-lengths according to Kayser; stellar lines: wave-lengths according to Rowland's tables for the solar spectrum.

VI. E. B. FROST: To these expressions of opinion may be added those of the circular letter of February 20, as setting forth the views of the writer, which have the approval of the director of the Yerkes Observatory.

It would now appear possible to find in the statements for the six spectrographs concerned a sufficient basis for beginning a scheme of coöperation, the incidental details of which may be worked out subsequently and gradually.

It seems to express the average opinion (1) that a list of ten stars should be adopted; (2) that three or more observations should be secured annually of each of these; (3) that a greater number of observations should be secured of Alpha *Boötis* and Alpha *Persei;* (4) that the observations of the other eight should be suitably distributed during the year, and should be made as nearly as convenient at the same dates by the different observers; (5) that the results should be published in a uniform manner.

From a consideration of what has *not* been stated in the responses, it would seem reasonable to draw certain inferences, *e.g.*, (*a*) that large latitude should be reserved to each observer in his mode of observing, choice of comparison spectrum, selection of star lines, and method of reduction; (*b*) that the suggested coöperation should not involve so much observation and reduction as to be a burden upon the participators and interfere with their regular programs of work.

Referring, then, to the summary of opinions, it would seem well: 1. To adopt as the star list that previously submitted, with the omissions suggested by Professor Campbell. This would accordingly include:

Alpha *Arietis*	Alpha *Boötis*
Alpha *Persei*	Beta *Ophiuchi*
Beta *Leporis*	Gamma *Aquilae*
Beta *Geminorum*	Epsilon *Pegasi*
Alpha *Crateris*	Gamma *Piscis*

For those who could not well secure the southern stars, substitutes could be chosen, if desired, from Professor Bélopolsky's supplementary list. It will be recalled that the list of ten stars intentionally includes faint as well as bright stars, and those in widely different declinations.

2. It has already been the practice at the Yerkes Observatory to employ Alpha *Boötis* as a control star when the Moon or planets are not readily available; and doubtless all will gladly concur with Professor Vogel as to it and Alpha *Persei.*

3. For the sake of definiteness in carrying out the suggestions of Messrs. Bélopolsky and Deslandres, it is now proposed that the three observations of

a star be secured as follows: one plate as nearly as possible at the date of zero Earth velocity, practically at the time of the star's opposition to the Sun, and the other two plates at dates as nearly as possible thirty days before and after this date.

4. In so far as possible all will no doubt adopt the form of publication proposed by Professor Vogel. It should be said, however, that it may be somewhat inconvenient for some observers to reduce separately the measures with the plate under the microscope in the direct (readings and wave-lengths increasing) and in the reversed positions. Nevertheless, additional pains will be willingly taken for the sake of uniformity.

The early publication of previous determinations of velocities of any of the stars on the list, even if the reductions are provisional and not entirely definitive, will be of assistance in the furtherance of the scheme of coöperation. It would also appear advisable that the results that may be obtained from the observations of the "fundamental velocity stars" during the remainder of 1902 should be published as promptly as possible in 1903.

To avoid further delay, will you be kind enough to reply as promptly as possible to this letter, signifying your adhesion to the general plan here mentioned, if you feel prepared to do so, and offering such further modifications or additional suggestions as seems to you likely to contribute to the success of our common enterprise. Very respectfully, etc.

To these condensed statements from different observers should be added the following, received later, which are in reply to both circular letters:

VII. David Gill (August 11, 1902): The reason why I did not reply to your circular of February 20 is that our spectograph is under alteration by the Cambridge Instrument Co. . . . . I will coöperate as far as practicable in your scheme with much pleasure whenever the spectroscope arrives, only you cannot expect us to observe the spectra of stars like α *Persei*, whose meridian zenith distance here is 83°. I should have been glad to see a greater variety in the types; the stars of your final list are nearly all of the solar type. My notion is that all observers should use the iron comparison spectrum and another comparison spectrum which should be left to the observer. Would it not be well also that at each observatory some star should be selected to be observed nearly throughout the whole year, so as to determine the mean velocity of the Earth's motion independently by spectroscopic means—in other words, to determine the solar parallax by spectroscopic methods? This should give us a tolerably sound measurement of the fundamental accuracy of our work.

VIII. H. F. Newall (July 25, 1902): (1) I am very glad to join in the scheme of coöperation. (2) I agree in thinking it very desirable to give the

widest latitude to each observer as to the mode of observing, reduction, choice of comparison spectrum and star lines. The more variety we have the better. (3) It seems right for a beginning to limit the list to ten bright stars. This will make a beginning for the coöperation. . . . . It would undoubtedly be a convenience to have for reference and control a much larger list of stars, about whose velocity there is something like agreement among observers. Would not the case be met by putting certain stars (say thirty or forty) on a list, inviting observers to publish any observations they may make, as soon as possible, but imposing no obligations upon an observer about making observations other than what each is satisfied with for a provisional determination (say from three plates, at any epochs)? Would it not be well to agree upon a common name by which to designate the ten stars on the list in order to facilitate indexing and references? There are objections to "fundamental" and "standard;" the word velocity can hardly be omitted. I would suggest "Velocity-Reference-Stars." (4) It is to be hoped that, for uniformity in publication, something like Professor Vogel's suggestions will be adopted. I venture to make one or two additions: (*a*) Hour-angle at mid-exposure, as well as G. M. T. (*b*) Linear dispersion in tenth-meters per mm, and in km per second per micrometer revolution. (*c*) Range of spectrum over which measurements have been made. (*d*) Comparison spectrum. (*e*) Slit-width."

The replies to the second circular letter received from the others may be condensed as follows:

A. Bélopolsky (July 17, 1902): I adopt the proposed scheme of observations of "fundamental velocity stars" at once, and will observe as rapidly as possible the following stars: α *Arietis*, α *Persei* (ι *Aurigae*), β *Geminorum* (ε *Leonis*), α *Boötis*, β *Ophiuchi*, γ *Aquilae*, ε *Pegasi*, γ *Piscis* (or γ *Cephei*). The stars in parenthesis I shall regard as supplementary stars, substitutes for those too far south for observation here. I shall observe each star three times — at opposition and about thirty days before and after opposition. In general I will adhere as closely to the program as is possible.

H. C. Vogel (July 25, 1902): (1) I agree to the proposed list of ten stars, and (2) I am also of the opinion that the greatest possible freedom should be allowed to each observer in respect to the mode of observing, choice of comparison spectrum, selection of star lines, and method of reduction. Therefore I shall have to regard as to be recommended but not as binding the suggestion of Messrs. Bélopolsky and Deslandres that the plates be obtained thirty days before, at, and thirty days after the star's opposition. I would suggest in simplification of the form of publication I proposed, that the measurements on the plate in the two positions under the microscope need not be given separately. As a substitute for items 4 and 5 (p. 173)

read as follows: Velocity of the star in kilometers (mean of measures with violet toward right and violet toward left under the microscope), reduced to the Sun with the constants of Schlesinger's tables. To item 3 (p. 173) I would add: The data shall refer to a definitely specified line approximately in the middle of the portion of the star's spectrum measured.

E. B. Frost: Observations have been in progress with the Bruce spectrograph of the Yerkes Observatory since July, following as closely as possible the program given in the second circular letter as representing as nearly as possible the combined views of all concerned. Attention will also be given here to the supplementary stars suggested by M. Bélopolsky. α *Boötis* and α *Persei* are being frequently observed.

Communications in addition to those cited above have not been received from Messrs. Campbell, Deslandres and Lord.

It is to be hoped that the experience in this simple plan of coöperation may be satisfactory enough to lead to its further development in the future.

Yerkes Observatory,
September 23, 1902.

COMET PHOTOGRAPHY WITH THE TWO-FOOT REFLECTOR.

By G. W. Ritchey.

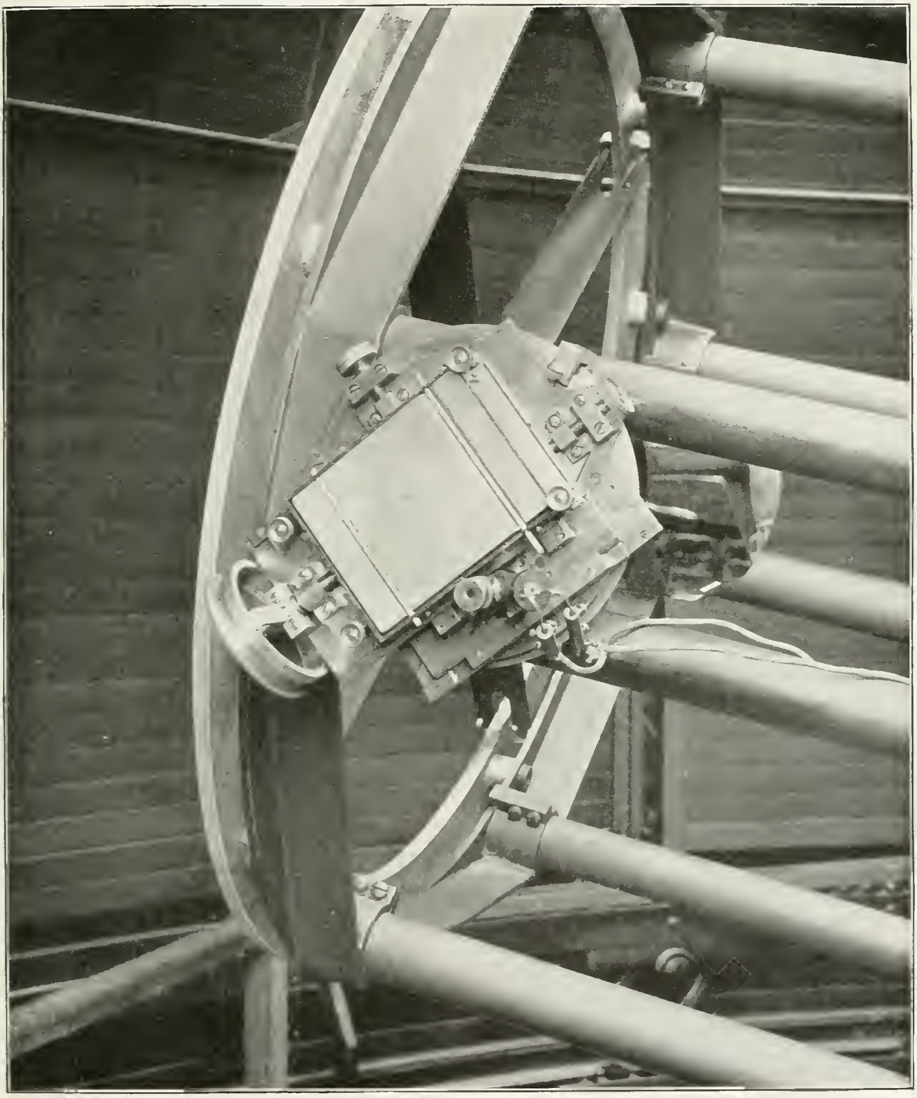
In photographing comets the usual method of guiding is by watching the nucleus of the comet in an auxiliary telescope, rigidly connected to the photographic one, and keeping this nucleus carefully bisected by the cross-wires in the eyepiece of the guiding telescope throughout the exposure, by means of the telescope's slow-motions.

In photographing stars and nebulæ the use of the double-slide plate-carrier presents many advantages over the use of a guiding telescope and the slow-motions; these advantages become more marked as larger photographic telescopes are used; indeed, for very large instruments the double-slide plate-carrier is absolutely indispensable; by its use the finest photographs yet obtained of such objects as the star-clusters and the nebulæ have been secured. This is due primarily to the fact that the incessant small corrections in guiding, the necessity for which becomes so apparent with large instruments, can be introduced with extreme accuracy and quickness. A star just outside the field being photographed is kept at the intersection of the cross-wires of the plate-carrier, by turning, when necessary, the milled heads of the fine screws which move the slides. It should be noticed that the small brilliant star image is a much better object for guiding than the usually faint and diffuse comet nucleus. But it is evident that this method of guiding is not available, without modification, in comet photography, on account of the motion of the comet with reference to the surrounding stars.

The writer, assisted by Mr. F. G. Pease, has obtained a series of photographs of Comet *b*, 1902 (Perrine), with the two-foot reflector and its double-slide plate-carrier, in the following manner.

An additional slide carrying the photographic plate was super-

PLATE VI



Double-Slide Plate-Carrier Adapted for Comet Photography

posed upon the two slides used in guiding, as shown in Plate VI. The two smaller milled heads are the ones used in keeping the guiding star exactly at the intersection of the cross-wires in the eyepiece, which is also shown. The large graduated head to the left is used to move the supplementary slide and plate-holder at such a rate and in such a direction as to compensate for the motion of the comet with reference to the star used in guiding. As will be seen in the illustration, the entire plate-carrier is rotated in position-angle to secure the proper direction of motion of the plate.

The direction and rate of motion to be given to the supplementary slide may be calculated in advance from a knowledge of the comet's orbit, or may be determined with great precision and ease by making a preparatory photograph as follows. An exposure of 30 seconds' duration is made on the comet, the observer guiding in the usual way with the double-slide plate-carrier, keeping a suitable star stationary on the cross-wires. The plate is then covered for 30 minutes, at the end of which time the guiding star is again kept at the intersection of the cross-wires while a second exposure of 30 seconds is made. Again the plate is covered for 30 minutes and a third exposure is then made. The exact durations of these exposures and of the intervals between them are carefully noted. The driving clock is now stopped and the stars in the field are allowed to trail. This preparatory plate is now developed, and is measured while still wet. The three exposures and the two intervals enable the observer to determine whether the direction and rate of the comet's motion are changing rapidly enough to effect perceptibly the sharpness of the photograph, which is made immediately after, with an exposure of an hour or more, and to allow for such changes if necessary.

Two improvements could easily be made upon the apparatus as shown in the illustration. First, instead of rotating the entire plate-carrier for position-angle as shown here, the supplementary slide only could be mounted on a plate which could be rotated. The original slides would thus remain, one in right-ascension, the

other in declination, which arrangement makes guiding on a star somewhat easier. Second, a small motor could be used to move the supplementary slide at the proper speed; as this motion need not be continuous, but can be intermittent, a very simple motor would suffice; the principal difficulty would be in regulating the speed properly, since this speed must be changed night after night. The present method of giving this motion by hand is simple and satisfactory; once in fifteen or twenty seconds the observer looks away from the guiding eyepiece, for two or three seconds only, to give the necessary movement to the graduated head; an assistant calls out the time for such movement; and to allow for the fractional parts of seconds which are generally necessary, a schedule is prepared on paper in advance, so that no cumulative error can occur during the exposure.

By these simple means the great advantages of the double-slide plate-carrier are made available for the photography of comets; and under favorable circumstances it should be possible to secure photographs of these objects as perfect in detail as the best photographs of star-clusters and nebulæ.

Yerkes Observatory,
October 1, 1902.

Minor Contributions and Notes.

NOTE ON FOCUSING PRISMATIC AND GRATING CAMERAS IN ECLIPSE WORK.

Mr. W. J. Humphreys, after having discussed the spectroscopic results obtained during the solar eclipse of May 18, 1901,[1] gives some suggestions very worthy of being considered in future eclipse work. But there is one point in which I do not agree with him ; and as I think I have good reasons to fear that if his advice were followed, some very remarkable phenomena would never come out so distinctly as the available instruments could show them, I am bound to submit my opinion to the notice of eclipse observers.

Mr. Humphreys says : " . . . . But since it is so essential to have exact definition, I would strongly urge final focusing, just before totality, on the narrow crescent of the Sun."

Now, I think final focusing on the spectrum of the narrow crescent must be very precarious. The Fraunhofer lines may for a few moments appear rather sharply defined, but they are very variable then and soon become faint and hazy, the spectrum of the remaining crescent of the photosphere being overlapped by the flash-spectrum and the chromospheric arcs. And focusing on the chromospheric arcs is quite impossible as long as we do not exactly know what we *ought* to see. There are at least two reasons why these arcs cannot be well-defined objects. First, the chromosphere itself is not sharply limited on the outer side, and secondly we are by no means sure that each arc, corresponding to a definite Fraunhofer line, really consists of monochromatic light. On the contrary, there is strong evidence that every chromospheric line is a narrow band with blurred edges and a dark core.[2]

As the existence of these double bands, appearing on the plates of the Dutch eclipse expedition to Sumatra, was required and foreseen by

[1] Astrophysical Journal, 15, 313-332, 1902.

[2] *Preliminary Report of the Dutch Expedition to Karang Sago* (*Sumatra*) *for the Observation of the Total Solar Eclipse of May, 1901*, by W. H. Julius, J. H. Wilterdink, and A. A. Nyland, Amsterdam, 1902.

a theory covering a great many solar phenomena,[1] it is an important question to ascertain whether the duplicate is real or not, and future observers should no longer bid fair to miss it by focusing, as may often have happened.

Indeed, there is an almost general complaint to be found in the eclipse literature, that the plates appeared not to have been exactly in focus.

In those cases, however, where the focusing had been very carefully carried out beforehand, the want of distinctness on the eclipse plates may just have revealed the true nature of the spectrum ; but most observers will not have accepted this result, believing that the chromospheric lines ought to appear nearly monochromatic, and they have, of course, preferred to focus on the phenomenon itself until the arcs came out as single as possible.

The above considerations induce me decidedly to advise focusing by some method independent of the eclipse phenomena.

In case a perfectly reliable collimator should not be available, star spectra should be used. (At Karang Sago Professor Nyland focused on the spectrums of Arcturus the night before the eclipse.) It may happen, however, that considerable differences in the temperature at night and day time, or other circumstances, make it desirable to control the focus a few hours before totality. Then the following device is recommended, which is so simple that it cannot be new.

At a considerable distance from the grating or prism-camera (say twenty-five or fifty times the focal length) a large black screen with a slit of proper dimensions is erected, in a convenient direction, such that the siderostat mirror may be so adjusted as to reflect the light from the slit into the instrument. The slit may be illuminated by means of a plane mirror placed behind the screen and reflecting a bright part of the sky or direct sunlight. Some more black screens with openings on the path of the beam will do for sufficiently excluding extraneous light.

Let the distance along the beam between the slit and the objective be p.

Now the Fraunhofer lines are made to appear as sharp as possible in the plane of the cross-wires of the ocular or on the photographic

[1] W. H. JULIUS, "On the Origin of Double Lines in the Spectrum of the Chromosphere, Due to Anomalous Dispersion of the Light from the Photosphere." ASTROPHYSICAL JOURNAL, 15, 28–37, 1902.

plate. Let the distance between this conjugate focus and the second nodal point of the objective be called q. Then the focal length is

$$f=\frac{pq}{p+q},$$

so that, in order to bring the wires or the plate exactly in the principal focus, we only have to move them toward the lens over a distance

$$d=q-f=\frac{q^2}{p+q}.$$

This is easily done by means of a short scale and vernier on the sliding tubes, provided the value of the quotient be known with sufficient accuracy. Now, for measuring the long distance p, a surveyor's chain gives ample exactness; and if we have taken p about fifty times as long as q, d is of the order of magnitude $\frac{q}{51}$. Consequently a slight error in the evaluation of q has an insignificant influence on d.

W. H. Julius.

Utrecht, August 1902.

ON THE CHANGE IN THE FOCUS FOR *NOVA PERSEI.*

In the Astrophysical Journal for October 1901 (**14**, 151–157), and in *Astronomische Nachrichten*, **159**, 49–58, 1902, I have given some measures for the focus of *Nova Persei* compared with that for a fixed star. It was shown in these papers that there was no difference whatever between the focus for the *Nova* and that for an ordinary star. The spectrum of the *Nova* showed the nebular lines, and it was thought that there might be a difference in its focus, like that shown in the planetary nebulæ (Astrophysical Journal, *loc. cit.*), where the focus is ¼ inch (6 mm) greater than for a star.

Here are the results of these tests for focus:

				Scale reading in.	
1901	August	12	*Nova* - - -	2.25	(4 obs.)
			B.D. 43°732 - -	2.26	(4 obs.)
			Nova — 43°732 =	— 0.01	(0.25 mm)
	September	3	*Nova* - - -	2.29	(5 obs.)
			B.D. 43°732 - -	2.30	(5 obs.)
			Nova — 43°732 =	— 0.01	

					Scale reading in.	
1902	January	31	*Nova* - - -		2.10	(5 obs.)
			B.D. 43°732 - -		2.11	(5 obs.)
				Nova — 43°732 =	— 0.01	
	January	31	*Nova* - - -		2.10	(5 obs.)
			B.D. 43°720 - -		2.12	(5 obs.)
				Nova — 43°720 =	— 0.02 (0.5 mm)	

Powers of from 700 to 1300 diameters were used in these tests. The persistent minus sign (showing a shorter focus for the *Nova*) is doubtless accidental.

It was, however, suggested that an increase in focus might present itself later on in the case of this star. Since then a careful watch has been kept to see when the phenomenon occurred, if it should occur.

This change has now taken place.

After its emergence from the region of the Sun this year, the *Nova* has been observed since July 14. The star had faded considerably and has since faded still further, until it is now but slightly brighter than the tenth magnitude. No change in its focus seemed to have taken place in the meantime.

The note-book says on July 14: "The *Nova* is bluish white; there does not seem to be any difference of focus." No scale readings.

On August 29 careful measures for focus were made of the *Nova* and the star *B.D.* 43°739, with the following result:

Scale reading for focus of *Nova* -	2.31 inches	(3 obs.)
Scale reading for focus of 43°739 -	2.29 inches	(5 obs.)
Nova — 43°739 =	+ 0.02	

This difference was so small that it was not considered evidence of change.

The note accompanying these measures says: "The *Nova* has a pale whitish color, and it does not seem so well defined as the star."

This lack of any decided color in the *Nova* has been present for some time since then; previously it had been greenish or bluish-white.

Following are some of the subsequent notes:

1902 September 1. It is a pale whitish color.
8. The light is whitish with suggestion of greenish tinge.
9. It has a pale whitish color.
15. The *Nova* is of a whitish color, with a slight bluish-green cast.

1902 September 16. The *Nova* is whitish and dim.

18. It has a pale white color.

29. The *Nova* is dull and bluish-white. In the finder it does not appear so sharp as the comparison star, and looks slightly hazy at best.

Observations on the following date show that the color seems to have changed as well as the focus:

October 6. The *Nova* is very bluish-white. It is bluer than usual. When the star is in focus the *Nova* is out of focus, and has a small point of light in the center—somewhat resembling *Nova Aurigae* in the latter part of 1892.

With a power of 700, the focus for the *Nova* and for the star *B.D.* 43°.739 was very carefully determined. The difference is very decided. Similar measures were made on October 7 and 13.

The results are as follows:

	Oct. 6 In.		Oct. 7 In.		Oct. 13 In.	
Focus for *Nova*	2.43	(8 obs.)	2.40	(6 obs.)	2.34	(5 obs.)
Focus for star	2.19	(8 obs.)	2.21	(6 obs.)	2.12	(5 obs.)
Focus for *Nova*—focus for star	+0.24	(6.1 mm)	+0.19	(4.8 mm)	+0.22	(5.6 mm)

Mean for the three nights, +0.22 inches (5.6 mm)

The focus has therefore changed to correspond with that of the planetary nebulæ, and is now about 0.2 inch greater than that for a star. This change has been very recent—certainly since August 29, and doubtless within a much shorter interval than that, as I think I should have noticed a difference of anything like this amount in the frequent examinations of the star.

E. E. Barnard.

Yerkes Observatory,
October 7, 1902.

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In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

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THE NEBULÆ IN THE VICINITY OF *NOVA PERSEI.*

By H. Seeliger.

The remarkable phenomena in the nebulæ near *Nova Persei,* the existence of which has been established by the photographs at the Heidelberg, Yerkes, and Lick observatories since the autumn of 1901, and which disclosed an apparent rapid motion, permit of the simplest explanation in the following well-known manner. At the time of its outburst there left the *Nova* a light-wave, powerful but decreasing in intensity comparatively quickly, which for a short time illuminated the surrounding cloud-structures, and, according to the measure of its progress, successively rendered visible new portions of the surroundings of the *Nova.* We designate these cloud-structures as nebulous, but without wishing to imply thereby anything as to their physical constitution. There can be no doubt that this exceedingly simple and obvious hypothesis, first published by Professor Kapteyn, is adequate for explaining all the essentials of the phenomena observed.

The photographs show, in addition to striking condensations and spots of light at different distances from the *Nova,* circular rings of light which are separated from each other by dark interspaces, and diffuse spots which are irregular and apparently of slight stability. This indicates that the material is not uniformly

distributed about the *Nova*, but that in many places it is arranged in flat layers and filamentary streaks; whereby of course only those portions of the matter can be taken into consideration which are sufficiently luminous to act upon the photographic plate. This is at least the simplest hypothesis. We could, indeed, also assume what to a limited degree would be certainly appropriate, that during the time of the greatest brilliancy of the *Nova* the light was not emitted equally in all directions. There would be in the neighborhood of the *Nova* absorbing accumulations which transmitted the light only in certain definite directions without weakening it too greatly. The *Nova* would also not have an equal luminosity at all the points of its surface, as we may assume *a priori*, and this may have caused a difference in the radiation in different directions which was quite independent of the time, to which we shall revert below.

The phenomena exhibited by the nebulæ in the neighborhood of the *Nova* are probably not yet at an end, and the observed data so far accumulated are not yet published in sufficient detail, so that it is not yet proper to undertake numerical computations. It appears to me, however, not to be inappropriate to discuss briefly the points of view as to principle which here come into question, since a lack of clearness on several points has become apparent. I therefore take the liberty of first treating the question as to what the observations indicate as to the form and the motion of the nebula in the neighborhood of the *Nova*, and what conclusions may be drawn therefrom. It will appear from this that the assumption that simple reflections are alone concerned is so general that an open contradiction of the observations could hardly be thought of.

Let us pass a rectangular system of coördinates through the *Nova* (N) as origin, with the X-axis in the direction of the very distant Sun, while the Y and Z-axes are perpendicular thereto. The light from N which falls upon the body, situated at a distance r, and is reflected by it to the Earth, will evidently require for its passage the time

$$\frac{1}{V}(r - x + \Delta),$$

where V is the velocity of light and Δ is the distance from the *Nova* to the Sun. If we disregard the insignificant variation in Δ, and compute the time t from the instant when the light from N reaches the Sun by the direct way, we have

$$r - x = \sqrt{x^2 + y^2 + z^2} - x = Vt = p \ . \qquad (1)$$

Now, if N was only for a short time, Δt, bright enough so that the reflected light could be sufficiently strong, then at the time t only those bodies would be visible in the neighborhood of the *Nova* which lie between the two paraboloids of revolution

$$r - x = Vt \quad \text{and} \quad r - x = V(t + \Delta t) \ .$$

$Vt = p$ is the parameter of the generating parabola, whose focus lies at N. Δt would probably amount to several days, since in this time the *Nova* declined several magnitudes in brightness.

If now a layer of nebular stratum, which therefore has a relatively small thickness and may be considered as having its mass distributed approximately in a plane, is illuminated by N, then at the time t those particles will appear upon the plate and form a curve which must lie near the intersection of the surface with the paraboloid. I will call this curve briefly the "observed curve." It will change its form and size with time, and we include all the data which can be yielded by the measurement of the plates if we assume that we fully know the equation of the observed curve

$$\phi\,(y, z, t) = 0 \ ,$$

or, solved for t,

$$t = f\,(y, z) \ . \qquad (2)$$

We must here assume that y and z are expressed in angular measure, say in seconds of arc. Further, y and z must also satisfy equation (1). Here x must be expressed on the same scale, therefore also in seconds of arc, but nothing is known as to V if the parallax π of the *Nova* is unknown. If π is expressed in seconds and the light equation of the Sun is $498^s.5$, we shall have

$$V = 63{,}300 = v\pi \ .$$

Hence for (1) we must write

$$\sqrt{x^2 + y^2 + z^2} - x = v\pi t \qquad (3)$$

We can now determine for each π the surface which is represented by the nebular layer, for it passes through the intersection of (3) and (2), the latter of which may be regarded as the equation of a right cylinder. The coördinates x, y and z of the required surface must, for every t, within a certain finite region, satisfy the equations (2) and (3). If we eliminate t from (2) and (3) we obtain as the equation of the required surface

$$\sqrt{x^2+y^2+z^2} - x - v\pi f(y, z) = 0 \ . \tag{4}$$

For every assumed value of π we can find the corresponding surface which represents the illuminated stratum of nebula. The parallax of the *Nova* therefore remains undetermined, and it is not determinable from measurements of the photographs which are before us without the aid of further and quite arbitrary hypotheses.

Kapteyn[1] assumes that the observed curves are circles whose centers do not coincide with N. This assumption agrees very well with the outer nebulous streaks according to photographs at the Yerkes Observatory, particularly toward the west and toward the southwest on the plate of September 20th, 1901, and also fairly well toward the north on the plates taken in January and February, 1902. He adds further that the distance ζ of the center of the circle from N changes proportionately to the time without change of direction, and similarly for the radius of the circle R. These assumptions do not seem to me to be very well founded, judging by the copies of the plates accessible to me, but nevertheless this assumption will not be further discussed, and we shall merely draw the inferences that follow from it.

If we place

$$\zeta = ct \ , \quad R = \gamma t \ ,$$

the equation of the apparent curve becomes

$$y^2 + (r - ct)^2 = \gamma t^2 \tag{5}$$

and, after some easy reductions, the equation of the required surface is found to be

$$(m^2 - n^2)(y^2 + z^2) + 4 \{xznc(m^2 + n^2) - c^2n^2z^2 - m^2n^2x^2\} = 0 \ ,$$

[1] *A. N.*, 157, 201, 1901.

where for brevity we have placed

$$m^2 = \gamma^2 - c^2 \quad \text{and} \quad n = v\pi \ .$$

This equation represents a right cone whose vertex is at *N*. Its axis lies in the *XZ* plane, making with the *X* axis an angle α, which is determined by

$$\sin \alpha = \frac{2cn}{\sqrt{(m^2 + n^2)^2 + 4c^2n^2}} \ ; \quad \cos \alpha = \frac{m^2 + n^2}{\sqrt{(m^2 + n^2)^2 + 4c^2n^2}} \ ;$$

$$\tan \alpha = \frac{2cv\pi}{\gamma^2 - c^2 + v^2\pi^2} \ .$$

The angle f between a light-ray of the cone and its axis is found to be

$$\tan f = \frac{2\gamma v\pi}{\gamma^2 - c^2 - v^2\pi^2} \ .$$

These results are all that can be obtained with certainty from any measurements; the parallax π of the *Nova* remains wholly undetermined.

Kapteyn's assumption corresponds to $f = 90°$, and is equivalent to the entirely arbitrary assumption

$$v\pi = \sqrt{\gamma^2 - c^2} \ .$$

If the observed curves do not satisfy equation (5), another surface of course at once takes the place of the cone. We remark incidentally that the result could also be otherwise interpreted in case the definitive measurement of the plates actually led to conical surfaces of the kind above mentioned. If the outburst of the *Nova* was produced by the entrance of a dark celestial body into a cosmical cloud, then the amount of heat generated would by no means be equal for all points of the surface; but it would be the greatest where the principal resistance was encountered, and would decrease in all directions from there. We might accordingly ultimately picture the situation in this way: the light reflected from the neighborhood of the *Nova* came from a certain polar zone inasmuch as the light proceeding from the other parts of the surface of the *Nova* was too faint to be noticed. The body of the *Nova* would therefore produce something like a shadow cone, and if the intersection of this cone with the paraboloid was occupied by particles sufficiently reflect-

ing the light, the observed curve (5) would result. We are here only mentioning one of the possibilities at hand, without investigating its greater or less probability. If we seek to explain in this way the most remote parts of the nebula in the west and southwest, and later in the north, it follows that the brightest parts of the *Nova* lay on the side away from the Sun. Under certain assumptions this would further lead us to infer that the relative motion of the *Nova* with respect to the nebula was as if the *Nova* was receding from the Sun and the nebula approaching.

Let us now consider a streak of nebulosity, a body of essentially linear extent.

The projection of the intersection of the wisp with the paraboloid will appear illuminated at the time t, and will look like a bright spot. Such a bright spot will move further along the projection of the wisp, and the measurement of the plate will yield equations of the form

$$\left.\begin{aligned} y &= \phi\,(t) \\ z &= \psi\,(t) \end{aligned}\right\} . \qquad (6)$$

The x coördinate of this point will follow, as before, from the equation of the paraboloid,

$$\sqrt{x^2 + y^2 + z^2} - x = v\pi t\ ,$$

and will be

$$x = \frac{y^2 + z^2 - v^2\pi^2 t^2}{2\,v\pi t}\ . \qquad (7)$$

Equations (6) and (7) represent the equations of the wisp of nebula in terms of the parameter t, and a definite and always real value of x belongs to every value of π. In this case π is therefore again wholly indeterminate. It is hardly necessary to remark that the moving bright spot will retain almost the same form if the mass is distributed homogeneously along the wisp; otherwise its form must change.

These remarks doubtless suffice to show that one cannot doubt that there are possibilities enough available for explaining all the phenomena in the vicinity of the *Nova*. For the present nothing further is intended to be presented by this discussion.

Dr. Louis Bell has recently expressed the opinion[1] that there

[1] ASTROPHYSICAL JOURNAL, 16, 38, 1902.

are difficulties in the way of the simple reflection theory, which are partly of a physical nature. He therefore gave preference to the view that the effects about the *Nova* are not purely optical, but are principally electrical actions, which however must follow the same geometrical laws. Mr. Bell brings forward three points which I should now like to discuss.

"First, reflected light, whether reflected in the ordinary way from heterogeneous surfaces or from small particles, would be polarized, and Perrine's report on this feature of the case indicates absence of polarization." To this we would remark as follows: Up to the present no details have been published as to Perrine's experiments. It is well known that it is generally a very difficult matter to prove the existence of polarized light from cosmical bodies, even in cases where there can be no doubt as to the presence of partial polarization. The following facts may be recalled.

According to Secchi[1] and others the full Moon exhibits no polarization, but with increasing phase-angle it becomes demonstrable, and reaches a maximum in the first and last quarters. At this time it is nearly uniformly distributed over the entire lunar surface; but its amount is decidedly dependent upon the nature of the surface. Polarized light has indeed never been detected on the lunar mountains, while the *maria* exhibit comparatively strong polarization. Great difficulties are met with in analogous investigations of the planets, and I am not familiar with any numerical results. Although the heads of comets show polarized light, it is in very small quantity. In a head of Coggia's comet (1874) Zenker[2] was able to detect it, but he could not determine its percentage. He says "the polarization was certainly very slight, but nevertheless I did not estimate it to be any greater in the case of *Jupiter*." The question as to the emission of polarized light by the zodiacal light was a matter of controversy for several decades, the individual results being decidedly diverse, sometimes entirely negative and sometimes indicating faint polarization. A certain conclusion was brought to the question by the work of A. W. Wright,[3] who found: (1) That the zodiacal light is polarized in a plane passing through the Sun;

[1] *A. N.*, 52, 93, 1860. [2] *A. N.*, 84, 173, 1874. [3] *Am. Jour.* (3) 8, 39, 1874.

(2) that the amount is probably 15, but scarcely 20 per cent. Ordinary clouds, as is well known, exhibit no polarization, but the light of the blue sky exhibits it in a high degree—at points 90 degrees from the Sun amounting under some circumstances to 88 per cent. In this respect the matter has been very accurately studied and the facts established.[1] Terrestrial objects giving diffuse reflection obviously can be much more readily and accurately investigated in respect to polarization, and a considerable number of experiments of this sort are available. A. W. Wright investigated a number of rocks, and he found between 5 and 26 per cent. of polarized light present; *e. g.*, for common dust 15.5, for syenite 16.4, for gneiss 8.3, for granite 11.8, for sandstone 12.1, for meteorites 11.7 per cent., etc. Finely divided powder was also studied by Henry Wright,[2] who found that no trace of polarization could be proven to be present.

The state of the case seems to be, as is of itself plausible, that a great quantity of discrete particles exhibit no demonstrable polarization as long as the dimensions of the particles are appreciably greater than the wave-length of light, and a certain irregularity in shape seems to be favorable for the absence of polarization. So-called turbid media (blue sky acts as such) consist of such small particles, compared with which the wave-length of light is not small, and therefore the light reflected from them may under some circumstances be strongly polarized.

In this state of affairs and in view of the great difficulties attending the analysis of a source of light so exceedingly faint as that of the nebula near *Nova Persei*, we should expect in advance a negative result from the experiments of Mr. Perrine; the result obtained could therefore be hardly surprising, as we ought to expect that the nebula would act in a similar manner to the turbid media. If the experiments of Mr. Perrine require us to exclude this possibility, it would naturally be a very interesting result, but without special importance for the question in hand. We are absolutely unable to judge of the accuracy or

[1] See the summary by JENSEN, *Meteor. Zeitschrift*, 18, 547, 1901.

[2] "Die diffuse Reflexion des Lichtes an matten Oberflächen," *Münchener* Doctor dissertation, 1899, and *Ann. der Phys.*, 1, 17, 1900.

inaccuracy of the result found on the basis of the investigations published. But even if it was not permissible to raise such doubts, the position with respect to the *Nova* of the portions of the nebula investigated would have to be taken into account; for the blue sky itself does not exhibit strongly polarized light at all distances from the Sun.

The second objection of Mr. Bell is this: "Second, reflection does not adequately explain the very remarkable persistence of some regions of strong nebulosity at a small angular distance from the *Nova*. Especially the nebular peak nearly south of the *Nova* has an intensity all out of proportion to that of the outer ring, while both on the reflection hypothesis should be at similar radial distances. If they are, then the ring must represent a condition of matter having a very small albedo compared with that in the other region."

These remarks seem to be based upon a misunderstanding. The surface brightness of a portion of the nebula is dependent upon the density, the distribution of matter, and the albedo of the particles at the place in question, and on the absorption which the light from the portion of the nebula suffers on its way to the Sun, on the phase-angle and on the distance from the source of light. The very bright spot already mentioned is situated at a very small angular distance from the *Nova*, and it must, according to our foregoing considerations, lie on the side of the *Nova* away from the Sun and not far from our line of sight through the *Nova*. In other words, since the spot lies on the paraboloid, its parabolic true anomaly a must be very small. If we compare the surface brightness h in this spot with that of a portion of the outer ring, for which let the true anomaly be a_1, and if we notice that the phase-angles in the two cases are similarly a and a_1, and if $f(a)$ represents the dependence of the brightness on the phase-angle, then, other things being equal, we shall have

$$\frac{h}{h_1} = \frac{f(a)}{f(a_1)} \left[\frac{\cos \frac{1}{2} a}{\cos \frac{1}{2} a_1} \right]^4 .$$

$f(\alpha)$ decreases quite considerably with increasing α, and similarly the second factor is much greater than 1. We may place $\cos \frac{1}{2}\alpha = 1$, while α may be much greater than 90°, for the outermost ring perhaps 120°. On this assumption the second factor becomes 16, so that $\frac{h}{h_1}$ may easily be several digits, even on only moderately favorable assumptions. Mr. Bell's assumption "should be at similar radial distances," therefore rests upon an oversight. Quite aside from the possibility of the free choice of the density, of the albedo, or of suitable conditions of absorption, we can understand why just these spots in the immediate vicinity of the *Nova* are particularly bright.

The third and last objection of Mr. Bell is this: "Third, at the radius of 210 light-days denoted by the ring of September 20, reflection does not adequately account for the brightness of the nebular matter observed," etc.

In respect to this I base my view on one of my papers which was published several months before the discovery of the nebula near the *Nova*, and upon my note in *Astronomische Nachrichten*, where I reach results different from those of Mr. Bell. It is probably unnecessary to discuss the matter anew here. It needs only to be remarked that in the choice of the albedo values of the particles constituting the nebula, of the density of the distribution of the mass, and of the thickness of the reflecting layers, we have at hand a means of determining the absorption produced by any superposed masses of nebula to correspond to the observations, as easily follows from the formula which I have elsewhere given. There is no necessity of making any *a priori* assumptions as to the albedo, since we know absolutely nothing as to the physical nature of the matter in the vicinity of the *Nova*. We must abstain from making a comparison with the residual particles of gas in exhausted Geissler or Hittorf tubes, not only because of the considerably different conditions of pressure in spite of the greatest exhaustions attained, but also, and especially, on account of the extremely low temperature which small cosmical masses must have in empty space, which are not to be compared with those under which laboratory experiments have

hitherto been conducted. For the present, therefore, the application of the results of laboratory experiments for explaining the phenomena of the nebula near *Nova Persei* has scarcely more justification than the invocation of a whole series of vague hypotheses. In my view we certainly should not decide to employ such hypotheses before the reflection theory has been entirely shipwrecked. For this reason I do not wish to enter into a more detailed discussion of the views expressed by Mr. Bell.

MÜNCHEN, October 1, 1902.

THE MOVEMENTS IN THE NEBULA SURROUNDING *NOVA PERSEI.*

By Arthur R. Hinks.

It seems to be quite certain that the apparent velocities in the nebula surrounding *Nova Persei* are of the order of the velocity of light; they can hardly be much less, and it is perhaps scarcely necessary at present to worry about how to explain them if they should be greater.

Assuming then, for the moment, that we are concerned with a velocity equal to that of light, we have to decide between two possibilities. Either the structures which appear in motion are material ejected from the star with the velocity of light, and continuing to move without any very apparent acceleration or retardation; or they are due to the lighting up of successive parts of a nebula already in position around the stars, by some influence proceeding outward with the velocity of light. It is with one or two aspects of the latter possibility that this note is concerned.

Various suggestions have been made as to the nature of the influence proceeding from the star which might develop an already existing nebula in the fashion which has been observed. Simple reflection, luminosity excited in tenuous gas by an electromagnetic wave, luminosity due to the recombination of tenuous gas after dissociation by a light-wave passing through it, luminosity following bombardment by streams of projected ions, have all been suggested. It was pointed out in a discussion at the recent meeting of the British Association in Belfast that they all come to practically the same thing. They are all concerned with the lighting up of an already existing distribution of matter by an influence traveling outward with the velocity of light. In any case the resulting apparent structure would be much the same, and we need not try to discriminate between the various suggestions as to the exact nature of the physics involved, until

we are satisfied that the "lighting up" hypothesis is competent to explain things generally.

It has been alleged against the lighting-up hypothesis that it will not explain the persistence of certain definite forms, the cusps or arrow heads, which are the most striking features of the moving nebula. It seems to me, however, that if one follows out in detail a case of "lighting-up" one finds that no such difficulty occurs. Kapteyn remarks that all points of the nebula which light up simultaneously as the result of an instantaneous burst of light lie on the surface of an ellipsoid of revolution, whose foci are at the star and the Earth. We may treat the portion of the ellipsoid near the star as a paraboloid; and it is easy to see that the retardation, that is, the interval between the arrival at the Earth of light which has come direct and the arrival of light which has come *via* a point on the paraboloid, is equal to the time which light takes to traverse the semi-latus-rectum of the paraboloid. If then light travels from the star at S in a direction which makes an angle ϕ with the line SE to the Earth, and lights up a point P at a distance R light-years from S, the "retardation" of the lighting up of P is the length in light-years of the semi-latus-rectum of the parabola through P, with S as focus and SE as axis; and that is $R\ (1-\cos\phi)$.

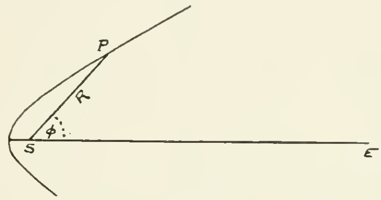


FIG. 1.

Now imagine that nebulous matter is disposed in a circle with the star as center, with a radius of R light-years, and that the plane of the circle makes an angle θ with the line of sight. Different radii of this circle will make angles ϕ with this line, which lie between θ and $\pi-\theta$; and corresponding points on the circle will appear successively with retardations lying between $R\ (1-\cos\theta)$ and $R\ (1+\cos\theta)$ light-years. Take a particular case. Suppose $\theta=30°$ and $R=3.73$ light-years (so that $R\ (1-\cos\theta)$ is equal to half a year.) The circle viewed from the earth will be projected into an ellipse. If ψ is the angular distance on the circle between any point P and the point of the

ring nearest the Earth, which projects into an end of the minor axis, then ϕ is found for the point P from the relation $\cos\phi = \cos\theta\cos\psi$. Taking points at equal intervals of 20° round the circle we have the following table:

$\theta = 30°$ $R = 3.73$ light-years.

ψ	$1 - \cos\psi\cos\theta$	Retardation
0°	0.134	0.50 years
20	0.186	0.69
40	0.337	1.26
60	0.567	2.12
80	0.849	3.17
100	1.151	4.29
120	1.433	5.35
140	1.663	6.20
160	1.814	6.77
180	1.866	6.96

It follows that if the star lights up for an instant we shall have this sequence of events. Six months afterward a point of nebulosity will appear at *B*. This will divide into two points which travel round the ellipse in opposite directions, at first rapidly, then slowing as they approach the ends of the major axis, and finally quickening as they come together again after six and a half years at the end of the minor axis opposite *B*.

Proceeding now to the case of a sudden outburst of light which dies away gradually. Our points become lines, which move round the ellipse as before; they are brightest at the head, and fall off in intensity as the star fell off. This is the result for a narrow circular distribution of nebula. But suppose now that the nebular ring has some breadth in the plane of the circle; and consider it divided into a number of narrow concentric elements. Each element will contribute a pair of moving lines as above; and further, as we go outward the head of each successive elementary line will be set a little back from the one that precedes it, since its light-radius R, and consequently its retardation, is slightly greater. We shall therefore get as an aggregate effect two moving cusps or arrow-heads as in Fig. 3.

It seems that the lighting-up hypothesis is competent to explain how two cusps of nebulosity may appear to move in

opposite directions and retain their form. Some such effect will be produced by the lighting up of any wisp of nebula, and I am far from suggesting that the examination of our simple example, the flat circular ring, goes any way toward discovering what is the real form of the *Nova Persei* nebula. It can do no more than point out how immensely the retardation complicates the effect of a light explosion among wisps of nebulosity. For instance, if we assume that our flat circular ring has a breadth of half a light-year, it is easy to show that the history of the moving cusps which result from it is given briefly in Fig. 4.

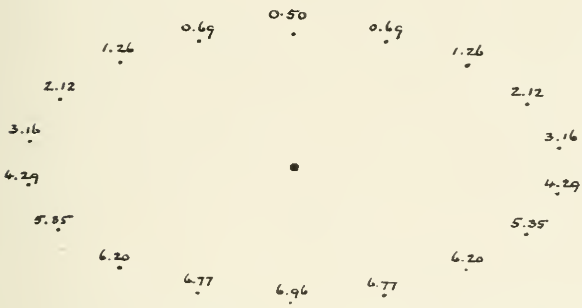


FIG. 2.—Retardation, in years, of light reflected from successive points, 20° apart, round a circular ring, about *Nova* as center, inclined 30° to the line of sight, and of radius 3.73 light-years.

The cusps start by being sharp; become blunted as they approach the apsides of the ellipse; and finally become more and more acute as they draw together again.

The early stages are not altogether unlike those shown in Ritchey's drawings of 1901, September 20, and 1902, February 8, for the condensations lettered *a* and *e*, which moved during that interval from the positions shown in black to the positions shown in outline in Fig. 5.

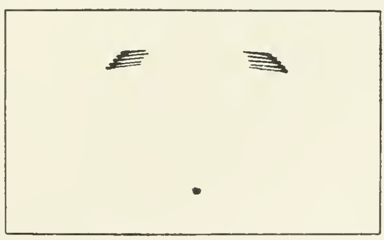


FIG. 3.

These quick-moving cusps of light were probably formed in a wisp of nebula that lay well on our side of the star, since they lit up very soon after the direct light of the star reached us, although they are at a considerable angular distance from it. It is perhaps more likely than not that these wisps are disposed all

round the star in spirals or other curves; and if this is so, it is not impossible that we may see things gradually develop something after the fashion of our example; that we may see the cusps which are now separating on one side of the star come together again on the other. And if such things have happened round other *Novæ*, it is perhaps not even now too late to photograph them in their last stages.

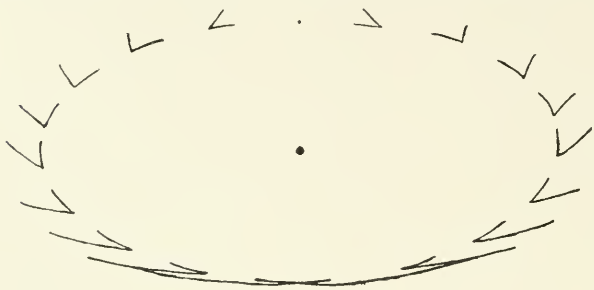


Fig. 4.

It is very easy, with this lighting-up hypothesis, to work out what will happen for a particular distribution of nebula. Probably it would be very troublesome to work back from a set of observed appearances, and make out the distribution of nebula which might have given rise to them. At any rate the time for that is not yet. But I should like to suggest that, if anything is to be made of the problem, a knowledge of all the outlines of the nebula will be at least as necessary as knowledge of the positions and movements of certain defined and measurable points; and that probably nothing could be better for this purpose than more drawings from photographs, made very carefully to scale, after the fashion of Mr. Ritchey's admirable drawings which have already appeared in the Astrophysical Journal.

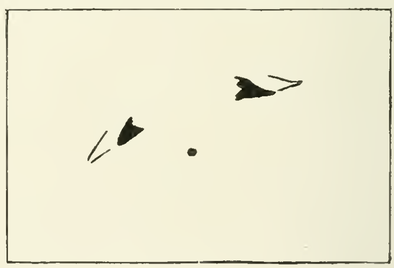


Fig. 5.

Cambridge Observatory, England,
September 24, 1902.

MINIMUM SUN-SPOTS AND TERRESTRIAL MAGNETISM.

By A. L. Cortie, S.J.

The relation which exists between the diurnal range of magnetic declination and horizontal force and the period of solar spot frequency has been fully discussed by Mr. Ellis for the period 1841–1896 by means of the Greenwich magnetic curves and the tables of Sun-spot frequency prepared by Dr. Rudolf Wolf. (*Proc. R. S.*, **63**, 64). The author sums up the results of his study of the subject as follows: "Considering that the irregularities in the length of the Sun-spot period are so entirely synchronous with similar irregularities in the magnetic period, and also that the elevation or depression of the maximum points of the Sun-spot curve is accompanied by similar elevations and depressions of the two magnetic curves, it would seem, in the face of such evidence, that the supposition that such agreement is probably only accidental coincidence can scarcely be maintained, and there would appear to be no escape from the conclusion that such close correspondence, both in period and activity, indicates a more or less direct relation between the two phenomena, or otherwise the existence of some common cause producing both." Moreover, the discussion showed that the strict relation as to intensity and duration of the periods was almost identical, whether the curves of magnetic diurnal range were derived from quiet days only or from all days, quiet or disturbed. That this relation between the two phenomena cannot be one of efficient cause and effect has been theoretically proved by Lord Kelvin, and observationally by Father Sidgreaves. In his paper on the "Connexion between Solar Spots and Earth-Magnetic Storms" (*Memoirs R. A. S.*, **54**), Father Sidgreaves classified and studied all the magnetic storms and all the greater solar spots for the years 1881–1896. The tables which illustrate the paper show that years of greater solar activity are accompanied by greater magnetic storms, while in years of solar minimum there are but few storms.

These facts demonstrate the close connection that exists between solar and magnetic storms, in addition to and beyond the consonant mean periodic fluctuation of the two phenomena established by Mr. Ellis. But on the other hand, in every class of solar spot magnitude there was at least one spot unaccompanied by a corresponding magnetic storm, and besides, there were correlatively great magnetic storms which occurred during periods of absolute solar quiet. These results are adverse to any theory which would place the cause of magnetic storms, and by the cause we mean the efficient cause, anywhere on or in the vicinity of the Sun. Father Sidgreaves proposes a theory according to which streams of electrified corpuscles moving with high velocity in interplanetary space would act sometimes as the hot vapors issuing from the photosphere, darkening them electrostatically, and sometimes on the Earth magnetically, and when the corpuscles were very numerous on both Sun and Earth. But though solar spots and their allied phenomena of faculæ and prominences are not the efficient cause of magnetic storms on Earth, may they not be a primary instrumental cause? This position appears to be that taken up by Professor Young in the latest edition of his work, *The Sun*, where he likens the action of the spot to the pulling of a trigger which causes the flight of the rifle-bullet, inasmuch as it releases the potential energy stored up in the powder, and causes the explosion which ensues. A similar action would be that by which the pressing of a button causes the launching of the huge mass of metal contained in the hull of a warship. Sometimes the trigger is pulled but the expected explosion does not occur, owing to some casual defect, and so, too, analogously the solar spot may appear, but no answering magnetic storm occurs. As examples of the direct action of solar storms on terrestrial magnetism, Professor Young gives in his book two instances which occurred on August 3 and 5, 1872, in which striking and extraordinary reversals of the C line, *Ha*, in a solar spot were exactly coincident in time with movements in the magnets. But with all deference to the opinion of so eminent a solar observer as Professor Young, the magnetic curves do not appear to warrant anything more than the deduction of a mere coincidence in time

between the chief paroxysms of the solar *Hα* reversals and the corresponding swings of the magnets. If the magnetic movements at these times had occurred during an otherwise magnetically calm day, the evidence of cause and effect would have been very weighty, but the oscillations of the magnets observed at the stated times are neither peculiar nor unusual during the course of magnetic storms. The second instance given, that of August 5, is stronger than the first, but even here the coincidences occurred at the tag-end of the magnetic storm which had run its course during the two preceding days. There was undoubtedly a general connection between the two phenomena, but, at least in the opinion of the writer, not the intimate connection claimed.

But to return to the general question as to the kind of connection that obtains between Sun-spots and terrestrial magnetism. If the solar spot be the primary instrumental cause in the production of a magnetic storm, its action ought not to be frustrated in a great number of cases, nor ought it to act capriciously and without method as to the order of the occurrence of the two phenomena, or to the time, before the reputed cause works its effect. A spot of any large area ought to be accompanied by a bigger magnetic movement, and it ought reasonably to be expected that it should occur when the spot was most active. It seemed possible to derive some further knowledge of the mode of action of the Sun-spots and faculæ by instituting a detailed comparative study of the solar surface and magnetic curves, and not merely a comparison of means for long periods, in which the principle of compensation is apt to mask the individual departures from the mean values. Moreover, if a period of minimum solar spots and magnetic storms were selected for study it would be more possible to determine the connection, if any, between the spots that appeared and any abnormal ranges in the magnetic elements. For though such ranges might be comparatively small at times of great disturbance, they would be very noticeable at periods of calm. Again, at periods of maximum solar and magnetic activity the storms are so mixed up the one with the other that it becomes a difficult matter to assign individual magnetic disturbances to their solar concomitants.

Such a discussion, even if it did not do much to elucidate the matter, might serve to corroborate past results from more recent observations. Accordingly, the three years, 1899–1901 inclusively, were selected, the material for the study of the solar surface being the Stonyhurst drawings supplemented by a very careful series of eye observations by Mr. Hadden, of Alta, Ia. For the magnetic disturbances the Stonyhurst series of curves for the declination elements were taken as sufficient for the purposes of comparison. The character of the movements for each day for the three elements is also given in the Stonyhurst annual reports. In addition the mean daily disk areas of the spots and faculæ, expressed in millionths of the Sun's apparent disk, were taken for the thirty-eight solar rotations covering the period from the results published in the *Monthly Notices R. A. S.* **61**, Nos. 1 and 8, and **62**, No. 5. These form the third and fourth columns of the annexed table, the fifth and sixth columns showing the mean diurnal range of the declination magnet for each corresponding rotation, and the greatest diurnal range during the rotation.

The general connection between the state of the solar surface and the intensity of the magnetic declination for the three years is well shown in the Stonyhurst annual reports. The mean daily disk areas of the spots reckoned in terms of the $\frac{1}{5000}$ of the apparent disk are 0.74, 0.55, and 0.29, with the corresponding mean diurnal ranges 12.′9, 9.′7, and 9.′1. We may remark that there is not such a marked decline in the magnetic as in the solar disk area for the two years 1900 and 1901, though the general decline affects both. But when the individual solar rotations from which the average results are drawn are studied, the existence of great anomalies is detected, as the following table will show.

The lack of perfect accord in the majority of cases is still more apparent when the daily solar observations are compared with the daily readings of the magnetic curves. In the year 1899 there were seven days on which the magnetic disturbances were classed as relatively great, in the year 1900 two, and none at all in the year 1901. The first greater movement of the magnetic needles for 1899 occurred on January 28, when there were some

Rotation		Greenwich		Stonyhurst	
		Mean of daily disk area		Mean of magnetic diurnal range	Greatest range
Number	Begins	Spots	Faculæ		
606	1899 Jan. 15	271	399	10.3	32.0
607	Feb. 11	154	402	15.5	28.7
608	March 10	38	218	16.2	37.5
609	April 7	485	441	15.4	29.6
610	May 4	159	344	14.7	37.5
611	May 31	61	211	12.7	20.5
612	June 27	173	410	15.6	65.0
613	July 24	448	439	12.6	21.5
614	Aug. 21	21	345	12.4	18.6
615	Sept. 17	3	161	12.8	23.5
616	Oct. 14	54	160	11.7	45.0
617	Nov. 10	129	152	9.9	25.0
618	Dec. 8	67	233	9.1	17.0
619	1900 Jan. 4	99	161	12.4	36.0
620	Jan. 31	133	327	9.2	30.5
621	Feb. 28	155	215	12.2	40.0
622	March 27	124	221	10.8	17.0
623	April 23	253	312	11.7	52.0
624	May 21	91	241	10.5	14.0
625	June 17	163	104	10.4	14.0
626	July 14	58	126	10.6	16.5
627	Aug. 10	28	36	11.9	20.0
628	Sept. 6	21	95	8.7	14.5
629	Oct. 4	210	88	9.1	19.3
630	Oct. 31	18	66	5.9	10.5
631	Nov. 27	0	14	4.2	8.2
632	Dec. 25	1	0	7.3	18.0
633	1901 Jan. 21	8	13	6.9	19.0
634	Feb. 17	22	29	8.5	26.0
635	March 17	0	7	11.4	28.0
636	April 13	0	0	10.4	19.3
637	May 10	339	53	11.8	40.0
638	June 6	70	34	10.7	20.0
639	July 4	1	47	10.8	16.0
640	July 31	0	6	11.1	28.0
641	Aug. 27	0	2	10.3	30.5
642	Sept. 23	10	8	9.0	24.0
643	Oct. 21	29	94	7.0	13.0
644	Nov. 17	51	4	5.0	12.0

small spots on the Sun. If this be reckoned as a possible connection, the second greater movements of February 12 cannot be so, as they took place when the solar surface was perfectly quiet. The two chief solar outbursts of the year occurred during March and June; the first consisting of a spot visible during one rotation from March 15 to March 27, and the second of a spot

formed on the invisible side of the Sun, which crossed the visible disk but once, also between June 23 and July 5. Two days of greater disturbance (March 21 and 23) accompanied the one spot when it had attained its greatest disk-area, and two others also (June 28 and 29) the second spot when it too passed the central meridian, after many reversals of the *Ha* line had been observed in it two days previously. The greatest range of the year of the declination magnet took place on the 29th. Between these two periods of solar activity a quiet time intervened in April and May. However, a few unimportant spots were on the Sun in the latter half of April and the first days of May, that might be possibly claimed as coincident with the magnetic storm and accompanying aurora of May 3. We have, therefore, of seven days of greater magnetic disturbance during the year, four coincident with greater spot-area, two possibly coincident with small spots, and one case of non-coincidence. After the middle of July the spots and faculæ became very scarce and small, with a quite remarkable absence of bright faculæ during August. After this period of calm the Sun's surface began again to be disturbed about the 20th of September, and during this period some moderate disturbances of the magnets and a corresponding aurora on the 26th were experienced. There had, however, been just such similar movements during August when the Sun was quite calm. Again in October a moderate sized spot with bright faculæ in which reversals of the *Ha* line were observed, was seen on the east limb of the Sun on the 23d, which was accompanied by a moderate disturbance with an extreme range of 45′, the second greatest swing of the year. But in December, when there was quite a small recrudescence of solar activity after another period of calm, in which there was a moderate disturbance of the magnets of similar character to that of October, with nothing on the Sun, the magnets were not only undisturbed, but were at the quietest period for the whole year. In January 1900, there were six moderate movements of the magnets, that of the 21st being accompanied by an aurora. All these, except the last, which was coincident with the appearance of a new spot, occurred during a period of calm. The only

moderate swing of February was coincident with a few spots of small area. These moderately large movements of the magnets, therefore, throw no light on the subject under discussion, except that they occur equally with and without spots. Their origin may be purely terrestrial.

The first great storm of the year 1900 occurred on March 13. A fine group of spots characterized by spectroscopic *Ha* reversals, had appeared about two days' distance west of the central meridian on March 6, and passed off the visible disk on the 11th. It was possibly still active on the 13th, when on the invisible hemisphere of the Sun. During the intervals March 26–April 17, and April 27–May 6, the solar surface was fairly active but the magnets were calm, with the exceptions of the dates May 4 and 5, when moderate and great storms were recorded as one group was passing round the west limb. An aurora was observed on May 1. This was the second greater disturbance of the year. Both, though recorded during an active solar period, do not seem to have any very close connection, either with the position of the spots on the Sun's disk, or with their more active phases.

Although in the year 1901 there was no magnetic disturbance that can be classed as great, yet there was a very fine spot on the Sun visible to the unaided eye, which lasted for nearly two solar rotations, and in its two appearances on the visible disk, contributed no less than 74 per cent. of the total spotted area for the year. It was, very probably, born just off the east limb of the Sun on the very day of the total eclipse, May 18, and its presence was marked by a coincident fine prominence, and by a disturbed area of hitherto unobserved character in the solar corona. The spot outburst had been prepared for by an appearence of bright faculæ in the very position in which it subsequently broke out, a whole rotation previously. There was no magnetic disturbance of any moment during either of this spot's transits across the visible hemisphere. The greatest magnetic oscillation of the year, however, occurred on May 10, coincident with a possible short-lived spot on the invisible hemisphere of the Sun, but when the visible disk was absolutely

calm, and fully seven days before the outbreak of the one great solar spot group of the year. When the spot was most active the magnets were absolutely quiet. In fact, as the table shows, between March and August, including the rotations 635 to 641, the mean diurnal range was almost constant, while the mean daily disk-area of the spots was fluctuating between 0 and 339 units. A full discussion of this spot group was given in the *Monthly Notices R. A. S.*, **62**, No. 7, and the conclusion there stated was that "the one great solar disturbance of the year, which showed itself in a spot visible to the naked eye, in a fine prominence, in bright faculæ, and in an unique coronal disturbance, was unaccompanied by any considerable magnetic storm, and seemingly had but a fortuitous connection with the slight and moderate disturbances which occurred during its existence."

The minimum of solar activity has persisted during the six months that have elapsed of the present year, the only spot of any size crossing the disk between March 5 and 13, unaccompanied by any striking magnetic disturbance. From this date to May 19 the Sun was absolutely clear of spots, and what faculæ there were, were very faint and unimportant. Yet on April 10 there was a magnetic disturbance which was relatively great, with a maximum swing of the declination needle of 38′.4, and in general character more intense than the short lived disturbances of May 10, 1901. These two cases, the one of a fine spot without any magnetic disturbance, and the other of the greatest magnetic disturbance of the six months without any accompanying spot at all, are sufficient of themselves to disprove any intimate connection of cause and effect between the two phenomena. Yet it may be possible, judging from the above detailed discussion of the minimum period, that Sun-spots are one of the instrumental causes of magnetic storms, though not the only one, but it is more likely that the two phenomena are correlated as two connected, though sometimes independent, effects of one common cause.

Stonyhurst College Observatory,
July 26, 1902.

SOLAR RESEARCH AT THE YERKES OBSERVATORY.

By George E. Hale.

The program of solar investigations outlined in this paper was prepared in substantially its present form in 1894, in connection with other plans for the work of the Yerkes Observatory. The necessity of constructing in our own shop the special instruments required for this work might not have involved any serious delay in the inception of the investigations. But certain demands, which for various reasons could not be set aside, compelled us to construct other instruments before making complete provision for solar research. It will be seen from what follows, however, that considerable solar work has already been done, and that the entire program will shortly be in effect. The principal investigations comprised in the program are as follows :

1. *Direct photography.*—Daily photographs of the Sun on a scale of seven inches (17.7 cm) to the diameter ; large scale photographs of spots and other regions.

2. *Monochromatic photography.*—Daily photographs with the spectroheliograph, for systematic study of the form, area, distribution, and motion of the calcium vapor in faculæ, chromosphere, and prominences. Comparative photographs taken simultaneously in various bright and dark lines, and other special researches.

3. Daily photographs of the spectra : (*a*) of Sun-spots, for the systematic study of the positions and intensities of the widened lines and the bright H and K lines ; (*b*) of various regions of the photosphere, for the study of the bright H and K lines and the detection of possible changes in the position or intensity of dark lines ; (*c*) a special series of photographs taken at the shortest practicable time intervals, near the Sun-spot maximum, in order to register, if possible, such remarkable changes in the reversing layer as are referred to on p. 220.

4. Special researches, radiometric, visual, and photographic,

on the spectrum of the reversing layer and the chromosphere with a large solar image and powerful grating spectroscope.

5. An investigation of the solar rotation, to be determined from displacements of certain narrow bright lines in the spectrum of the chromosphere and prominences, photographed with very high dispersion.

6. Radiometric investigations of various kinds, with particular reference to the level of Sun-spots.

7. Visual observations to supplement those made photographically.

DIRECT PHOTOGRAPHY.

The series of direct photographs of the Sun now in progress is made with the 12-inch (30.5 cm) refractor, which gives an image two inches (5.1 cm) in diameter. These will give place later to a series in which the diameter of the image will be seven inches. In view of the fact that the heliocentric position of all spots is determined at Greenwich, it is not expected to measure these plates. They are intended for use in connection with other photographs, especially those of spot spectra, for the identification of the spots, and the study of their structure. For the latter purpose they are to be supplemented by large scale photographs of special regions.

MONOCHROMATIC PHOTOGRAPHY.

The series of daily photographs of the Sun made with the spectroheliograph of the Kenwood Observatory covers the period, January 1891–June 1896. These photographs were taken with the aid of the 12-inch equatorial refractor referred to above. Had circumstances permitted, the series would have been continued with the same telescope after its removal to the Yerkes Observatory. But unfortunately this could not be done. The telescope was needed for micrometric, photometric, and other work of a general nature, which did not involve the use of large and heavy attachments. It was therefore necessary to remodel the telescope, as it had been especially designed to carry the large Kenwood spectroheliograph. In a series of photographs of this character, it is imperative, for purposes of measurement,

etc., that the entire disk of the Sun should appear on a single plate. Hence the old spectroheliograph, whose slits are only three inches long could not be used to make such a series with the 40-inch (102 cm) Yerkes refractor, which gives a solar image seven inches in diameter. A new and much larger spectroheliograph was accordingly designed for this telescope, while the Kenwood instrument, after the reconstruction required to adapt it to the large refractor, was employed as a solar spectroscope, and later as a spectroheliograph for photographing limited areas.

In designing the large spectroheliograph, the only difficulty arose from the large diameter of the solar image. For full illumination, with a solar image seven inches in diameter, collimator and camera lenses nearly ten inches (25 cm) in diameter would be required. Funds were not available for the purchase of such lenses, and in any event their great weight would have precluded their use. Accordingly Voigtländer portrait lenses, such as were formerly used in photographers' studios, were selected, and after an extensive search among dealers in photographic supplies, two such lenses, of 6½ inches (16.5 cm) clear aperture, were obtained at a small fraction of their original cost. The loss of light at the upper and lower ends of the slit with lenses of this aperture is not sufficient to affect the image at all seriously. At the same time the field is large enough to give an image well defined at the limb. It was decided to mount these lenses in parallel collimator and camera tubes, and to connect them by an optical train consisting of a plane mirror and two 60° prisms, giving a total deviation of 180° to the K line.

The problem of securing the necessary relative motion of instrument and solar image then presented itself. I long ago came to the conclusion that the best form of spectroheliograph is that in which the instrument is moved as a whole, the solar image and photographic plate remaining stationary. In the present case, however, such an arrangement was out of the question, as the motion of an instrument weighing several hundred pounds would set the entire telescope in vibration, and consequently ruin the photographs. Exposures on a trailing star

image, made when the telescope was driven in declination by the slow motion electric motor, indicated that it should be possible to produce with this motor a uniform motion of the solar image across the first slit. It would then only remain to cause a synchronous motion of the plate behind the second slit by a shaft led down the telescope tube from the same motor. This design would not have been chosen for a smaller spectroheliograph, but it seemed to be the best available for the instrument in question.

A photograph of the spectroheliograph, which was completed in 1899, is reproduced in Plate VII, Fig. 2. The above brief description should make its general construction clear. Such details as the method employed to eliminate distortion of the solar image arising from the strong curvature of the spectral lines, the method of setting the K line on the second slit, the adjustments of the mirror and prisms, the occulting disk moving with solar image for prominence photography, the mechanism of the plate-carriage and second slit, etc., need not be discussed here. A statement regarding the results obtained with the instrument will suffice for my present purpose.

Great care had been taken in constructing the spectroheliograph to guard against diffuse and reflected light, and the resulting photographs show that the precautions were effective. In spite of the great curvature of the spectral lines the solar image is circular and free from distortion The network of hot calcium vapor, first shown on the Kenwood photographs in all parts of the solar disk, was found to persist throughout the spot minimum, at a time of feeble solar activity. It occurs at the poles as well as at the equator, and may doubtless be considered a permanent feature of the solar structure. With the new spectroheliograph, by giving a suitable exposure, photographs were obtained showing this reticulation almost alone, with only a faint background, due to the white light of the solar disk. This fact affords the best evidence of the excellent contrast of negatives taken with the instrument,[1] which is not adequately shown in the reproduction of a part of one of the photographs (Fig. 1).

[1] In a recent number of the *Comptes Rendus* (September 29, 1902) M. Deslandres remarks:

"Ces deux spectrographes des formes et des vitesses de Meudon sont actuelle-

PLATE VII.

FIG. 1.—Sun-spot of March 6, 1900.
Photographed with the Yerkes Spectroheliograph.

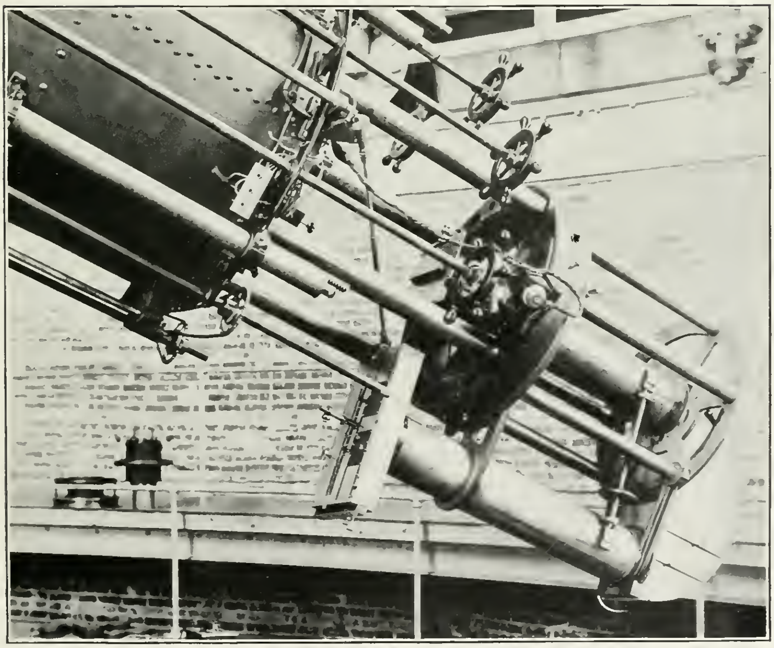


FIG. 2.—Spectroheliograph attached to 40-inch Yerkes Refractor.

A periodic error, probably due to the absence of suitable end-thrust bearings on the screw used to move the plate, was easily removed. There remained another periodic error which produced lines on the plates at regular intervals of about one-third of a millimeter. These were less conspicuous than the lines commonly present on plates taken with spectroheliographs driven by a clepsydra, but they were nevertheless objectionable, especially because of the serrated effect they tended to produce at the solar limb. It was also found, however, that even with perfectly uniform motion of the telescope and plate the limb was serrated whenever the telescope was swaying in the wind or when the seeing was poor. Such effects had been obtained with the Kenwood spectroheliograph, but on account of the smaller size of the telescope and of the solar image they were less noticeable.

After the construction of a horizontal reflecting telescope of 61½ feet (18.7 m) focal length had been decided upon, I concluded to transfer the large spectroheliograph from the 40-inch to the new telescope as soon as completed, and to use the remodeled Kenwood instrument, either as a spectroheliograph or spectrograph, for regular work with the large refractor. The great weight of the large spectroheliograph (about 700 pounds, or 317 kg) made it impossible to attach it to and detach it from the telescope very rapidly, and hence no spectroscopic work could be done with the telescope on days when the spectroheliograph was in use. With the horizontal telescope it can be moved into or out of position in a moment, and hence it will not interfere with the instant use

ment les seuls en service dans le monde entier; car le spectrographe des formes, ou spectrohéliographe, réalisé par Hale à Chicago, n'a pas été remonté lors de son transfert à l'Observatoire Yerkes en 1897. Mais j'ai appris récemment que les Anglais, sur l'initiative de Sir Norman Lockyer, ont commandé deux séries d'appareils similaires, qui seront placées en Angleterre et aux Indes." As M. Deslandres has made a similar statement in a previous article, it is perhaps worth while (without further mention of the Yerkes Observatory) to recall the fact that he has overlooked the systematic work of Mr. Evershed, who first used a spectroheliograph in 1893 (some months before M. Deslandres' first spectroheliograph was constructed), and that of Dr. Kempf, which has been in progress for several years at the Potsdam Observatory, and is reported annually by Director Vogel in the *Vierteljahrschrift der Astronomischen Gesellschaft.* Experimental work with a spectroheliograph has also been in progress for several years at Sir Norman Lockyer's Observatory.

of the large spectroscopes and other attachments provided for this instrument. At present, therefore, spectroheliographic work with the 40-inch telescope is confined to the limited regions which can be photographed with the Kenwood instrument.

THE SPECTRA OF SUN-SPOTS.

The remarkable peculiarities of the spectra of Sun-spots seem to deserve more attention than they have hitherto received from spectroscopists. With the exceptions noted below, all observations of the widened lines have been made visually, and for the most part they have been confined to a limited number of the most widened lines in the spectra. Professor Young's important observations of spot spectra, especially those made at Mount Sherman in 1872, comprise a valuable record of all the widened lines then visible. If systematic observations of this kind could have been carried on daily throughout an entire Sun-spot cycle, it is probable that our knowledge of the nature of Sun-spots would have been considerably increased. On account of the multitude of widened lines present, and the consequent difficulty of recording all of them within the time available for observation, it would be difficult to make such a series. Nevertheless, a series in which these conditions were partially fulfilled was carried on by Mr. Maunder at Greenwich, principally during the years 1877 to 1883, and his results constitute one of the most important sources of information regarding Sun-spot spectra.[1] Another important series is that of Father Cortie, who recorded the widened lines in the region C to D during the years 1880–1889.[2] In the observations systematically conducted under the direction of Sir Norman Lockyer for many years, attention has been confined to the six "most widened lines" between D and *b* and the six most widened lines between *b* and F.[3] The program of solar observations prepared by the Observatories Committee of the Royal Society for the Astrophysical Observatory at Kodaikanal, India, provides for a similar series of observations. It also states "that other widened lines should be noted."

[1] *Greenwich Spectroscopic and Photographic Results.*

[2] *Memoirs R. A. S.*, Vol. L, pp. 30–56. [3] *Proceedings of the Royal Society.*

In view of the importance of the conclusions which may be based on the study of Sun-spot spectra, it has seemed to me that every effort should be made to record systematically, not merely the twelve most widened lines, but all of the lines which are distinctly affected in the spectra. In my work at the Kenwood Observatory from 1891 to 1896, I found that recourse should be had to photography, if possible, on account of the large amount of time required to make a complete record visually. Experiments in the photography of spot spectra were accordingly instituted, and some degree of success was attained. The 2-inch solar image given by the 12-inch refractor was too small, however, to permit any but the strongest of the widened lines to be photographed. Experiments with an enlarged solar image were therefore made, but as the instrumental conditions were not favorable, it was decided to postpone the work until it could be undertaken with the 40-inch refractor of the Yerkes Observatory. The difficulty arising from the use of a small solar image is doubtless what interfered so seriously with the photographic experiments on spot spectra made by Father Sidgreaves at Stonyhurst. It was probably because of the encroachment of photospheric light on the spot spectrum that he was led for a time to doubt the objective existence of widened lines. With the larger solar image given by the 23-inch (58.4 cm) Princeton refractor, Professor Young in 1893 obtained photographs of spot spectra which showed a considerable number of widened lines. An engraving from one of these photographs is given on p. 217 of the (1898) revised edition of Professor Young's *General Astronomy*. This photograph shows, as was well known from visual observations, that while many lines are widened in the spectra of Sun-spots, others are materially reduced in intensity.

The following program of observations of Sun-spots, prepared by Mr. C. Michie Smith for the Kodaikanal Observatory, is given in his report on the Kodaikanal and Madras observatories for the period April 1 to December 31, 1901:

(*a*) A daily examination of the Sun's surface for spots.

(*b*) When a spot of sufficient size is present, one or more photographs of

the spectrum with the necessary comparison spectra will be taken. It is intended to take photographs of as large a part of the spectrum as possible, so that the taking of the photographs will occupy a considerable time; only a small part of the spectrum can be taken at a time.

(*c*) If it be found impracticable to photograph the whole of the visible spectrum, the photographs will be supplemented by eye-observations.

(*d*) The photographs will be at once developed.

(*e*) The measurement and reduction of the negatives will, as far as possible, be kept up to date, but as there will always be plenty of cloudy days on which this work can be done, the first duty on bright days will always be the making of observations.

The program of Sun-spot observations which the Observatories Committee of the Royal Society has substituted for the program proposed by Mr. Smith, gives first place to the visual observations of the twelve most widened lines referred to above. It also provides that "after the above requirements are fulfilled, it is desirable that if possible photographs should be taken of Sun-spot spectra, for which, it is to be noted, comparison spectra, other than the solar spectrum, are unnecessary." It should be added that Mr. Smith's general solar program, as well as that of the Royal Society Committee, provides also for photography with the spectroheliograph.

A program of Sun-spot observations which I prepared in 1894 is as follows:

1. Daily photographs of the Sun, on a scale of seven inches to the Sun's diameter, to identify spots and to give their general form and heliographic position.

2. Enlarged photographs of spots, to record details of structure.

3. Monochromatic photographs with the spectroheliograph to show the distribution of calcium vapor.

4. Photographs of the spectra of spots, supplemented by visual observations.

5. Photographs of the H and K lines, with very high dispersion, at regularly spaced points over the entire solar surface, to show the radial motion of the calcium vapor, particularly in the vicinity of Sun-spots. In certain cases, where special accuracy is required, these photographs are to include comparison spectra.

6. Measurements of the heat radiation of spots, and also of the photosphere near the spot and at the center of the Sun.

In a discussion of the observations the changes in intensity of

the spot lines would thus be considered in connection with the heliographic position of the spots, their form and structure, the distribution and motion of calcium vapor in their vicinity, and the amount of their heat radiation.

In the present preliminary paper, I wish to consider briefly only that part of this program which relates to the photographic spectra of spots. Up to the present time all of the photographs have been made by Mr. Ellerman with the remodeled Kenwood solar spectograph attached to the 40-inch telescope. For the most part they have been taken in the second spectrum of a large plane grating having 20,000 lines to the inch (7,874 to the cm). The camera, which is of 3¼ inches (8.3 cm) aperture, has a focal length of 42½ inches (108 cm). With this scale all of the more conspicuous of the widened lines are easily recorded. Photographs showing some of these widened lines are reproduced in Plate VIII[1]. One of these is of special interest, as it gives a photographic record of many of the "bands" seen for the first time by Maunder on November 27, 1880, and frequently observed by him during the three succeeding years. The following table contains the wave-lengths of some of these bands as measured on one of our photographs by Mr. Barrett, together with the positions of the corresponding bands as determined by Maunder in April 1882.[2]

Three facts have become clear from the work so far accomplished: (1) that a considerably greater linear dispersion will be required in order to record photographically the faintest widened lines and other of the less conspicuous features of the spectrum; (2) that a solar image having a diameter of more than seven inches would be very advantageous, especially for the smaller spots; (3) that in order to determine, with high precision, whether the spot lines are displaced from the normal positions of the corresponding solar lines, a comparison spectrum will be essential. The new apparatus which has been constructed for the work will therefore comprise a coelostat

[1] Unfortunately these reproductions fail to bring out clearly the widened lines, which are well shown on the original negatives.

[2] *Greenwich Spectroscopic and Photographic Results*, 1882, p. 10.

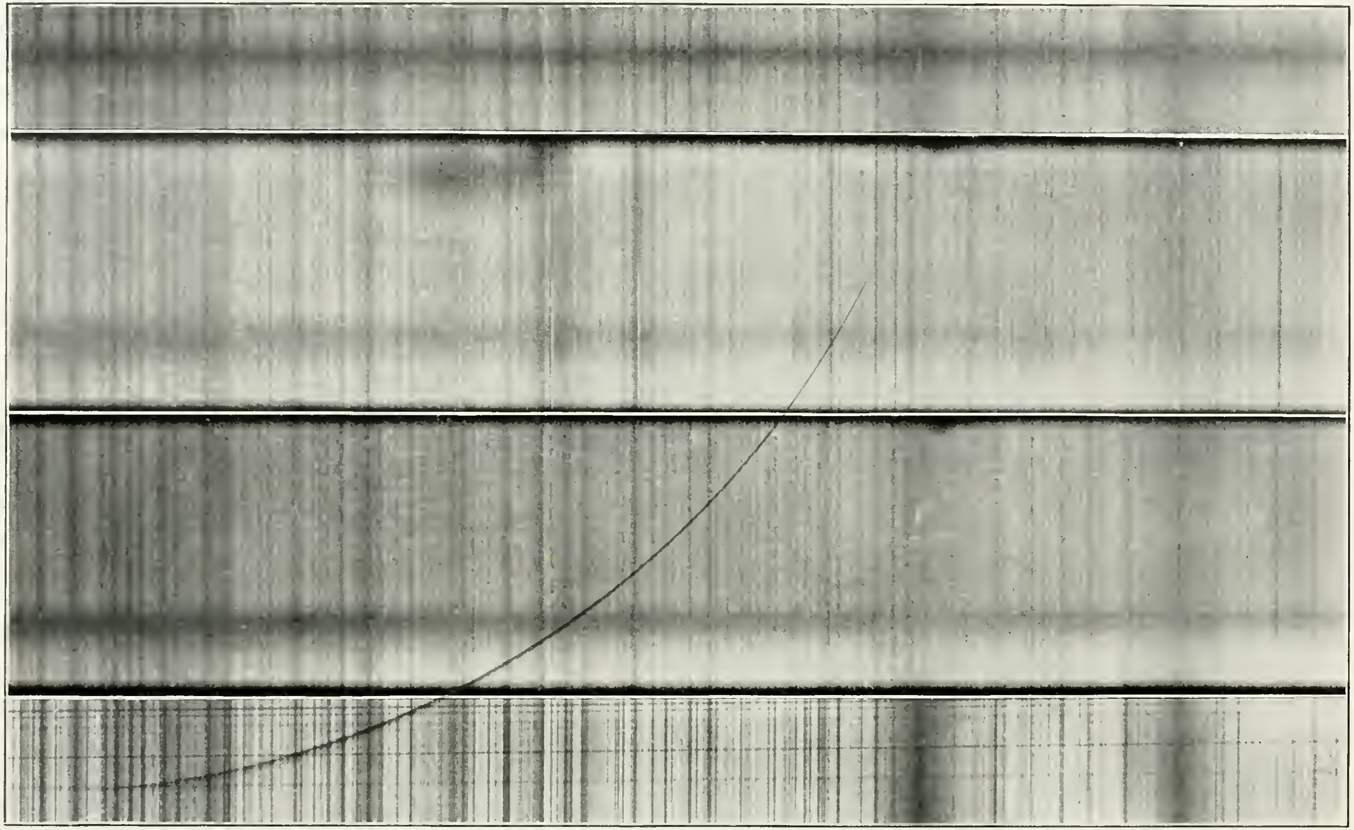
Yerkes Photograph			Maunder	
Wave-length	Width	Character	Wave-length	Width
5096.33	0.54	double	5094.0 } triple 5096.5 }	
5100.65		very indistinct		
5101.66		broad, hazy	5101.0	0.2
5111.86	0.57	double, perhaps triple		
5113.28	0.96	triple ?	5113.8	0.8
5114.58	0.45	double		
5116.60	1.28	triple	5116.0	0.8
5118.88	1.68	triple	5117.8	0.8
5120.45	0.48	narrow	5120.1	0.8
5122.36		narrow		
5134.76	0.86	triple		
5135.29		narrow		
5136.38	0.67	triple? ?	5136.4	0.3
5138.75	0.59	band	5138.3	0.3
5140.44		narrow		
5141.38	0.51	band		
5144.03	0.67	double		
5147.72	0.64	double ?		
5150.03	1.08	double	5150.2	1.0
5156.80	1.02	triple	5156.2	0.7
5157.86		narrow		
5160.15	0.86	double	5159.6	0.6
5163.06	0.76	double	5162.3	0.8
5163.72	0.37	narrow band		

reflecting telescope of 165 feet (50.3 m) focal length, giving a solar image about 20 inches (50 cm) in diameter, and a large spectroscope, with which a concave grating of 21 feet (6.4 m) radius, or a plane grating with collimator of 18 feet (5.5 m) focal length, and cameras of about equal focal length, can be employed.

A REMARKABLE DISTURBANCE OF THE REVERSING LAYER.

In February 1894, while engaged in making a series of photographs of the solar spectrum with the spectrograph attached to the 12-inch Kenwood refractor, Mr. Ellerman unconsciously recorded a phenomenon which appears to be quite unique. A series of exposures was made on a single plate, in order to find the relative exposure times required in spectra of the first, second, third, and fourth orders of the grating. The 51 mm image of the Sun was so adjusted that the image of a spot fell exactly on the slit. A diaphragm limited the width of each

PLATE IX.



1. Spectrum just before the Disturbance. 2. "Abnormal" Spectrum. 3. "Intermediate" Spectrum. 4. Normal Solar Spectrum.

A Remarkable Disturbance of the Reversing Layer.

strip of spectrum to 6.5mm, corresponding to about one-eighth of the Sun's diameter or 4⅛′ of arc. Unfortunately, as the photographs were intended merely for the purpose of ascertaining the relative exposure times, and not for spectroscopic study of the Sun, no record of the plate was made in the note-book. As the peculiarities of the spectra were not noticed until some months later, we have no way of determining the date on which the photographs were made or of identifying the spot in which the disturbance centered. A large spot in the southern hemisphere, which may have been the one in question, is first shown near the east limb on a spectroheliograph plate taken on February 16. Subsequent plates in the daily series show the changes in this spot as it passed across the disk, but give no evidence of any unusual disturbance. The available evidence on the spectrum plates, indicates, however, that the phenomenon was very short-lived, and for this reason it might easily have escaped detection. Furthermore, the bright H and K lines, instead of showing a great increase of intensity, as in certain eruptions recorded with the spectroheliograph, in this case disappeared entirely. Hence any record of the disturbance made with this instrument would have involved the *disappearance* of the bright calcium region surrounding and partly covering the spot.

The photographs are reproduced in Plates IX and X. Fig. 1 is the spectrum, showing few, if any, deviations from the normal, which was taken just before the disturbance occurred. All of the spectra were in the third order of a plane grating having 14438 lines to the inch (5684 to the cm), with a camera of 3¼ inches aperture and 42½ inches focal length. The spectra were all overexposed, and consequently show little contrast. Nevertheless the ordinary reversals of the H and K lines can be seen over and near the spot band in Fig. 1. A few moments later, as Fig. 2 shows, the disturbance was at its height. In the seven spectra photographed before the disturbance occurred, and also in the four following ones, the continuous band of absorption due to the spot is clearly shown. In this abnormal spectrum, however, though the band is clearly visible at the two extremities of the spectrum, it is very faint in the region of H and K. It

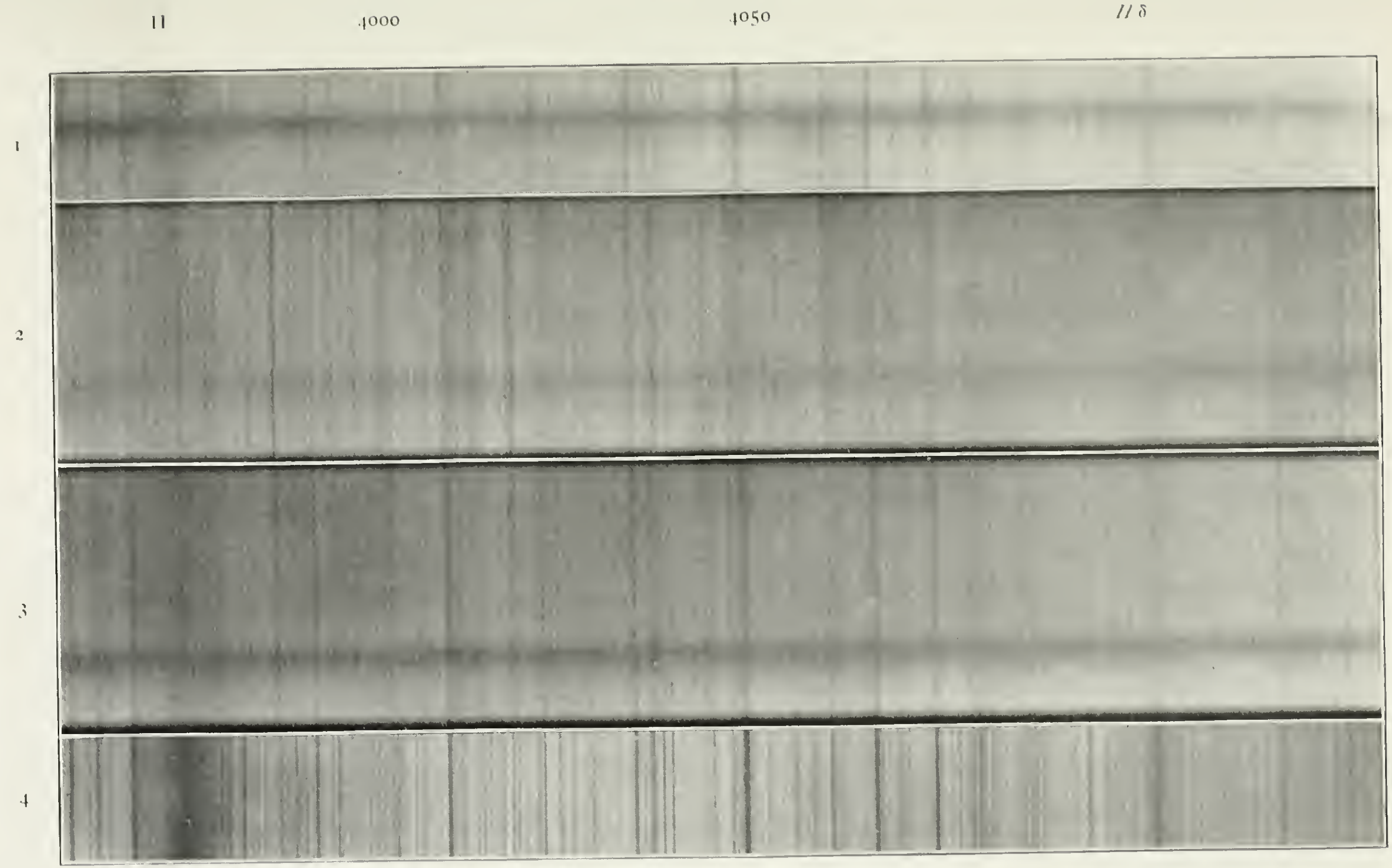
is a curious fact that the greatest changes in selective absorption also occurred in the neighborhood of H and K, and that at the two ends of the negative the lines differ but little from those of the normal solar spectrum. Two narrow bright lines at λ 3884.64 and λ 3896.21 form a striking feature of this spectrum. Indeed, it was through the presence of these lines that the peculiarity of the spectrum was first recognized.

Fig. 3 shows the spectrum as photographed a few moments later. As the slit apparently remained at about the same point on the spot throughout the twelve exposures obtained on the plate, it is extremely probable that the change in the spectrum represents a later stage of a short-lived phenomenon. This spectrum is intermediate in character between the abnormal spectrum of Fig. 2 and the normal solar spectrum. The bright lines, which may be too faint to appear in the illustration, are shorter than before, and their wave-lengths have changed to λ3884.28 and λ3895.98 respectively. Strong dark lines of the normal solar spectrum, which are faint in the abnormal spectrum, have regained some of their intensity, while other lines, peculiar to the disturbance, are much less pronounced than before.

The next photograph was taken in the fourth order spectrum, and shows only the normal spectrum, with the H and K lines once more reversed across the spot. The dark shades at H and K, which are so inconspicuous in the spectra of the disturbance, have also resumed their usual appearance. This spectrum is not given here, but a photograph of the normal spectrum, taken with proper exposure on another date, is reproduced in Fig. 4 for comparison.

Every spectroscopist will understand why I have hesitated to publish these spectra. So far as I know, the phenomenon is quite without precedent, and I could hardly believe it possible that the solar spectrum should undergo so complete a change throughout an area whose length was at least one-eighth of the Sun's diameter. Had the disturbance been confined to the spot it would have seemed far less remarkable, though even then without precedent; but the photographs show it to extend far beyond the spot, with few, if any, indications that the width of

PLATE X.



1. Spectrum just before the Disturbance. 2. "Abnormal" Spectrum. 3. "Intermediate" Spectrum. 4. Normal Solar Spectrum.

A Remarkable Disturbance of the Reversing Layer.

the spectrum was sufficient to include the entire disturbed area. Even after the two peculiar bright lines had led to an examination of the plate, we were inclined to assume that the effect was produced by the chance superposition of two spectra, photographed in different positions of the grating. It soon appeared, however, that the spectra could not be accounted for in this way. Copies were then sent to several spectroscopists for examination, with the request that an explanation referring the phenomenon to some origin other than solar be supplied, if possible. As no such explanation was forthcoming, I measured the lines of one of the spectra, and intended to measure those of the other. I was prevented from doing so, however, by an eye affection, and accordingly requested Mr. Adams to make the necessary measures. His results, which include both spectra, are given in the following table. The intensities of the lines, which are on Rowland's scale, were estimated independently for the two spectra.

An examination of these results will show that most of the lines of the "abnormal" and "intermediate" spectra are solar lines in Rowland's table, but in many cases so changed in intensity as to be quite unrecognizable. From the limit of the spectrum in the ultra-violet to about λ 3865 the abnormal spectrum corresponds fairly well with Rowland's map. Hence, comparatively few lines in the intermediate spectrum were measured above this point. The spectra now begin to show marked differences, but these relate to the intensity rather than to the position of the lines. The group of lines which forms a blend at about λ 3878.5 in the solar spectrum, with a combined intensity of 22, is absent from the abnormal spectrum, though it appear with full intensity in the intermediate spectrum. Similar cases occur at λλ 3883.34, 3889.05, 3903.11, 3921.71, 3948.91, 3953.02, 3961.67, etc. The last-named line is one of a strong pair between H and K in the second subordinate series of aluminium. In the spark between aluminium poles in water, and in other cases in the laboratory, I have found these lines to vary together. In the present instance, however, the other member of the pair (at λ 3944.19) is but little reduced in intensity in the abnormal spectrum, while its

Wave-length			Intensity			Substance
Rowland	ntermediate	Abnormal	Rowland	Inter-mediate	Abnormal	
3747.098	Spectrum poor	3747.10	5	..	6	*Fe*, –
3748.408	few lines measured	3748.38	10	..	10	*Fe*
3758.375	....	3758.32	15	..	15	*Fe*
3760.196	....	3760.21	5	..	5	*Fe*
3761.487	3761.45	3761.47	9	9	8	*Ti*, *Fe*
3762.435	....	3762.47	4	..	4	–, *C*, *C*
3763.945	....	3763.97	10	..	10	*Fe*
3765.689	....	3765.71	6	..	8	*Fe*
3779.598	....	3779.52	4	..	5	*Fe*
3783.674	....	3783.67	6	..	6	*Ni*
3788.046	3788.01	....	9	10	..	*Fe*
3793.656	......	3793.68	8	..	2	–, –, *Fe*, *Ni*
3795.150	....	3795.13	10	..	7	*V*, *Fe*, –
3799.693	....	3799.66	7	..	8	*Fe*
3801.873	....	3801.96	5	..	3	*Fe*, *Fe*
3805.486	3805.49	3805.50	6	5	6	*Fe*
3809.724	....	3809.76	4	..	5	*Mn*
3811.989	....	3811.95	4	..	3	*Fe*, –
3813.100	3813.10	3813.03	5	5	5	*Fe*
3814.698	3814.67	3814.73	8	9	8	*Fe-C*, *C*
....	....	3815.77	..	..	6	......
3817.72	....	3817.72	3	..	4	*Fe-C*
3820.586	....	3820.58	25	..	20	*Fe-C*
3826.027	....	3826.07	20	..	20	*Fe*
3827.980	....	3827.95	8	..	8	*Fe*
3835.509	....	3835.47	1	..	2	*C* (*H?*)
3838.435	....	3838.43	25	..	25	*Mg-C*
3840.580	....	3840.62	8	..	9	*Fe-C*
3843.195	....	3843.19	2	..	2	*Fe-C*
Blend?	....	3845.44	..	..	7	......
3846.924	....	3846.85	8	..	10	*C*, *C*, *Fe*, *C*
3848.007	....	3848.08	2+	..	5	*C*, *C*, *C*
3850.118	3850.11	3850.07	10	10	10	*Fe*
....	....	3850.83	..	..	4	......
....	....	3851.72	..	..	3	......
3852.714	....	3852.72	4	..	4	*Fe*
3853.620	....	3853.59	2	..	2	*C*
3854.707	....	3854.66	2	..	5	*C*
....	....	3855.61	..	..	3	......
3856.58	....	3856.58	10	..	9	*Fe*, *C?*
3858.924	....	3858.94	4	..	7	*C*,-, *C?*
3860.055	....	3860.11	20	..	20	*Fe -C*
3861.769	....	3861.67	5	..	8	*C*, *C*, *C*
3862.660	....	3862.69	3	..	3	*C?*, –
3863.875	....	3863.92	4	..	3	*C*, *Fe*
3865.674	3865.64	3865.66	7	8	6	*Fe-C*
....	....	3866.24	..	8	..	......
3867.36?	3867.28	....	3	6	..	*Fe-C*
3867.832	....	3867.83	2+	..	3	*C-V*, *C-*
Blend?	3869.54	....	6	7	..	*Fe--C*, *C*, *C C*
3869.725	....	3869.77	5	..	8	*Fe-C*, *C*, *C*
....	....	3871.16	..	..	5	......

Wave-length			Intensity			Substance
Rowland	Intermediate	Abnormal	Rowland	Inter-mediate	Abnormal	
3871.963	....	3871.96	2	..	7	*Fe*
Blend?	3872.85	3872.80	..	15	5	*Fe, C*
....	....	3873.32	..	..	3	......
3874.091	3874.10	....	4	9	..	*Co-C*
3875.025	3875.05	....	5	6	..	*C, C, V*
3875.355	....	3875.30	5	..	6	*V, Ti-C, C*
Blend?	3876.04	....	..	..	..	......
3876.194	....	3876.11	5	..	8	*Fe*
3877.051	....	3877.06	8	..	7	*Co-C, C*
3875.152	....	3878.11	8	..	10	*Fe-C*
Blend	3878.47	....	22	25	..	*Fe, Co*
3878.767	....	3878.75	11	..	12	*Fe, Co-Fe*
3879.716	....	3879.73	1	..	6	*C*
Carb'n iden-	....	3880.24	..	..	9	......
tifications	....	3880.60	..	..	8	......
possible	....	3881.05	..	..	6	... ..
....	....	3882.11	..	..	8	......
....	....	3882.82	..	..	7	......
Blend	3883.34	....	..	12	..	......
3883.462	....	3883.46	3	..	12	*C–, C*
....	3884.28*	....	..	..	..	......
3884.518	3884.51	....	2	6	..	*Fe*
....	....	3884.67*	..	..	..	......
3884.780	3884.85	....	2	3	..	*Ca, Fe*
3885.327	3885.33	....	4	4	..	*Fe, FeCr*
....	....	3885.44	..	..	5	......
3886.434	3886.44	3886.38	15	12	12	*Fe*
3887.196	3887.21	3887.20	7	8	7	*Fe*
....	....	3888.03	..	..	4	......
3888.560	....	3888.50	2	..	4	......
Blend	3889.05	....	..	15	..	*Fe, Mn*
Blend	....	3889.26	..	..	8	*Fe, Mn*
....	....	3889.71	..	..	3	......
3890.538	....	3890.49	2	..	3	*Fe-Zr*
3890.986	....	3891.02	3	..	9	*Fe*
3892.069	3892.04	....	4	5	..	*Fe*
....	....	3892.22	..	..	3	......
3892.698	....	3892.75	2	..	2	*Mn*
3893.542	3893.59	3893.50	4	4	4	*Fe*
3894.181	3894.18	3894.10	10	8	5	*Fe,-Cr, Co*
3895.145	3895.22	....	3	3	..	*Co, Ce*
3895.803	3895.79	....	7	12	..	*Fe*
....	3895.98*	....	..	..	..	......
....	....	3896.21*	..	..	..	......
3897.596	....	3897.58	2	..	2	*Fe*
3898.131	3898.10	3898.10	10	10	12	*Fe, V, Fe*
....	3898.65	....	..	..	..	......
3899.213	3899.30	....	5	4	..	*Fe, –*
3899.850	3899.87	3899.90	8	8	8	*Fe*
3900.681	3900.68	3900.68	5	5	6	*Ti-Fe-Zr*
3901.735	......	3901.77	2	..	2	......
3902.030	3902.08	....	4	4	..	*–, Fe*
3902.399	....	3902.39	3	..	8	*V*

*Bright line.

Wave-length			Intensity			Substance
Rowland	Intermediate	Abnormal	Rowland	Inter-mediate	Abnormal	
3903.090	3903.11	....	10	12	..	*Fe-Cr*
3903.153	....	3903.21	13	..	12	*Fe – Cr*, –, –
3904.023	3903.98	3904.06	8	5	7	–, *Fe*
3905.660	3905.64	....	12	20	..	*Si*
Blend	... ...	3905.81	21	..	20	*Si*, –
3906.703	....	3906.75	14	..	4	*Fe, Fe*
Blend?	3907.08	....	5	6	..	*Fe*, –
3908.077	3908.08	....	5	8	..	*Fe*
3908.410	....	3908.41	1	.	15	–
3908.900	....	3908.90	4	..	8	*Cr*
3909.919	3909.93	3909.88	12	12	7	*Fe, Fe, Co-Ca*
3910.469	3910.49	....	2	4	..	–
3910.670	....	3910.67	2	..	6	–
3910.984	3910.98	....	4	5	..	*Fe-V*
Blend?	....	3911.20	..	..	3	*Fe-V*, –
3912.127	3912.14	....	2	3	..	*Cr?*
Blend?	....	3912.36	4	..	7	*Cr?*, *V?*
3913.123	3913.11	....	2	2	..	*Ni*
3913.395	....	3913.37	1	..	9	–
3913.683	3913.63	....	9	7	..	*Ti-Fe, Fe*
3914.493	3914.50	3914.48	7	8	5	*Fe?*, *Ti*,–, *Ni?*
3914.880	....	3914.92	0	..	2	*Fe*
3915.847	....	3915.87	9	..	7	*Fe, Cr*, –, *Cr* –
3916.545	3916.53	....	3	..	4	–
3916.879	....	3916.84	5	..	10	*Fe*
3917.307	3917.34	3917.25	7	10	3	*Co, Fe*
3917.731	....	3917.67	0	..	5	*Cr*
....	....	3918.11	..	..	3	......
3918.514	3918.53	3918.50	8	7	8	*Fe, Fe*
3919.258	3919.27	3919.24	6	6	3	*Fe, Cr*
3920.410	3920.38	3920.42	10	10	10	*Fe*
3920.984	3921.00	....	2	2	..	......
3921.188	....	3921.21	3	..	3	*Cr-Nd*
Blend?	3921.71	....	9	14	..	*Ti,La*–,*Zr-Mn*
3921.855	... ...	3921.87	4	..	20	*Zr-Mn*
3923.054	3923.06	3923.03	12	12	12	*Fe*
....	....	3924.10	..	..	6	......
3924.673	3924.67	....	4	4	..	*Ti*
....	....	3924.89	..	..	4	......
3925.347	....	3925.34	4	.	3	......
3925.771	3925.70	3925.76	6	5	8	–, *Fe*
3926.123	3926.15	3926.14	7	5	6	*Fe*, –
3927.585	3927.63	....	1	4	..	–
....	....	3927.77	..	..	25	......
3928.783	....	3928.77	3	..	3	*Cr*
3929.260	....	3929.20	2	..	2	*Fe-Co*
3930.450	3930.46	3930.45	8	15	28	*Fe*
....	....	3931.49	..	..	4	......
....	3932.02	3932.09	..	2	3	......
....	3932.29	....	..	2	..	......
....	....	3932.46	..	..	5	......
....	....	3932.97	..	..	4	......
....	3934.29	....	..	4	..	......

Wave-length			Intensity			Substance
Rowland	Intermediate	Abnormal	Rowland	Intermediate	Abnormal	
....	....	3934.43	..	..	3	......
....	....	3935.27	..	..	2	......
....	3937.23	....	..	4	..	......
....	....	3937.39	..	..	10	......
3938.552	3938.58	3938.54	4	5	4	–
....	....	3938.89	..	..	5	......
3939.288 ?	....	3939.31	0	..	3	–
....	3940.25	3940.32		7	12	......
3941.025	3941.02	....	5	3	..	*Fe, Co*
3941.637	....	3841.56	3	..	3	*Cr*
Blend ?	....	3941.99	3	..	2	*Co,–*
3942.558	3942.61	....	5	4	..	*–, Fe*
3943.370	....	3943.42	7	..	2	*–, –, Fe*
3943.721	......	3943.77	1	..	2	–
3944.160	3944.19	3944.16	15	15	12	*Al*
3944.884	....	3944.90	2	..	3	*Fe?*
3945.365	....	3945.31	7	..	6	*Fe, –, Co*
3945.993	3945.98	4946.00	1	4	9	*Mn?*
3947.142	3947.17	3947.09	3	4	4	*Fe*
3947.624	3947.58	3947.70*	6	5	8	*–, Fe*
3948.246	3948.30	....	5	..	4	*Fe*
Blend	3948.91	....	13	15	..	*Ti, Fe, Ca, etc.*
Blend	....	3949.15	..	..	9	......
3949.372	3949.33	....	1	2	..	–
3950.102	....	3950.07	5	..	2	*Fe*
Blend ?	3950.33	....	..	10	..	......
3950.497	......	3950.51	2	..	13	*Y*
3951.296	3951.29	3951.30	6	10	15	*Cr, Fe*
3952.754	....	3952.74	4	..	4	*Fe*
Blend	3953.02	....	17	15	..	*Fe, Mn, Co, Cr*
Blend	....	3953.24	10	..	7	*Mn, Co, Cr*
3954.002	....	3953.96	3	..	3	*Fe*
....	3954.15	....	..	6	..	......
....	....	3954.40	..	..	13	......
3954.857	3954.84	....	1	2	..	*Fe?*
3955.461	3955.45	....	6	6	..	*–, Fe*
....	....	3955.58	..	..	5	......
3956.099	3956.10	....	3	3	..	*Fe*
Blend ?	3956.50	3956.42	8	6	8	*Co–Ti, Fe*
3957.177	....	3957.18	7	..	6	*Fe–Ca*
....	....	3957.94	..	..	7	......
3958.355	3958.41	....	5	8	..	*Ti, Zr*
3958.877	3958.86	....	..	2	..	*Fe*
3959.135	....	3959.45	0	..	2	–
3959.972	3959.99	....	1	<2	..	–
....	....	3960.24	..	..	4	......
....	....	3960.96	..	..	2	......
....	....	3961.56	..	..	6	......
3961.674	3961.62	....	20	20	..	*Al*
3962.287	....	3962.26	3	..	11	*Fe?, –*
3963.831	3963.90	3963.83	3	3	5	*Cr*
3964.663	3964.67	3964.75	3	2	2	*Fe*
3965.366	3965.40	....	0	1	..	*–Co*

* Probably Fe λ3947.675 alone.

Wave-length.			Intensity.			Substance
Rowland	Intermediate	Abnormal	Rowland	Inter-mediate	Abnormal	
3965.655[1]	....	3965.67	2	..	6	*Fe*,–
3966.212	....	3966.17	3	..	3	*Fe*
Blend	3966.80	....	6	6	..	*Ni*, *Fe*
3966.966	....	3967.02	1	..	6	......
3968.625?	3968.62	3968.68	(700)	7	7	*Ca*
....	....	3969.00	..	..	8	......
3969.413	3969. 4	....	10	4	..	*Fe*
Blend	3970.05	....	7	8	..	*Cr*, *H*
3970.177	....	3970.26	5	..	10	*H*
3971.475	....	3971.47	5	..	5	*Fe*
....	....	3971.88	..	..	2	......
3972.313	3972.30	....	2	12	..	*Ni*
3972.639	....	3972.61	2	..	12	......
3973.285	....	3973.30	2	..	2	*Co*, *Co*
3973.772	....	3973.74	6	..	15	*Ni*, *Zr*, *Fe*, *Ca*
3974.774	....	3974.80	2	..	3	*Ni*
3975.350	....	3975.37	2	..	5	*Fe*
3975.985	....	3975.91	2	..	2	*Fe*–*Mn*
Blend	3976.72	3976.71	5	4	5	*Fe*, *Fe*
3977.891	3977.90	....	6	8	..	*Fe*
....	....	3978.30	..	..	7	......
3978.809	3978.72	3978.80	3	4	3	*Co*, *Cr*
3979.664	3979.67	3979.66	4	3	4	*Co*
....	....	3980.39	..	..	2	......
3981.249	....	3981.17	4	..	4	......
3981.917	3981.89	3981.98	4	13	30	*Ti*
3984.091	3984.07	3984.09	7	6	8	*Cr*, *Fe*
3984.806	3984.81	....	2	2	..	*Ce*–*Zr*
....	....	3984.97	..	..	4	......
....	......	3985.38	..	..	4	......
3985.526	3985.50	....	6	5	..	*Mn*, *Fe*
3986.321	3986.34	3986.32	3	4	3	*Fe*
3986.903	3986.90	....	6	8	..	—
3987.241	3987.26	3987.25	5	8	8	– , *Mn*, *Co*
3988.114	....	3988.05	0	..	2	–
3988.660	....	3988.69	1+	..	2	......
3989.175	3989.18	3989.15	5	4	3	–; –
3989.912	3990.00	3989.91	4	4	11	*Ti*
3991.333	3991.29	....	3	3	..	*Cr*, *Zr*
3992.971	3992.93	3993.05	3	4	10	*V*–*Cr*
3993.246	3993.29	....	2	2	..	*Fe*
3994.219	3994.22	....	6	4	..	*Cr*, *Ni*, *Fe*
3995.431	3995.43	3995.46	7	5	6	–, *Co*
3996.140	....	3996.17	3	..	3	*Fe*
....	3996.57	....	..	4	..	......
....	....	3996.80	..	..	9	......
3997.577	3997.61	3997.56	6	9	4	......
3998.129	3998.12	3998.08	8	6	8	*Co*, *Fe*
3998.790	3998.78	3998.75	4	4	4	*Ti*
3999.144	....	3999.17	1+	..	2	*Zr*, *Fe*, *Ce*
4000.507	....	4000.49	4	..	10	*Fe*, *Fe*
4001.315	4001.32	....	3	5	..	–
4002.227	....	4002.20	..	..	2	......

Wave-length			Intensity			Substance
Rowland	Intermediate	Abnormal	Rowland	Inter-mediate	Abnormal	
4002.652?	....	4002.71	0	..	8	*Fe–Ti*
4003.076	4003.09	....	2	2	..	–
4003.912	4003.94	4003.92	3	4	3	*Ce-Ti-Fe*
4005.408	....	4005.41	7	..	10	*Fe*
4005.856	4005.86	4005.87	3	25	5	–
4006.411	4006.35	4006.45	3	2	5	*Ni Fe*
Blend	4006.91	....	6	7	..	*Fe*, –
4007.429	4007.49	4007.44	3	3	3	*Fe*
4008.075	....	4008.09	0	..	4	*Co* – ?
4008.748	....	4008.75	0	..	7	–
4009.056	4009.09	....	5	4	..	–, *Ti*
4009.864	....	4009.85	3	..	3	*Fe*
4010.004	4010.98	4011.03	..	5	3	......
4012.513	4012.58	4012.46	5	4	5	*Nd, Zr, Ti*
....	....	4013.09	..	..	2	......
4013.902	4013.89	4013.95	8	12	15	*Ti-Fe, Fe*
4014.677	4014.66	4014.70	5	9	20	*Fe*
....	....	4015.12	..	..	2	......
4015.760	4015.78	....	3	3	..	–
Blend?	....	4016.46	2+	..	2	–, *Fe*
4017.308	4017.33	4017.33	4	3	4	*Fe*
4017.655	4017.69	4017.70	3	2	2	*Ni? Ni?*
4018.420*	4018.36	4018.42	3	12	8	*Fe*
4019.201	....	4019.22	1	..	7	*Ni-Ce*
....	4019.35	....	..	4	..	......
Blend	4020.46	4020.54	6	10	4	*Mn*, –, *Sc, Fe*
4021.057	4021.09	4021.03	3	2	4	*Co*
4022.018	4021.95	4022.02	5	6	8	*Fe*
....	....	4023.38	..	..	10	......
4023.533	4023.56	....	3	3	..	*Co* –
4024.216	4024.22	4024.23	3	2	2	*Zr, Fe*
4024.815	4024.80	4024.79	7	7	8	*Ti, Fe*
4025.286	4025.29	....	3	3	..	*Ti*
4025.579	....	4025.58	1	..	2	*Cr*
4025.972	....	4025.96	2	..	3	*Co-La*
4027.189	4027.21	4027.18	1	2	3	*Co*
4027.822	....	4027.87	1	..	2	–
4028.497	4028.49	4028.50	4	6	4	*Ti*
....	....	4029.27	..	..	3	......
4029.796	4029.81	4029.77	5	5	5	*Fe-Zr*
4030.918	4030.93	4030.89	9	12	10	*Mn*
4031.904	4031.87	4031.93	4	4	5	*Fe-La, Mn*
4032.117	4032.09	....	2	2	..	*Fe*
4032.729	4032.70	4032.80	6	5	4	*Fe, Fe*
4033.224	4033.24	4033.22	7	12	3	*Fe-Mn*
4033.773	4033.72	4033.81	2	3	15	*Mn, Mn*
4034.456	....	4034.46	2	..	7	–, –
4034.644	4034.63	....	6	10	..	*Mn-Fe*
....	....	4035.32	..	..	2	......
4035.806	4035.81	4035.82	6	12	6	*Co, Mn*
4036.522	....	4036.52	0	..	2	–
....	....	4037.15	..	..	2	......
....	....	4037.72	..	..	3	......

*Perhaps a blend in intermediate spectrum.

Wave-length			Intensity			Substance
Rowland	Intermediate	Abnormal	Rowland	Intermediate	Abnormal	
4039.244	4039.29	....	1	2	..	*Cr*
4039.727	....	4039.74	0	..	2	–
4040.792	4040.78	4040.80	3	6	20	*Fe*
4041.432*	4041.50	4041.43	3	6	3	*Fe*
4042.743	....	4042.75	0	..	2	*Cr, Nd*
Blend?	4044.09	4044.19	5	20	15	*Fe*, –
4044.736	4044.72	4044.71	4	6	4	–, *Fe*
4045.975	4046.01	4045.94	30	30	5	*Fe*
....	....	4046.77	..	..	3	......
....	....	4047.73	..	..	5	......
4048.224	4048.22	4048.26	1	2	4	–
4048.883	4048.91	4048.81	6	6	7	*Zr, Mn-Cr*
....	....	4049.33	..	..	5	......
4049.799	4049.85	....	2	2	..	–, –
4050.830	4050.81	4050.84	2	4	7	*Fe*
4052.070	4052.05	....	3	2	..	*Fe*
Blend	4052.72	....	7	4	..	*Mn, Fe*, –
4053.424	....	4053.43	2	..	5	*Fe*
4053.981	....	4053.94	3	..	3	*Fe-Ti*,
4054.999	4054.96	4055.04	5	8	14	*Fe, Fe-Ti*
4055.701	4055.71	4055.73	6	7	6	*Mn*
4057.466	....	4057.39	4	..	15	*Co, Fe*
4057.668	4057.66	....	7	10	..	–
4058.372	4058.34	4058.34	4	3	4	*Co-Fe*
4058.998	4058.99	4058.92	6	4	10	*Fe, Cr, Mn*
4059.535	4059.53	....	1	6	..	*Mn*
....	....	4059.70	..	..	12	......
....	....	4060.69	..	..	5	......
4061.244	....	4061.26	3	..	5	*Nd*
4061.881	....	4061.86	2	..	2	*Mn*
....	4062.10	....	..	4	..	......
4062.599	4062.60	4062.62	5	4	5	*Fe*
4063.436	....	4063.38	4	..	12	*Fe*
4063.759	4063.74	....	20	20	..	*Fe*
Blend?	....	4064.41	5	..	8	–, *Ti, Ni, Fe*
Blend?	....	4066.47	4	..	8	–,–, *Co*
4067.139	4067.07	4067.09	5	3	5	*Fe*
4067.429	4067.39	....	3	3	..	*Fe*
4068.137	4068.15	4068.11	6	7	3	*Fe-Mn*
4069.221	4069.21	4069.19	2	2	3	......
4070.431	4070.43	4070.44	3	3	6	*Mn*
4070.930	4070.92	....	4	4	..	*Fe*
4071.908	4071.87	4071.91	15	15	15	*Fe*
4072.655	4072.66	4072.64	2	3	3	*Fe*
4073.921	4073.90	4073.87	4	4	4	*Fe*
4074.902	4074.91	4074.90	5	6	5	–, *Fe*
....	4076.24	....	..	4	..	......
4076.823	4076.84	4076.78	9	7	7	*Fe-Zr, Fe*,–
4077.885	4077.88	4077.82	8	10	7	*Sr*
4078.565	4078.56	4078.58	7	6	8	*Fe, Ti*
4079.570	4079.50	....	3	7	..	*Mn*
4079.996	4079.97	4079.98	3	2	5	*Fe*
4080.368	4080.34	....	3	4	..	*Fe, Nd*

*Probably in a blend intermediate spectrum.

Wave-length			Intensity			Substance
Rowland	Intermediate	Abnormal	Rowland	Inter-mediate	Abnormal	
....	....	....	..	..	..	......
....	....	....	..	..	..	......
....	....	4081.77	..	..	2	......
4082.264	4082.24	....	2	4	..	*Fe*
4082.589	....	4082.61	3	..	6	*Sc–Fe–Ti*
4083.095	4083.08	....	4	5	..	*Mn*
4083.783	4083.84	4083.77	7	11	8	*Fe,Mn,Y,Fe*
4084.148	....	4084.17	0	..	2	–
4084.647	4084.60	4084.67	5	6	5	*Fe*
....	....	4085.27	..	..	7	......
4085.455	4085.46	....	5	5	..	–, *Fe*
4086.469	....	4086.40	3	..	3	*Co* –
4087.252	4087.27	4087.23	3	3	3	*Fe*
4088.713	....	4088.72	3	..	6	*Fe*
4089.374	4089.31	4089.36	3	4	3	*Fe*
4090.228	....	4090.19	2	..	3	*Fe*
4091.109	4091.08	....	3	2	..	–
4091.711	4091.77	....	3	4	..	*Fe*
4092.626	4092.64	4092.68	9	9	8	*Fe,Co,Mn,V,Ca*
4095.094	4095.10	4095.09	4	4	4	*Ca*
4096.213	4096.26	4096.19	6	6	7	*Fe,Fe*, –
4097.241	4097.23	4097.23	3	4	3	*Fe*
4098.335	....	4098.28	5	..	4	*Fe*
4098.708	4098.70	....	6	2	..	*Ca?*, –
4099.207	....	4099.23	0	..	4	–
Blend ?	4100.00	....	2+	2	..	*V*, –
4102.000?	4101.95	....	40	7	..	*H,In*
4103.097	4103.10	4103.09	5	6	5	*Si,Mn*
4104.288	4104.29	....	5	4	..	*Fe*
Blend ?	4105.23	4105.23	3	4	3	–, *V*
4106.502	4106.49	....	4	4	..	*Fe,Fe*
4107.649	4107.65	4107.65	5	5	4	*Ce–Fe–Zn*
4108.687	4108.73	....	2	2	..	–
4109.215	....	4109.17	3	..	7	*Fe*
4109.934	4109.96	4110.00	5	6	8	*V,Fe*
4111.940	4111.93	4111.92	4	4	4	*V*
4113.104	4113.09	4113.16	4	4	4	–, *Fe*
4114.606	4114.64	....	4	4	..	*Fe*
4115.330	4115.34	4115.28	3	2	7	*V*
....	....	4116.02	..	..	2	......
4116.746	4116.79	....	2+	4	..	*V,V,Nd?*
4118.008	4118.01	....	2	2	..	–
4118.708	4118.66	....	5	6	..	*Fe*
4119.973	4119.01	4118.96	..	8	10	*Co,Fe*
4120.368	4120.37	....	4	4	..	*Fe*
4121.477	4121.52	4121.44	6	5	2	*Cr–Co*
4121.963	4121.95	....	3	2	..	*Fe,Cr*
4122.673	....	4122.67	3	..	2	*Fe*
4122.710	4122.78	....	4	4	..	*Fe*, –
4123.907	4123.95	4123.92	5	2	4	*Fe*
4126.344	4126.34	4126.34	4	3	4	*Fe*
4127.862	4127.86	4127.86	8	7	..	*Fe,Fe*
4128.251	4128.27	....	6	3	..	*V*

Wave-length			Intensity			Substance
Rowland	Intermediate	Abnormal	Rowland	Intermediate	Abnormal	
4130.196	4130.21	....	2	3	..	*Fe*
4132.212	4132.20	4132.24	12	10	10	*V,Fe*
4133.062	4133.08	4133.05	4	5	3	*Fe*
4134.010	4134.02	4134.06	3	4	3	*Fe*
Blend	4134.72	....	12	9	10	*Fe?,V-Fe?,* –
4137.156	4137.20	....	6	3	..	*Fe*
4140.089	4140.10	....	6	4	..	*Fe*
4142.686	4142.70	....	4	4	..	*Cr*, –
4143.603	4143.63	....	6	4	..	*Fe*, –
4144.038	4144.08	....	15	12	..	*Fe*
4146.225	4146.26	....	3	4	..	*Fe*
4147.836	4147.83	....	4	6	..	*Fe*
4149.533	4149.51	....	4	4	..	*Fe*

companion has disappeared. On the other hand, such lines as λλ 3898.13, 3920.41, 3923.05, etc., remain of about uniform intensity, while λλ 3930.45, 3940.25, 3951.30, 3981.92, 3992.97 and others are strongest in the abnormal spectrum. Again, there are many cases where lines are stronger in the intermediate spectrum than in the normal or abnormal spectra. At the less refrangible end of the plate, the lines of both the intermediate and abnormal spectra resemble much more closely the lines of the normal solar spectrum. In both the intermediate and abnormal spectra, several lines are strengthened where they cross the spot band.

In examining the photographs, the greatly reduced intensity of the broad dark bands at H and K will be noticed. Under a microscope the original negative shows the K band to be broken up into a number of fine lines, those on the less refrangible side being the more conspicuous. At H there are some strong metallic lines on the violet side, but on the red side there seem to be two or three fine lines resembling those at K. This is of special interest in view of the fact that Jewell found the shading of the H and K lines broken up into lines on one of Rowland's photographs of the solar spectrum; he particularly remarks that the general shading of H and K on this plate is unusually weak.[1] On another occasion, with an arc produced by an extremely powerful

[1] Astrophysical Journal, 8, 51, 1898.

current, Jewell also succeeded in resolving the shadings of the calcium lines. In the present spectra, however, there is no such uniformity in the spacing of the lines as Rowland's photograph shows.

I had intended to photograph this region in the spectra of certain stars for comparison with the above results, and to make some laboratory investigations with the same end in view. As the Sun-spot maximum is approaching, however, I have thought it best to publish the results as they stand. In view of the importance of recording other similar phenomena, it is to be hoped that a plan can be arranged, perhaps through co-operation, by means of which photographs of the solar spectrum, in the neighborhood of Sun-spots, can be taken daily at very short time intervals.

YERKES OBSERVATORY,
November 6, 1902.

DETERMINATION OF THE INTENSITY-RATIOS OF THE PRINCIPAL LINES IN THE SPECTRA OF SEVERAL GASEOUS NEBULÆ.

By J. Scheiner and J. Wilsing.

At the request of the editors of the Astrophysical Journal, we communicate here a short abstract of our detailed paper on the intensity-ratios of the principal lines of the nebular spectrums recently published in *Astronomische Nachrichten*.[1]

The measures were made with a spectro-photometer, constructed on Crova's principle, attached to the great refractor. By a special arrangement of the eyepiece of the spectro-photometer we were able to compare the nebular lines with objects almost exactly like them in appearance. The intensity of the photometer lamp was determined on every observing night by comparison with a constant source of light (benzine lamp), so that the observations could also be utilized for deriving the relative brightness of the first nebular line in the different nebulæ.

Rigorous photometric comparisons are only possible between radiations of precisely the same wave-lengths, a condition which is fulfilled by the arrangement of our observations. But this carries with it the fact that we do not obtain the desired intensity-ratios of the three lines, $\frac{J_1}{J_2}$ and $\frac{J_1}{J_3}$, directly, but in terms of the corresponding ratios in the spectrum of the photometer lamp. The latter are, however, of a purely accidental character, dependent upon the temperature of the carbon filament of the incandescent lamp used as source, and upon the dispersion of the spectrograph. They are designated below as $\frac{1}{\alpha_2}$ and $\frac{1}{\alpha_3}$.

The following table contains a summary for the two observers of the values of

$$\log \alpha_2 \frac{J_1}{J_2}, \quad \log \alpha_3 \frac{J_1}{J_3}$$

[1] *A. N.*, 159, 181, 1902.

PLATE VIII.

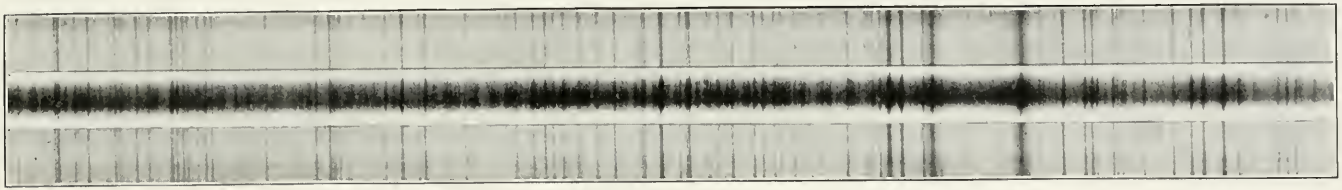


FIG. 1.—"Bands" and Widened Lines in the Spectrum of a Sun-spot.

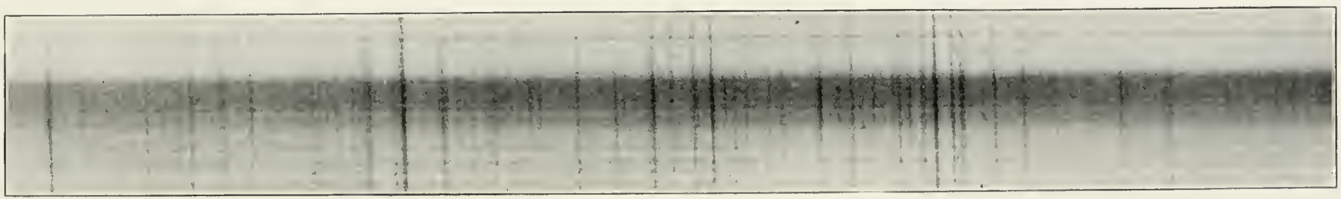


FIG. 2.—Widened Lines in the Spectrum of a Sun-spot.

and their corresponding natural numbers, together with the mean values for the different nebulæ:

1901	Wilsing		Scheiner		Wilsing		Scheiner	
	$\log a_2 \frac{J_1}{J_2}$	$a_2 \frac{J_1}{J_2}$	$\log a_2 \frac{J_1}{J_2}$	$a_2 \frac{J_1}{J_2}$	$\log a_3 \frac{J_1}{J_3}$	$a_3 \frac{J_1}{J_3}$	$\log a_3 \frac{J_1}{J_3}$	$a_3 \frac{J_1}{J_3}$
				G.C. 4234				
May 20	0.428	2.7	0.387	2.4	0.637	4.3	0.607	4.0
20	0.439	2.7	0.362	2.3	0.787	6.1	0.661	4.6
21	0.515	3.3	0.407	2.6	0.943	8.8	0.748	5.6
21	0.383	2.4	0.450	2.8	0.717	5.2	0.672	4.7
22	0.447	2.8	0.330	2.1	0.849	7.1	0.627	4.2
22	[0.601]	[4.0]	0.411	2.6	0.752	5.6	0.763	5.8
28	0.451	2.8	0.329	2.1	0.794	6.2	0.901	8.0
June 5	0.503	3.2	0.371	2.3	0.836	6.9	0.802	6.3
7	0.549	3.5	0.354	2.3	0.957	9.1	0.766	5.8
	0.464	2.91	0.377	2.38	0.808	6.43	0.727	5.33
				G.C. 4373				
May 21	0.411	2.6	0.467	2.9	0.517	3.3	0.528	3.4
21	0.465	2.9	0.401	2.5	0.509	3.2	0.454	2.8
22	0.450	2.8	0.422	2.6	0.591	3.9	0.702	5.0
28	0.515	3.3	0.365	2.3	0.642	4.4	0.550	3.5
June 5	0.568	3.7	.....	...	0.620	4.2	.....	...
	0.482	3.03	0.414	2.59	0.576	3.77	0.559	3.62
				G.C. 4390				
June 5	0.432	2.7	0.365	2.3	0.681	4.8	0.712	5.1
7	0.483	3.0	0.487	3.1	[1.124]	...	0.823	6.7
July 10	0.293	2.0	0.218	1.7	0.698	5.0	0.642	4.4
11	0.482	3.0	0.390	2.5	0.683	4.8	0.734	5.4
Aug. 10	.....	...	0.261	1.8	.....	...	0.628	4.2
	0.423	2.65	0.344	2.21	0.687	4.86	0.708	5.10
				N.G.C. 6790				
July 11	0.372	2.4	0.386	2.4	0.884	7.7	1.108	12.8
12	0.537	3.4	0.327	2.1	0.994	9.9	0.932	8.6
Aug. 9	.....	...	0.416	2.6	.....	...	1.133	13.6
10	.....	...	0.273	1.9	.....	...	0.901	8.0
Nov. 9	0.417	2.6	.....	...	.....	...	.....	....
	0.442	2.77	0.351	2.24	0.939	8.69	1.019	10.40

1901	Wilsing		Scheiner		Wilsing		Scheiner	
	$\log a_2 \frac{J_1}{J_2}$	$a_2 \frac{J_1}{J_2}$	$\log a_2 \frac{J_1}{J_2}$	$a_2 \frac{J_1}{J_2}$	$\log a_3 \frac{J_1}{J_3}$	$a_3 \frac{J_1}{J_3}$	$\log a_3 \frac{J_1}{J_3}$	$a_3 \frac{J_1}{J_3}$
				G.C. 4514				
May 22	0.355	2.3	0.401	2.5	0.599	4.0	0.607	4.0
28	0.356	2.3	0.403	2.5	0.603	4.0	0.617	4.1
June 5	0.400	2.5	0.342	2.2	0.664	4.6	0.667	4.6
7	0.448	2.8	0.402	2.5	0.613	4.1	0.712	5.2
July 11	0.417	2.6	.....	...	.....	...	.....	...
12	0.502	3.2	0.399	2.5	0.657	4.5	0.610	4.1
16	0.470	3.0	0.533	3.4	0.765	5.8	0.722	5.3
	0.421	2.64	0.413	2.59	0.650	4.47	0.657	4.54
				N.G.C. 6891				
July 12	0.501	3.2	0.479	3.0	0.736	5.45	0.823	6.7
16	0.453	2.8	0.502	3.2	.....	....	.....	...
Aug. 9	.....	...	0.362	2.3	.....	....	0.605	4.0
10	.....	...	0.424	2.7	.....	....	.....	..
. 9	0.440	2.8	0.266	1.8	.....	....	0.919	8.3
	0.465	2.92	0.407	2.55	0.736	5.45	0.782	6.05
				N.G.C. 7027				
July 16	0.457	2.9	0.446	2.8	1.023	10.5	0.993	9.8
Aug. 9	.....	...	[0.668]	[4.6]	.....	....	0.934	8.6
10	.....	...	0.390	2.5	.....	....	0.859	7.2
Oct. 16	0.481	3.0	0.374	2.4	1.104	12.7	0.918	8.3
16	0.594	3.9	0.400	2.5	0.947	8.9	0.932	8.6
Dec. 12	.....	...	0.381	2.4	.....	....	1.018	10.4
28	0.388	2.4	.....	...	0.857	7.2	.....	...
	0.480	3.02	0.400	2.51	0.983	9.62	0.942	8.75
				G.C. 4964				
Aug. 9	.....	...	0.515	3.3	.....	...	0.743	5.5
10	.....	...	0.334	2.2	.....	...	0.630	4.3
Nov. 9	0.504	3.2	0.357	2.3	0.898	7.9	0.797	6.3
9	0.493	3.1	.....	...	0.861	7.3	.....	...
Dec. 12	.....	...	0.378	2.4	.....	...	....	...
1902								
Jan. 31	0.530	3.4	[0.605]	[4.0]	0.956	9.0	[1.161]	[14.5]
31	0.503	3.2	0.495	3.1	0.769	5.9	1.056	11.4
	0.508	3.22	0.416	2.61	0.871	7.43	0.807	6.41

1901	Wilsing		Scheiner		Wilsing		Scheiner	
	$\log a_2 \frac{J_1}{J_2}$	$a_2 \frac{J_1}{J_2}$	$\log a_2 \frac{J_1}{J_2}$	$a_2 \frac{J_1}{J_2}$	$\log a_3 \frac{J_1}{J_3}$	$a_3 \frac{J_1}{J_3}$	$\log a_3 \frac{J_1}{J_3}$	$a_3 \frac{J_1}{J_3}$
1901	*Orion* nebula. Trapezium							
Nov. 9	0.468	2.9	0.303	2.0	0.379	2.4	0.451	2.8
Dec. 5	0.528	3.4	.....	...	0.503	3.2	.....	...
5	0.450	2.8	.....	...	0.434	2.7	.....	...
12	.....	...	0.383	2.4	.....	...	0.409	2.6
28	0.461	2.9	0.345	2.2	0.461	2.9	0.408	2.6
1902								
Jan. 31	0.541	3.5	0.463	2.9	0.512	3.3	[0.669]	[4.7]
Feb. 21	[0.644]	[4.4]	0.474	3.0	[0.688]	[4.9]	0.521	3.3
26	0.432	2.7	0.456	2.9	0.417	2.6	0.436	2.7
26	0.399	2.5	0.421	2.6	0.427	2.7	0.306	2.0
28	0.462	2.9	0.366	2.3	0.476	3.0	0.439	2.7
	0.468	2.94	0.401	2.52	0.451	2.83	0.424	2.65
1901	*Orion* nebula. South edge of the Huyghens region							
Dec. 28	0.370	2.3	0.418	2.6	0.390	2.5	0.477	3.0
28	0.410	2.6	0.436	2.7	0.478	3.0	0.519	3.3
1902								
Jan. 31	0.596	3.9	0.352	2.3	0.227	1.7	0.415	2.6
Feb. 21	0.442	2.8	0.459	2.9	0.319	2.1	0.391	2.5
24	0.318	2.1	0.371	2.4	0.181	1.5	0.278	1.9
25	0.557	3.6	0.460	2.9	0.295	2.0	0.395	2.5
25	0.472	3.0	0.451	2.8	0.349	2.2	0.363	2.3
26	0.602	4.0	0.406	2.6	0.364	2.3	0.316	2.1
	0.471	2.96	0.419	2.62	0.325	2.11	0.398	2.50
1902	*Orion* nebula. North edge of the Huyghens region.							
Feb. 21	0.580	3.8	0.479	3.0	0.499	3.2	0.400	2.5
24	0.365	2.3	0.365	2.3	0.317	2.1	0.291	2.0
26	0.588	3.9	0.408	2.6	0.395	2.5	0.370	2.3
28	0.543	3.5	0.420	2.6	0.362	2.3	0.338	2.2
28	0.453	2.8	0.545	3.5	0.395	2.5	0.471	3.0
	0.506	3.21	0.443	2.77	0.394	2.48	0.374	2.37
1902	*Orion* nebula. West edge of the Huyghens region							
Feb. 26	0.282	1.9	0.357	2.3	0.312	2.1	0.524	3.3
28	0.527	3.4	0.478	3.0	0.555	3.6	0.547	3.5
	0.405	2.54	0.418	2.62	0.434	2.72	0.536	3.44

1902	Wilsing		Scheiner		Wilsing		Scheiner	
	$\log a_2 \frac{J_1}{J_2}$	$a_2 \frac{J_1}{J_2}$	$\log a_2 \frac{J_1}{J_2}$	$a_2 \frac{J_1}{J_2}$	$\log a_3 \frac{J_1}{J_3}$	$a_3 \frac{J_1}{J_3}$	$\log a_3 \frac{J_1}{J_3}$	$a_3 \frac{J_1}{J_3}$
1902	*Orion* nebula. Trapezium (40 cm diaphragm).							
Feb. 21	0.471	3.0	0.460	2.9	0.452	2.8	0.460	2.9
21	0.462	2.9	0.434	2.7	0.507	3.2	0.451	2.8
24	0.530	3.4	0.355	2.3	0.521	3.3	0.431	2.7
24	0.583	3.8	0.493	3.1	0.535	3.4	0.349	2.2
	0.512	3.25	0.435	2.72	0.504	3.19	0.423	2.65

The values of this table are now suitable for furnishing the precision of the observations, with general statements of which we shall here content ourselves. The results are as follows for the probable errors, expressed in magnitudes, of the intensity ratios for the mean of an evening, consisting of four settings of the intensity-circle for lines 2 and 3, and eight for line 1. These ratios may be designated briefly as ½ and ⅓.

Observer	½	⅓
Wilsing	$\pm 0^{m}.125$	$\pm 0^{m}.134$
Scheiner	± 0.105	± 0.152

While the probable errors of the two ratios are nearly the same for Wilsing, that for ⅓ is considerably greater than that for ½ for Scheiner. In the mean the probable errors are nearly the same for the two observers—for Wilsing $\pm 0^{m}.130$, for Scheiner $\pm 0^{m}.129$.

It was to be expected that the probable error would come out appreciably larger than for other photometric observations in which the observer on principle does not go below a sufficient brightness. The satisfactory value of the probable error, in spite of this, is to be attributed to the fact that with our relatively large slit-width we were working with appreciable surfaces.

The data of the foregoing table are united into mean values in the following tables A and B, where they are also converted into magnitudes.

TABLE A.

Nebula.	Wilsing		Scheiner		W.–S. in Mag.	Corr. $\alpha_2 \frac{J_1}{J_2}$		Corr. Mag.		Corr. Mean	
	$\alpha_2 \frac{J_1}{J_2}$	Mag.	$\alpha_2 \frac{J_1}{J_2}$	Mag.		W.	S.	W.	S.	$\alpha_2 \frac{J_1}{J_2}$	Mag.
G.C. 4234	2.97	1.16	2.38	0.94	+0.22	2.49	2.32	0.99	0.91	2.40	0.95
G.C. 4373	3.03	1.21	2.59	1.04	+0.17	2.61	2.52	1.04	1.01	2.57	1.03
G.C. 4390	2.65	1.06	2.21	0.86	+0.20	2.27	2.15	0.89	0.83	2.21	0.86
N.G.C. 6790	2.77	1.11	2.24	0.88	+0.23	2.38	2.19	0.94	0.85	2.28	0.90
G.C. 4514	2.64	1.05	2.59	1.03	+0.02	2.25	2.52	0.88	1.00	2.38	0.94
N.G.C. 6891	2.92	1.16	2.55	1.02	+0.14	2.49	2.49	0.99	0.99	2.49	0.99
N.G.C. 7027	3.02	1.20	2.51	1.00	+0.20	2.58	2.45	1.03	0.97	2.51	1.00
G.C. 4964	3.22	1.27	2.61	1.04	+0.23	2.75	2.54	1.10	1.01	2.64	1.06
Orion, Trapezium	2.94	1.17	2.52	1.00	+0.17	2.51	2.45	1.00	0.97	2.48	0.99
Orion, South	2.96	1.18	2.62	1.05	+0.13	2.54	2.56	1.01	1.02	2.55	1.02
Orion, North	3.21	1.27	2.77	1.11	+0.16	2.75	2.70	1.10	1.08	2.73	1.09
Orion, West	2.54	1.01	2.62	1.05	−0.04	2.17	2.56	0.84	1.02	2.36	0.93
Orion, Tr. (Diaph.)	3.25	1.28	2.72	1.09	+0.19	2.78	2.66	1.11	1.06	2.72	1.09
Mean	2.93	1.16	2.52	1.00	+0.16	2.51	2.45	0.99	0.97	2.49	0.99

TABLE B.

Nebula	Wilsing		Scheiner		Mean		W.–S. in Mag.
	$\alpha_3 \frac{J_1}{J_3}$	Mag.	$\alpha_3 \frac{J_1}{J_3}$	Mag.	$\alpha_3 \frac{J_1}{J_3}$	Mag.	
G.C. 4234	6.43	2.02	5.33	1.82	5.86	1.92	+0.20
G.C. 4373	3.77	1.44	3.62	1.40	3.70	1.42	+0.04
G.C. 4390	4.86	1.72	5.10	1.77	5.02	1.75	−0.05
N.G.C. 6790	8.69	2.35	10.40	2.55	9.55	2.45	−0.20
G.C. 4514	4.47	1.63	4.54	1.64	4.53	1.64	−0.01
N.G.C. 6891	5.45	1.84	6.05	1.96	5.75	1.90	−0.12
N.G.C. 7027	9.62	2.46	8.75	2.36	9.20	2.41	+0.10
G.C. 4964	7.43	2.18	6.41	2.02	6.92	2.10	+0.16
Orion, Trapezium	2.83	1.13	2.65	1.06	2.75	1.10	+0.07
Orion, South	2.11	0.81	2.50	1.00	2.31	0.91	−0.19
Orion, North	2.48	0.99	2.37	0.94	2.44	0.97	+0.05
Orion, West	2.72	1.09	3.44	1.34	3.08	1.22	−0.25
Orion, Tr. (Diaphragm)	3.19	1.26	2.65	1.06	2.91	1.16	+0.20
Mean	4.07	1.53	4.25	1.57	4.17	1.55	−0.04

Now, in the first place, the ratio of the first to the second line (Table A) exhibits an almost absolute constancy in the different nebulæ, so that differences cannot be derived from the nine observed nebulæ. An entirely different behavior is shown

in Table B in respect to the ratio of the first to the third line. Here very pronounced differences from the mean, rising to a magnitude, or to nine times the probable error, occur for both observers, and always in the same sense. It cannot be doubted that at least the larger of these differences are to be considered as real. If we now arrange the nebulæ according to the size of the ratio, we see that the third line is relatively faintest in the small nebula *N.G.C.* 6790, and relatively brightest in the *Orion* nebula, as follows:

N.G.C. 6790	$2^m.45$	*G.C.* 4390	$1^m.75$
N.G.C. 7027	2.41	*G.C.* 4514	1.64
G.C. 4964	2.10	*G.C.* 4373	1.42
G.C. 4234	1.92	*Orion* nebula	1.10
N.G.C. 6891	1.90		

Accordingly we may state this proposition: In the nine nebulæ we have investigated the ratio of brightness is constant between the first and second lines, but strongly varies between the first and third lines.

This is precisely the result reached by Keeler from his estimates of brightness, which is now confirmed by our measures. We call attention to the fact that this result is favorable to the view that the first and second nebular lines belong to the same element, at present still unknown, and that the hydrogen does not shine in the different nebulæ under the same physical conditions (relative quantity?). The often expressed view, recently again advanced by M. Bélopolsky, that the first and second nebular lines belong to a modified hydrogen spectrum is less favored by our result, although not contradicted by it.

The final means of Table B agree for the two observers within the probable errors, so that there is no constant difference between our measures. It is, therefore, the more surprising that the means in Table A show a clearly pronounced difference, Wilsing–Scheiner $= + 0^m.17$, although the measured difference of brightness of $1/2$ is considerably smaller than $1/3$. Now, the only difference between the measures of $1/2$ and $1/3$ consisted in the fact that line 1 remained visible in the field while the settings were being made on line 2, which was not the case for settings on

line 3; whence we must infer that the cause of the constant difference lies here. We therefore subsequently investigated this matter with ratios of brightness of $0^{m}.8$ and $3^{m}.1$ between the first and second nebular lines, which were artificially produced. The numerous measures do not reveal a dependence on the absolute intensities, but do indicate a marked effect on the settings on line 2 due to the visibility of line 1. This effect was:

For $1/2 = 0^{m}.8$		For $1/2 = 3^{m}.1$	
W.	S.	W.	S.
$-0^{m}.125$	$-0^{m}.08$	$-0^{m}.342$	$-0^{m}.165$

There is, therefore, a marked increase of the influence with rise of the intensity-ratio. If we interpolate from the above values for the correct ratio for $1/2$, we obtain for the correction, by which the ratio found for $1/2$ is to be diminished,

for W. $-0^{m}.17$, for S. $-0^{m}.03$.

The personal difference W.-S. comes out $+0^{m}.14$, in good agreement with that deduced from the nebular observations. The values corrected in this way for the ratio $1/2$ are given in the last column of Table A, headed "Corr. Mean."

We now believe the mean values given in Tables A and B for the separate nebulæ are free from personal or physiological errors, when compared only among themselves for the elimination of a_2 and a_3. For the ratio $1/2$ it is sufficient to remark that it is to be regarded, from our observations, as constant for the different nebulæ.

If we call the ratio of brightness between the third and first lines in the *Orion* nebula (trapezium) 1, we get for the other nebulæ the following values, which may be compared with similar observations in future, for the purpose of establishing the fact of any changes, or to dispose of them as improbable.

Orion nebula (trapezium)	1.00	*N.G.G.* 4234	0.47
G.C. 4373	0.75	*G.C.* 4964	0.40
G.C. 4514	0.61	*N.G.C.* 7027	0.30
G.C. 4390	0.55	*N.G.C.* 6790	0.29
N.G.C. 6891	0.48		

With these numbers our problem may be regarded as solved, but it is naturally interesting to examine more closely the ques-

tion of the actual ratios of brightness of the three lines among themselves, in order also to obtain the values of a_2 and a_3. The rigorous solution of this problem is possible only with the help of the bolometer, by which the true energy-ratios at the proper places in the spectrum of the photometer lamp could be found. The determination of the "physiological" values of a_2 and a_3 is necessary for obtaining the physiological intensity-ratio of the lines, as it has entered into previous estimates.

As a mean for the two observers, the physiological intensity-ratio of the first to the second line came out as $0^m.5$, and for the first to the third line as $1^m.5$. Hence to us the ratio of the first to the second line would be as 4 : 1 ($1^m.5$) for all nebulæ, while the ratios for the first to the third lines would appear to us as follows in the different nebulæ:

Nebula	⅓
N.G.C. 6790........	40 : 1 ($4^m.0$)
N.G.C. 7027........	36 : 1 (3.9)
G.C. 4964........	27 : 1 (3.6)
G.C. 4234........	23 : 1 (3.4)
N.G.C. 6891........	23 : 1 (3.4)
G.C. 4390........	21 : 1 (3.3)
G.C. 4514........	17 : 1 (3.1)
G.C. 4373........	15 : 1 (2.9)
Orion (trapezium)...	11 : 1 (2.6)

We had the intention of settling by measures on the *Orion* nebula the question whether the intensity-ratio of the nebular lines is the same or different at different parts of the nebula. The question attracted especial attention a few years ago, and we add the following remarks to recall the circumstances. Huggins could not reach a positive opinion, while Vogel expressly remarked that the ratio of brightness of the lines was the same in all parts of the nebula. Keeler also arrived at the same result, but later he adopted the view of Campbell, who had found marked differences, especially near the star *Bond* No. 734 (*Scheiner* No. 260).

One of us (Scheiner) reached conclusions, from observations made in a modified manner, confirming the view originally expressed by Vogel, and he sought to explain the divergent

result of the American astronomers as an effect of the Purkinje phenomenon, since the observers concerned agreed in estimating the third line relatively brighter in the faintest parts of the nebula.

We have hitherto been unable to carry out our intention, as the light-power of the large spectro-photometer is not sufficient to allow lines even to be seen outside the Huyghens region. But in order to make at least a beginning in this direction, we made measures at the edges of the Huyghens region. We went so far that the second and third lines could only just be seen, and the measures could be made only with averted vision. For better exposition we now give a summary of the differences of the ratios between the Huyghens region near the trapezium and near the edges, in the sense trapezium — edges.

	Wilsing		Scheiner		
	½	⅓	½	⅓	Weight
South edge........	$-0^m.01$	$+0^m.32$	$-0^m.05$	$+0^m.06$	8
North edge........	-0.10	$+0.14$	-0.11	$+0.12$	5
West edge.........	$+0.16$	$+0.04$	-0.05	-0.28	2

In the mean for the three edges the third line was measured brighter than for the trapezium — $0^m.22$ by Wilsing, $0^m.04$ by Scheiner. The table shows an evident agreement in the sign, which is essentially negative for ½, and positive for ⅓. Since we have been able to establish the probability that the ratio ½ is everywhere constant, we must conclude that this agreement in sign is wholly accidental, especially in view of the small numbers in ½.

The same conclusion must therefore be drawn as to the agreement in sign for ⅓, and hence differences in the intensity-ratios within the Huyghens region are not to be inferred from our measures.

In order to decide whether the intensity-ratio of the lines depends in our measures upon the absolute intensity, we made a few measures in the neighborhood of the trapezium with the

objective stopped down by one-half. These yield the following differences, in the sense full aperture—half aperture:

Wilsing		Scheiner	
½	⅓	½	⅓
$-0^{m}.09$	$-0^{m}.00$	$-0^{m}.11$	$-0^{m}.13$

They therefore agree with the other measures within the limits of accuracy.

As was mentioned in the beginning, we have by the reduction to the benzine lamp rendered possible the expression in absolute measure of the brightness of the lines for the separate nebulæ, which therefore become mutually comparable. We have here the choice of taking the values for the brightest line or for the sum of the three lines. The latter plan would more nearly approach the ratio of the total brightnesses of the nebulæ, and would in that respect be preferable. But against this we have the fact that the influence of that part of the nebular light which produces the continuous spectrum is eliminated in spectro-photometric measures, while it enters with its full amount into the ordinary photometric measures. We therefore decided to employ only the relative brightness of the first line, and thus give homogeneous results. Calling the brightness of the first line in the brightest nebula 1.00, we get the following magnitudes for the nine nebulæ:

Nebula	W.	S.	Mean	W.-S.
G.C. 4390....	$1^{m}.00$	$1^{m}.00$	$1^{m}.00$	$0^{m}.00$
N.G.C. 7027....	1.85	1.88	1.87	−0.03
G.C. 4234....	1.88	2.25	2.07	−0.37
G.C. 4373....	2.21	2.40	2.31	−0.19
G.C. 4964....	2.60	2.76	2.68	−0.16
N.G.C. 6790....	2.86	2.59	2.73	+0.27
Orion(trapezium)	3.39	3.13	3.26	+0.26
G.C. 4514....	3.38	3.55	3.47	−0.17
N.G.C. 6891....	3.79	4.01	3.90	−0.22

The probable error of the mean for an evening is $\pm 0^{m}.21$ for Wilsing, and $\pm 0^{m}.23$ for Scheiner, which seems very satisfactory in view of the unfavorable circumstances. The agreement between the two observers is also surprisingly good and can be explained only by supposing that the effect of the different conditions of the seeing was almost always in the same sense for the two observers.

POTSDAM, October 1902.

NOTE ON THE WAVE-LENGTH OF THE MAGNESIUM LINE AT λ 4481.

By Henry Crew.

The prominence of this line in the spectra of many stars, notably those of the first class, and possibly those of the solar type, as well as the suggestion of Scheiner[1] that it might be employed as a criterion of stellar temperatures, makes its wave-length a matter of some interest to astrophysicists. I have, therefore, at the request of Professor E. B. Frost, measured the wave-length of this line as it appears in the metallic arc with a rotating electrode.

It will be remembered that the line does not appear in the ordinary arc or flame spectrum of magnesium. And, indeed, it appears in the rotating arc only when a comparatively large current is employed. If a current much less than two amperes for each contact between the fixed and rotating electrodes is employed, no trace of λ 4481 appears, and the spectrum is precisely that obtained with the carbon arc. Thus, if a cross of four equal arms is employed as a rotating electrode, a current of eight amperes is necessary to produce the line in question. If only one contact is used, two amperes will give the same spectrum, but will require four times the exposure. As is well known, the line is diffuse under all circumstances, and, so far as I am aware, is never reversed.

For purposes of measurement both the first and second orders of a ten-foot concave grating were employed. For purposes of comparison about a dozen of Rowland's standard iron lines and an equal number of Hasselberg's cobalt lines were selected. The values obtained are given in the following table. Each value is the result of three settings. It will be observed at once that the discrepancies are enormous when compared with those which would occur, under the same conditions, in the measurement of a moderately sharp line:

[1] Scheiner, *Sitzungsberichte der k. preuss. Akad. Wiss.*, March 1894; translated in *Astronomy and Astro-Physics*, 13, 569–71, 1894.

Number of Negative	Wave-length in Arc with Rotating Electrode	Wave-length in Spark of "Hedge-hog" Transformer	Remarks
211	4481.326		Negative made seven years ago.
478	.272		
484	.325		
484	.337		Measured by Mr. F. J. Truby.
498 (*a*)	.277		Arc working in an atmosphere of coal gas.
498 (*a*)	.293		Plate measured by Mr. A. A. Knowlton.
498 (*a*)	.260		Plate measured by Mr. A. A. Knowlton.
498 (*b*)	.318		Arc in air, on same plate.
503	.384		Cobalt used for comparison.
505	.390		Cobalt used for comparison.
506	.380		*Ca* and *Ba* in carbon arc used for comparison.
487		4481.364	
492 (*a*)		.302	
492 (*b*)		.256	
Mean	4481.324	4481.306	

It should perhaps be added that the comparison spectrum was put on each of these negatives by making half the exposure before, and half after, the magnesium exposure, the method now usually employed to detect any displacement of the instrument during exposure.

Scheiner[1] has measured this wave-length in eighteen stars of the first class and in eight stars of the second class, from which he derives the definitive value, 4481.52 (Potsdam scale) = 4481.43 (Rowland scale).

Adams[2] has measured fifteen plates taken from three stars in which this line is "sharp, narrow, and of great brilliancy." His result, after elimination of radial velocities, is 4481.400. As to the identity of this line, one is tempted to think that possibly the stellar line is the titanium line at λ 4481.438. But this possibility is practically disposed of by the following extract from a letter of Professor Frost:

It (the stellar line) cannot be the *Ti* line at λ 4481.438 which I constantly use as a comparison line, for that *Ti* line is not at all unusual in its behavior

[1] *Publicationen des Astrophysikalischen Obs. Potsdam*, VII, Th. II, pp. 315, 316.

[2] Astrophysical Journal, 15, 214–17, 1902.

in the arc and spark; and besides, the star line is very strong in stars otherwise showing no *Ti* lines.

Even if we assign the discrepancy in wave-lengths to errors in measurement of the arc line, as is probably the fact of the case, an interesting problem still remains, namely, *to discover the laboratory conditions under which Mg.* 4481 *becomes a sharp line, as in stellar spectra.*

NORTHWESTERN UNIVERSITY,
Evanston, Ill., November 10, 1892.

Minor Contributions and Notes.

PHOTOGRAPHS AND MEASURES OF THE NEBULA SURROUNDING *NOVA PERSEI.*[1]

The following negatives of *Nova Persei* have been obtained with the Crossley reflector since those described in *Lick Observatory Bulletin* No. 14.

No.	Date 1902	P. S. T. of Exposure	Duration of Exposure
8	January 31	$6^h\ 46^m$ to $11^h\ 46^m$	$9^h\ 45^m$
	February 2	7 5 " 11 50	
9	March 4	7 21 " 9 50	4 15
	" 6	8 30 " 10 16	
10	" 28	7 43 " 9 15	4 20
	" 29	7 42 " 8 53	
	" 30	7 42 " 9 19	
11	July 12	14 27 " 15 30	5 37
	" 13	14 14 " 15 30	
	" 14	13 54 " 15 31	
	" 15	13 49 " 15 30	

Negatives Nos. 9 and 10 were obtained under very poor conditions. The sky was hazy part of the time, and in some cases the exposure was entirely stopped by clouds. The effective exposure is, therefore, much less than is indicated by the times.

THE NEGATIVES.

No. 8.—Although the seeing on January 31 was poor and the star images not sharp in consequence, this plate shows the nebulosity very satisfactorily. Condensation *D* is considerably brighter than any other portion of the nebula, and I think it is brighter than it was in November and December. There appears to be little, if any, motion radially outward in its central portion where it is brightest, but the northwestern and southeastern wings show an apparent growth. Condensation *A* has undergone considerable change of form, and has continued its outward movement. The outer wisps of nebulosity, *E* and *F*, have undergone the greatest changes. The movements and increase in brightness of both masses, previously pointed out, continue. At this time both condensations are well marked. Excepting condensations

[1] *Lick Observatory, University of California, Bulletin* No. 23.

D and *A*, *E* is now the most conspicuous, being almost as bright as *A*. The structure of mass *F* has become more complicated. It consists of several short wisps approximately in the form of arcs of circles. The *Nova* has faded perceptibly in the interval of three weeks since the last photograph.

No. 9.—The *Nova* has decreased in brightness, and is only slightly brighter than the ninth magnitude star north preceding. There is but little diffused light about the image of the *Nova*, and some small faint masses of nebulosity can be seen to the south of it. Condensation *D* is almost as distinct as on the first plate in November, although the latter had a much greater effective exposure. Condensation *A* is faint but visible. It has become much more pointed, and has increased in size.

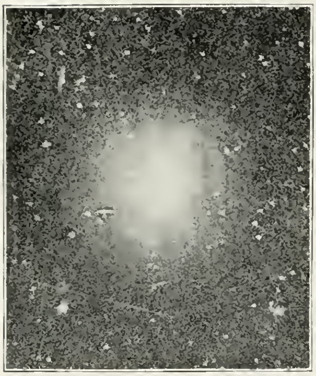
No. 10.—The decrease of light in the *Nova* continues, and the image is almost exactly the same size as that of the ninth magnitude star near by. The small masses of nebulosity to the south noted on the previous negative, closer than *D*, are also visible on this plate. Condensation *D* is fully as bright as on the previous plate. Condensation *A* has grown sharper, and but little fainter.

No. 11.—Although obtained under fair conditions of sky and seeing, this exposure of five and one-half hours does not show so much nebulosity as was to be expected. Little, if any, greater density is shown than on negatives Nos. 9 and 10. Condensation *D* is more dense than any other of the masses Just outside of the southwest wing of this condensation is a mass (*Y*) which, in the interval of 107 days since the last photograph, has moved out so as to be seen well separated from *D*. This new mass is nearly as bright as *D*. It gave some indications of its presence in the photographs of March. It is curved about the *Nova*, but not quite circularly, and extends over an arc of some 60°.

Condensation *A* still continues its rapid motion outward, and shows additional changes of structure. It is now much fainter than formerly. The *Nova* has decreased in brightness and is probably a half magnitude fainter than the 9.0 magnitude star near it.

With the fading of the *Nova* and the diffuse nebulosity about it, some appearances have become noticeable which were not so evident earlier. The outlines of condensation *D* are much more clearly marked, especially on the side nearest the *Nova*. Its appearance is strikingly suggestive of jets such as are to be seen in all very bright comets. Very faint extensions can be traced from both wings of the principal mass, one extending to the north and curving about the

PLATE XI.



March 9, 1901. Exposure 10 m.

The Nebulosity Around *Nova Persei.*

From negatives made with the Crossley Reflector of the Lick Observatory November 7 and 8, 1901. Exposure 7 h. 19 m.

Nova; the other extending toward the southeast in the direction of condensation *A*. In fact, a faint band of nebulosity can be traced from *D* entirely to *A*.

On this negative nothing can be seen of any of the outer wisps of nebulosity. Some of these may possibly be disclosed by copying the negative on slow plates — which has not yet been done.

On the negatives secured early in January there is a slender ray extending $1\frac{1}{2}'$ to the northeast from the *Nova*. On those plates it does not differ markedly from one of the diffraction rays from the star (due to the supports of the secondary mirror) except that it is broadened about midway, giving it a slightly arrow-headed appearance. As the star faded, however, and the diffraction rays grew less prominent, this ray became more conspicuous. The plates Nos. 9 and 10 show almost no diffraction rays and yet this appendage is there, apparently unchanged in brightness or position. The inner portion on these later negatives is very faint, or wanting entirely, leaving the outer portion almost an isolated mass. As there is no such appendage to the ninth magnitude star near by, this object must be real. The photograph of July 12–15 shows it in the same place, as an isolated mass, and with some structure.

Mr. H. K. Palmer and Mr. R. H. Curtiss, Fellows in Astronomy at the Lick Observatory, rendered efficient assistance in taking the photographs.

THE MEASURES.

Negatives made from the originals by successive copyings were used in making the measures in all cases except the last. The original negative of No. 11 was measured. The nebulosity being too faint to permit of magnification, the points to be measured were indicated on the back of the plate by some definite marks before placing it under the microscope. These marks could be measured very accurately, the settings seldom showing a range of $1''$; and the uncertainty lies in the determination of the proper point for the mark. The changes of form and appearance in the nebulosity render this very difficult, and the resulting positions are therefore subject to a large probable error. In some of the fainter and more poorly defined masses this uncertainty may amount to $10''$ or more. The later negatives, with their much shorter exposure, fail to extend the nebulosity as far as the earlier negatives, and thus, in such cases as the extremities of masses, the points selected are hardly comparable.

Table I contains the measurements of the negative of 1901 March 29. As the appearance of the nebula was so very different at this time from its appearance on the later photographs, no connection with the later series could be traced between the various condensations. A sufficient number of points was chosen arbitrarily in the two rings of nebulosity to represent fairly their forms and dimensions.

Table II contains the position angles of the various condensations, from November 7, 1901, to July 12, 1902.

Table III contains the distances, in seconds of arc, from the same plates.

Table IV contains the results of the measures of the axes of the outer area of the nebulosity on the plates which showed it sufficiently well. On the copied negatives used for measuring, the outlines of this area could be recognized fairly well by looking at the plates from a distance, and by the device of inclining the plate at a considerable angle with the normal to the line of sight. On only one plate could the outlines be traced to the north.

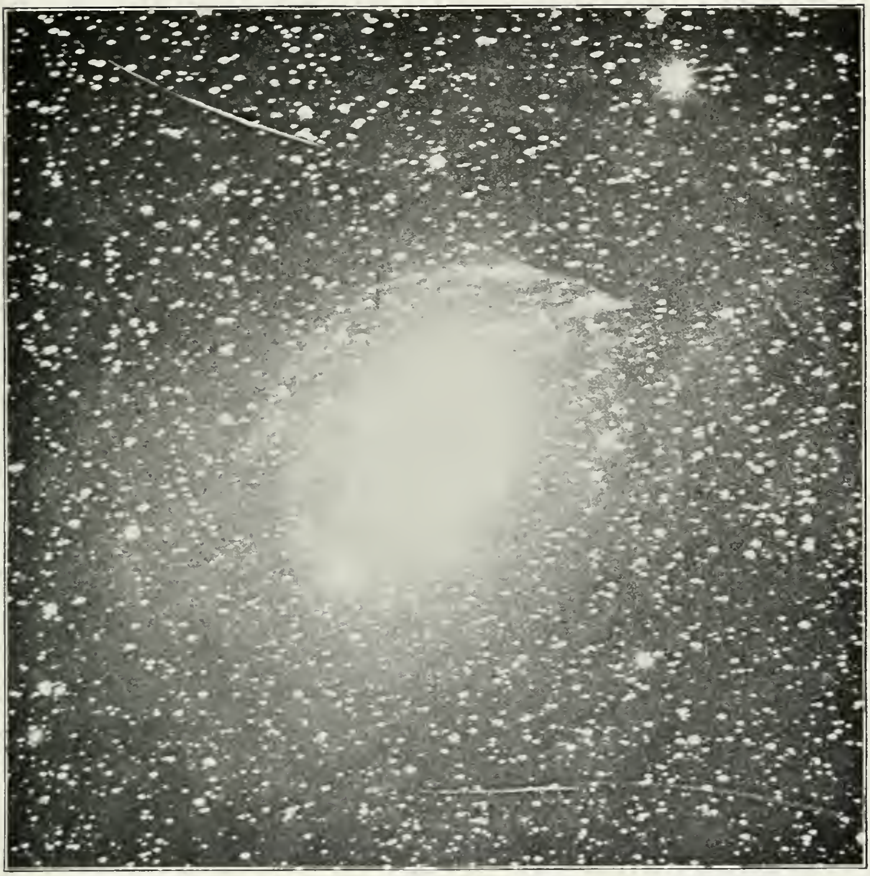
The extreme faintness of this nebulosity and the consequent uncertainty of the measures preclude the drawing of any certain conclusions from so short an interval as to whether the whole of this nebulosity is also expanding. However, the means of the results from these six plates can be considered to give a fairly correct idea of its general form and dimensions. It will be noticed that the area is an ellipse with the *Nova* near the southeast focus. The positions of the axes agree very closely with those of the outer ring of nebulosity on the plate of 1901 March 29.

The measures are referred to the *Nova*.

TABLE I.

Date	Inner ring		Outer ring		Arc to N. E.		Mass to S.	
	p	*s*	*p*	*s*	*p*	*s*	*p*	*s*
1901 March 29	25°.0	83″	1°.0	145″	6°.5	310″	171°.5	160″
	61.8	89	45.3	140	59.6	321	....	
	93.6	81	89.6	140	....		....	
	134.0	64	128.8	149	....		....	
	178.4	67	179.5	102	....		....	
	215.7	45	211.1	101	....		....	
	258.0	69	260.6	133	....		....	
	298.9	69	285.1	147	....		....	
	338.8	76	320.2	144	....		....	

PLATE XII.



The Nebulosity Around *Nova Persei.*

From a negative made with the Crossley Reflector of the Lick Observatory on November 12 and 13, 1901. Exposure 10 h.

TABLE II.

Condensation	1901				1902					
	Nov. 7-8	Nov. 12–13	Dec. 4	Dec. 8–11	Jan. 2–3	Jan. 10–11	Jan. 31 Feb. 2	Mar. 4–6	Mar. 28–30	July 12–15
A_1	124°1	124°2	124°0	124°4	123°1	122°9	123°0	121°8	120°7	.....
$_2$	131.0	130.2	129.5	128.8	127.2	127.1	126.5	125.8	124.5	122°8
$_3$	133.6	133.6	133.1	132.2	130.4	130.7	131.0	129.2	.....	.....
B_1	155.6	155.6	153.5	152.6	149.8	149.5	147.1	.....	141.1	.....
$_2$	162.7	163.8	162.9	160.8	156.8	155.8	153.4	.....	148.9	.....
C	192.6	192.6	188.3	187.1	(203.7)	.....	.....	.....	.....	.....
D_1	156.6	151.2	157.4	159.3	163.6	161.2	164.9	166.2	172.0	154.9
$_2$	214.6	211.3	210.8	212.4	210.5	215.8	216.0	211.4	214.8	183.7
$_3$	263.3	262.0	260.2	268.1	265.5	272.5	275.3	265.4	268.0	270.3
E_1	.....	.....	211.9	210.8	223.8	223.8	224.8	233.2	.....	.....
$_2$	.....	.....	.....	.....	.....	223.3	.....	.....	.....	.....
F_1	.....	344.4	.....	350.3	343.8	345.0	341.7	.....	.....	.....
$_2$	.....	351.0	.....	356.6	347.6	346.1	342.4	.....	.....	.....
$_3$	.....	.....	.....	359.5	351.2	351.2	345.9	.....	.....	.....
$_4$	.....	.....	.....	5.3	353.6	352.8	349.7	.....	.....	.....
$_5$	.....	.....	.....	.....	359.3	359.2	353.3	.....	.....	.....
$_6$	.....	.....	.....	.....	.....	.....	355.6	.....	.....	.....
G	272.5	270.7	271.4	271.6	275.5	278.4	276.0	.....	.....	.....
H	279.3	281.1	285.7	277.3	284.6	285.6	284.6	.....	.....	.....
I	358.0	1.0	358.4	355.7	353.8	352.6	353.0	347.5	.....	.....
J	120.4	119.4	117.4	119.5	119.2	120.3	120.2	119.0	.....	.....
K	137.4	149.1	.....	.....	.....	134.8	.....	.....	.....	.....
L	172.8	182.4	174.6	168.3	189.4	180.9	180.6	.....	.....	.....
M	244.5	242.1	244.2	244.0	251.1	259.2	248.6	.....	.....	.....
N	305.3	305.0	305.2	304.5	303.5	307.8	306.6	.....	.....	.....
O	.....	.....	.....	290.4	297.3	293.4	295.0	.....	.....	.....
P	.....	.....	.....	.....	.....	245.3	.....	.....	.....	.....
Q	.....	.....	.....	.....	32.4	32.7	29.8	31.8	30.7	32.4
R	.....	.....	.....	.....	.....	62.2	66.8	.....	.....	.....
S	.....	.....	.....	.....	.....	343.0	341.5	.....	.....	.....
T_1	.....	.....	142.1	138.0	133.3	133.4	132.6	130.2	128.6	.....
$_2$	.....	.....	.....	.....	140.0	142.7	139.9	133.6	.....	.....
U	.....	.....	.....	.....	.....	.....	329.7	.....	.....	.....
V	.....	.....	.....	.....	.....	261.0	.....	.....	.....	.....
W	.....	.....	.....	.....	.....	.....	337.8	.....	.....	.....
X	.....	.....	.....	.....	.....	.....	12.0	.....	.....	.....
Y	.....	.....	.....	.....	.....	.....	.....	.....	.....	141.2

TABLE III.

Condensation	1901				1902					
	Nov. 7–8	Nov. 12–13	Dec. 4	Dec. 8–11	Jan. 2–3	Jan. 10–11	Jan. 31 Feb. 2	Mar, 4–6	Mar. 28-30	July 12–15
A_1	531″	532″	559″	579″	587″	634″	644″	650″	699″	....
2	480	485	508	519	535	553	575	583	625	722″
3	412	413	427	437	470	485	483	500	....	....
B_1	415	432	425	439	453	458	497	....	556	....
2	397	395	410	427	442	449	475	....	518	....
C	371	375	414	414	(439)	....	....	....	....	....
D_1	130	133	129	149	150	150	153	143	137	169
2	100	90	92	102	105	97	103	101	99	104
3	175	168	180	205	201	210	213	209	212	241
E_1	....	....	818	808	839	845	866	890	....	....
2	....	....	....	....	....	792	....	....	....	....
F_1	....	873	....	921	895	920	981	....	....	....
2	....	813	....	937	947	961	938	....	....	....
3	....	....	....	894	912	928	947	....	....	....
4	....	....	....	959	973	981	973	....	....	....
5	....	....	....	....	942	957	962	....	....	....
6	....	....	....	....	....	....	815	....	....	....
G	441	424	437	470	499	513	506	....	....	....
H	293	294	363	327	379	395	401	....	....	....
I	388	387	395	410	428	431	444	466	....	....
J	432	439	441	462	487	493	516	536	....	....
K	333	294	....	....	....	410	....	....	....	....
L	248	233	248	279	274	282	303	....	....	....
M	390	383	388	415	421	457	450	....	....	....
N	374	422	425	450	450	479	495	....	....	....
O	....	....	....	225	251	245	258	....	....	....
P	....	....	....	....	....	368	....	....	....	....
Q	....	....	....	....	90	142	88	102	99	112
R	....	....	...	....	....	439	433	....	....	....
S	....	....	....	....	....	571	589	....	....	....
T_1	....	....	420	447	503	516	536	572	588	....
2	....	....	....	....	436	449	471	528	....	....
U	....	....	....	....	....	....	1034	....	....	....
V	....	....	....	....	....	833	....	....	....	....
W	....	....	....	....	....	....	442	....	....	....
X	....	....	....	....	....	....	1057	....	....	....
Y	....	....	....	....	....	....	....	....	....	190

PLATE XIII.

THE NEBULOSITY AROUND *NOVA PERSEI.*

From a negative made with the Crossley Reflector of the Lick Observatory on December 8 and 11, 1901. Exposure 10 h.

TABLE IV.

Date	Major axis				Minor axis			
1901	p	s	p	s	p	s	p	s
November 7–8	99°	1007″	279°	1239″	197°	821″	...	...
12–13	106	981	286	1162	198	860	13°	870″
December 8–11	122	1079	297	1015	209	808	...	...
1902								
January 2–3	118	1108	300	1022	210	849	...	...
10–11	124	1044	307	1185	214	914	...	...
Jan. 31–Feb. 2	121	1064	299	1267	206	894	...	...
Mean	115	1047	295	1148	207	858	...	...

Condensation A is the only one of those showing large motion which has retained its form throughout the series of observations sufficiently to enable a good determination of its motion to be made. The positions of the brightest and best defined portion of this mass (A_2) were plotted on a large scale. A straight line, in position angle 107°.5, seems to represent the direction of motion best. During the interval of 255 days, between November 7, 1901, and July 13, 1902, the displacement amounts to 258″, almost exactly 1″ per day. Projecting this point backward, we find that on February 17, 1901, its position would be, position angle 154°.0, and distance 252″. This does not indicate coincidence at the time of the outburst either with condensation D, or with the mass of nebulosity to the south of the rings on the negative of March 29, 1901.

FAINT STARS NEAR THE *NOVA*.

The long-exposure negatives show a number of faint stars near the *Nova*. Although the images are not perfect enough for any great accuracy of measurement, I have thought that it might be useful to give the coördinates of all within 1′ of the *Nova*, especially as most of them are not included in the charts and measures published by Aitken[1] and Barnard.[2] The results given in the accompanying table were obtained from measures of the plate of July 12–15, 1902. As the refraction corrections are less than the uncertainties of the measures they have not been applied. The magnitudes of the stars l, m, n, and p, were observed visually, and have been adopted in the table. The magnitudes of the others are based upon these.

[1] *Lick Observatory Bulletin*, No. 8.

[2] *Astronomische Nachrichten*, 159, 50, 1902.

The position of star *l* as obtained from the photograph does not agree with that given by Aitken. It has also been measured on the plate of March 4–6. The agreement between the positions from the two plates is as close as can be expected from the character of the images, and gives no certain indication of a change of position.

FAINT STARS NEAR *NOVA PERSEI.*

Star	p	s	Magnitude
a	31°5	57″2	15
b	32.2	48.0	18
c	32.2	33.9	16
d	67.6	61.5	18
e	68.4	19.5	15
f	78.9	63.4	18
g	148.8	51.3	17
h	224.5	42.0	16
i	265.6	45.4	18
j	290.7	19.6	18
k	300.9	14.2	18
l	338.5	45.6	16.5
(*l*)[1]	(337.9)	(47.1)	....
m	57.0	132.5	13.9
n	107.4	91.1	13.5
o	236.2	103.5	15
p	303.6	163.8	13.1

THE ILLUSTRATIONS.[2]

The plates accompanying this *Bulletin* are direct photographic reproductions from negatives, and are enlarged 2¼ diameters. The scale of the plates is 17″ to the millimeter. The negatives were obtained by successive copyings of the originals on slow plates.

The top of each plate is south.

There are several scratches in the glass of the original negatives which appear on the printed plates. One small one is to be seen on the plate of March 29, and two large ones on that of November 12–13.

The faint outer nebulosity is well shown, considering its elusive character, on several of the plates. The later plates show the appearance and growth of the wisps *E* and *F* near the outer edges of this region, to the southwest and north, respectively, of the Nova. The form of the bright inner mass *D* is best shown on the plate of January 10–11.

[1] Plate of March 4–6, 1902, for comparison.

[2] The original electrotypes have been kindly loaned by Professor Campbell for reproduction in this JOURNAL.

PLATE XIV.

THE NEBULOSITY AROUND *NOVA PERSEI.*

From a negative made with the Crossley Reflector of the Lick Observatory on January 10 and 11, 1902. Exposure 10 h. 30 m.

OBSERVATIONS OF THE NEBULOSITY ABOUT *NOVA PERSEI* FOR POLARIZATION EFFECTS.

The interpretation of the changes observed in the nebulosity surrounding *Nova Persei* would be much facilitated by spectroscopic or other physical observations. On account of the faintness of the nebulosity, spectroscopic investigations held out little hope of definite results, but the possibility of detecting polarization effects suggested itself. The success of the Crocker Expedition in detecting polarization in the light of the corona at the Sumatra eclipse strengthened the probability that such a method would be efficient in the case of the nebula. The conditions seemed even more favorable in the case of the nebula where the plane of polarization should be better defined than in the corona.

To this end a double-image prism was placed in the optical axis of the Crossley reflector, between the diagonal mirror and the plate. It was mounted in an auxiliary frame to permit its being set in all position angles. Following are the constants of the double-image prism and its position in the telescope:

Clear aperture	33 mm
Thickness of prism	23
Distance from the film of dry plate to the nearest surface of prism	66
Separation of images	6

The interposition of this prism was found to lengthen the focus of the telescope 10 mm.

The star images were considerably elongated by the prism in the direction of its principal plane. The field of the prism was sufficiently large to include the Nova and condensations *A* and *D*. The principal plane was made to pass through the *Nova* and *A*. As the plane passing through the *Nova* and *D* was at right angles to that passing through *A*, this position of the prism sufficed to test *D* also.

It is well known that there is but little effect produced upon polarized light by reflecting it from metallic surfaces. In the case of the parabolic mirror (silver on glass) there is practically no effect, as the reflection is so nearly normal. The diagonal plane mirror of the Newtonian form, however, has a small effect if the plane of polarization is not parallel (or perpendicular) to the plane of reflection of the diagonal. The mounting of the Crossley reflector is such that it was not possible to place these planes in the most favorable position without

reducing the exposure times at that season of the year so seriously as to endanger the securing of any results whatever. The eye-end of the telescope was used in a position, therefore, in which these planes made an angle of 45° with each other.

With this arrangement of telescope and prism a negative of the *Nova* was secured with the following exposures:

March 10 - - - - - - - - -	$3^h\ 8^m$
11 - - - - - - - - -	2 57
12 - - - - - - - - -	1 22
Total exposure - - - - -	$7^h\ 27^m$

The conditions were not good at any time, and on the last night clouds interfered seriously. The star images on the resulting negative are poor, owing to the causes mentioned. The negative shows the two condensations *A* and *D*. The other condensations were too faint to make any impression. Both images of *D* are very distinct, and there seems to be no appreciable difference of intensity. The images of *A* are both extremely faint. One of them is in the overlapping fields given by the prism where the film is considerably darkened by diffused light from the sky. The other image is in a region where only one beam from the prism fell upon the plate, and where the darkening of the film is much less. It is therefore much more difficult to judge of the relative intensities of these images. Allowing for these variations as well as possible, these two images appear to be of the same intensity.

After this photograph had been taken a method occurred to me of testing the effect of the mirrors — particularly the diagonal — upon polarized light. This was to pass polarized light through the telescope, and observe it with the double-image prism.

The plane unsilvered mirror of our heliostat was placed in front of the telescope in such a position that the light of a star was reflected from it into the telescope at the angle of maximum polarization (approximately 56° for flint glass). The light to be observed was, therefore, almost totally polarized. All other light was carefully excluded.

The heliostat mirror was only 7½ inches in diameter, but sufficient light was obtained by observing a first or second magnitude star. The plane of reflection of the diagonal was placed successively parallel to, at an angle of approximately 45° with, and perpendicular to the plane of polarization, and the effect was observed visually with the double-

PLATE XV.

THE NEBULOSITY AROUND *NOVA PERSEI.*

From a negative made with the Crossley Reflector of the Lick Observatory on January 31 and February 2, 1902. Exposure 9 h. 45 m.

image prism. In the two positions in which the plane of reflection of the diagonal was parallel and perpendicular to the plane of polarization, the extraordinary image could be almost wholly extinguished. When these two planes were at an angle of 45° (the position of the prism during the taking of the photograph) there was a depolarizing effect, the extraordinary image being possibly one-tenth the brightness of the ordinary image. α *Lyrae* was the source of light employed. These observations of the effect of the telescope upon polarized light enabled a more certain interpretation to be made of the evidence afforded by the photograph of the *Nova*.

The foregoing observations point to little or no polarization effect in the light from condensation *D*, and, with perhaps less certainty, in condensation *A*.

It does not follow that the nebula is not shining by reflected light; but in view of the result from the corona already referred to, we should be led to expect some polarization effects if the light were all or nearly all reflected.

Owing to the very unusual amount of stormy weather, the experiments on the effect on the mirrors upon polarized light were greatly delayed.

It is hoped to repeat these polarization observations, under more favorable conditions, as soon as the *Nova* is again in good position for observing.

An effort will also be made to secure some spectroscopic evidence with a spectrograph which has been designed for the problem.

THE CHARACTER OF THE PHENOMENA OBSERVED IN THE NEBULOSITY SURROUNDING *NOVA PERSEI*.

Several explanations have been advanced to account for the apparent motions observed in the nebula about *Nova Persei*. We shall point out some of the considerations which bear most strongly on the question of the character of these phenomena. The principal results of the observations are as follows:

1. There appear to be two pretty well defined areas of nebulosity: a bright inner ring or disk about 15′ in diameter; and a very faint outer ring about 30′ in diameter (January 1902).

2. The inner ring is expanding. Two series of wisps near the outer edge of the outer ring indicate expansion in that ring also. A comparison of photographs made in January, 1902, with a photograph of the *Nova* taken on March 29, 1901, which shows two faint rings of nebulosity, indicates the following daily rates of radial expan-

sion for the circumferences of the two areas, assuming the former to result from the latter:

Inner ring (average) - - - - - - - -	1!4
Outer ring (from two rings) - - - - -	2.8

The above rates of expansion would carry the inner ring back to the *Nova* on February 8, 1901 (deduced from the plates of March 29, 1901, and January 10–11, 1902). Both plates give the same date. The outer ring would in a similar way be carried back to the *Nova* on February 16 and 17, 1901 (deduced from the plates of March 29, 1901, and January 2–3, 1902). These results do not necessarily imply the earlier formation of the inner ring, but, considering the uncertainties of measurement, they point rather to a contemporary origin.

3. Both rings show considerable structure. Many of the separate condensations have individual motions, which are usually not radial, but contain large tangential components. Clockwise and counterclockwise motions are found in both rings.

4. The observations made with a double-image prism indicate little or no polarization effects in the two brightest condensations.

5. The inner ring and its condensations have shown a consistent decrease of light. In the outer ring, on the other hand, masses of nebulosity have become visible and have shown rapid increase in brightness, as well as changes of form.

The first explanation that naturally suggests itself is that the observed motions are due to real translations of matter. As far as the velocities alone are concerned, there would appear to be no positive objections to this view, so long as the velocities do not exceed that of light. The exhibitions of force with which we are familiar lead us to expect high velocities in the case of so stupendous an outburst as that of the *Nova*.

There are other objections to this explanation, however. The motions observed are not radial. Nearly all of them have large tangential components. It is difficult to account for these tangential components. A consideration of the conditions probably existing in the nebula, upon the assumption of an actual translation of matter, would lead us to expect a very rapid loss of light.

The inner ring has decreased in brightness, and some of its features have become too faint to record themselves on the photographs. Several masses, all in the outer ring, have been recorded only on the later photographs, and have grown both in brightness and size, a con-

dition difficult to explain on the above hypothesis. It is perhaps not inconceivable that the two rings represent different phenomena.

The idea that the observed phenomena in this nebulosity might be due to light waves seems to have suggested itself about the same time to many persons. It appears to have been first published by Kapteyn.[1]

While several hypotheses are possible, only one seems worth considering, viz., that finely divided solid or gaseous stationary matter, having the observed structure, is illuminated by light waves emanating from the *Nova*. The appearance and growth of the wisps in the outer ring of nebulosity are facts which seem to be well explained by this theory.

An accurate knowledge of the star's parallax would be a strong test of this theory. The assumed velocities of expansion for the outer ring would limit it to $0''.02$. The determinations made so far have generally been negative, indicating that it is small, and therefore not inconsistent with the above theory.

The forms of the two rings of nebulosity and their symmetrical arrangement with respect to the *Nova*, lead us to believe that the displacements of their outer portions are the maximum, and relatively comparable. If this is the case, and the two rings have been formed by the expansion of earlier ones, all of which appear to have emanated from the star about the time of its greatest brilliancy in February 1901, we have two widely different velocities indicated. Such a condition is inconsistent with our present knowledge of light.

As the increase of light in the *Nova* was much more rapid than its decrease, the outer surface of the light wave should be more sharply defined than the inner. There seems to be no evidence of any such difference. It is not quite clear how some of the best marked condensations could retain such distinctive forms and still be displaced, if the streams of matter giving rise to them are normal, or nearly so, to our line of sight, as they appear to be. It would seem also as if some of the condensations were bright enough to leave some trace after the principal wave of light had passed.

The bearing of the polarization observations on the theory of light waves has already been pointed out.

Another explanation is that the light of this nebulosity is inherent and due to incandescence resulting from some form of electrical excitation or other invisible radiations from the *Nova*. As yet there seems to be little direct evidence either for or against such a theory.

[1] *Astronomische Nachrichten*, 157, 201, 1901.

Our conception of the conditions most probably obtaining in such an outburst as that of *Nova Persei* would lead us to expect that the pressure effect shown to exist in heat and light, would be an active factor in producing the observed appearance. There is no evidence, however, of any acceleration in the velocity of condensention *A*, which is the only one sufficiently well observed. The measures indicate a slight retardation in the velocity of this condensation, but the large probable error makes it doubtful if there has been any real change.

C. D. PERRINE.

July 24, 1902.

ON A NEW OBJECTIVE METHOD FOR THE MEASUREMENT OF SPECTROGRAMS.[1]

The method is that used for several years past by Exner and Haschek in their work on the ultra-violet spark spectra of the elements; it was briefly described in *Sitzungsberichte d. k. Akademie zu Wien*, 1895, p. 913.

The spectrum is directly projected on a screen of fine drawing paper by a lantern provided with all the various adjustments necessary. The enlargement of the projected image on the screen may be controlled within narrow limits. The negative is held in a frame, which may be moved longitudinally by hand, vertically by a screw, and may be rotated so as to secure verticality of the projected lines.

The screen is made of fine drawing paper, tightly stretched, with due precautions to avoid warping of its support. Two rods of iron at the top rest upon rollers and permit a smooth movement of the screen as a whole in the direction of its length. Two lower brackets support a slotted bar, in which the lower side of the screen moves while keeping its plane unaltered. Three millimeter scales, of which only the middle one is used, are fastened to the screen at a distance of 30 cm from center to center. A distance of one meter at the center of the screen is all that is used in practice, as at the edges the definition is not so sharp. Reels carry a strip of paper, on which are accurately laid off one thousand standard lines, with their wave-lengths (Rowland). One hundred tenth-meters cover one meter on the screen. This moves so easily that it can be shifted within a tenth of a millimeter without trouble, corresponding to 0.01 tenth-meter. The position of the lines projected on the screen can be read off by estimation to 0.1^{mm}, corre-

[1] Abstract, by Heber D. Curtis, of a paper by Karl Kostersitz.

sponding to 0.01 tenth-meter, as mentioned above. In measurement standard lines are chosen, as close as possible to the lines under consideration, so as to make the measures depend upon a small section of the scale. The screen is then shifted so as to give the correct reading for this line. The proper degree of enlargement has been previously secured. The position of the desired line may be at once read off in Ångström units.

With especially strong or broad lines he used a "thread-shadow micrometer" (*Faden-schaltens Mikrometer*). This is simply a frame, over whose central opening a thread is stretched. This is placed over the line, and its shadow made to coincide with the pointed ends of the lines. The position of the shadow on the central scale is then read off.

Kayser holds that the method is less accurate than a comparator. Kostersitz claims that this contention is borne out neither by the character of the method as such, nor by the results already secured with it. Readings may be made with entire accuracy to 0.01 tenth-meter. To test the accuracy of the method a spectrum was remeasured after an interval of several weeks had elapsed. The mean of the differences, without regard to sign, for 103 lines was 0.013 tenth-meter.

The deviations were grouped as follows:

0.00 tenth-meter in 28 cases
0.01 tenth-meter in 32 cases
0.02 tenth-meter in 30 cases
0.03 tenth-meter in 10 cases
0.04 tenth-meter in 2 cases
0.05 tenth-meter in 1 case

No deviations greater than 0.05 occurred. This was based on a *single* direct reading for each line, not the mean of several. An investigation of 1,531 cases in spectra of various elements gives for the average deviation 0.015 tenth-meter. By the use of plate glass instead of ordinary glass, Exner and Haschek have since reduced this to 0.0127 tenth-meter.

A comparison is also given between the results of Rowland, Kayser, and Exner and Haschek, in measurements of the spectrum of platinum. Δ_1 denotes the mean and Δ_2 the maximum deviation.

	Δ_1	Δ_2
K.—R.	0.010	0.046 tenth-meter
E.H.—R.	0.015	0.044
E.H.—K.	0.014	0.058

The measurements of E. and H. were single, direct readings on the

spark spectrum, those of Kayser are the mean of six readings, on the sharper lines of the arc.

There is a great advantage also in speed, no computations being necessary in the determination of the wave-lengths. In the space of one and a half years the spectra of seventy-five elements were investigated, involving the positions of about 50,000 lines, a task which would have been nearly impossible by ordinary methods.

Among the advantages claimed are :

Quickness and accuracy.

Ease with which spectra may be examined as a whole, impurities and anomalies detected, and elements identified.

Errors in the scale have less effect than those of the micrometer screw ; and may be more easily controlled and allowed for.

Mistakes in scale-reading are less liable to occur than in the readings of a divided micrometer-screw head.

Ease of the observer ; less strain on the eyes.

The elimination, to a considerable extent, of the psychological peculiarities of readings made with microscopes.

The author's rather hopeful claims for the utility of the method in detecting shifts of spectral lines have yet to be proven by trial. The lack of normality of the prism spectrum might be obviated by projecting on a screen, as in the Exner-Haschek method, and enlarging or diminishing the image so as to coincide with a previously computed dispersion. When the comparison lines fell into their proper places on the screen, the positions of the lines of the star spectrum could be rapidly read off and compared with their computed places.

The difficulty caused by the relatively broad and hazy lines, as compared with the beautifully sharp lines of grating spectra, would be the greatest hindrance; only an actual trial could determine the possibility of accurate results from the Exner-Haschek method with star spectra.

KARL KOSTERSITZ.

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The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

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THE

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ON THE OPTICAL CONDITIONS REQUIRED TO SECURE MAXIMUM ACCURACY OF MEASUREMENT IN THE USE OF THE TELESCOPE AND SPECTROSCOPE.

By F. L. O. Wadsworth.

Preliminary Note.—The main results of the present paper were obtained in 1897 and were informally presented before the Astronomical and Astrophysical Conference at the dedication of the Yerkes Observatory in October of that year.[1] The general investigation on the "Conditions of Maximum Efficiency in Astrophotographic Work," of which these formed a part,[2] was interrupted by my departure from Yerkes Observatory in the winter of 1897–8, and since then my attention has been so continuously devoted to technical and engineering work that until very recently I have had no opportunity to put any of the results obtained at that time in form for publication. This will explain, I trust, the delay in the resumption and continuation of work begun more than five years ago.

In the use of any optical instrument there are three quantities which are more or less closely related and which together determine what is ordinarily termed "optical definition." These

[1] *Yerkes Observatory Bulletin* No. 2; Astrophysical Journal, 6, 150, 1897. See also papers "General Theory of Telescopic Images," *ibid.*, pp. 123, 127; and "Effect of Atmospheric Aberration on the Intensity of Telescopic Images," *ibid.*, 7, 70.

[2] See Note on the Result Concerning Diffraction Phenomena, *M. N.*, 58, 287 (*b*).

quantities are "resolution," "accuracy," and "contrast," and the corresponding characteristics of the instrument with reference to them have been termed "resolving power," "metrological power," and "discerning or delineating power." They all depend directly on the form and distribution of intensity in the physical image of the object under examination as formed at the focal plane of the instrument, and differ from each other only in the way in which this form and distribution affect the particular use to which the instrument is to be applied. Thus in the discovery of double stars and general spectrum analysis, resolution is of most importance; in meridian circle and heliometric work and in determinations of absolute and relative wave-lengths, accuracy is the first consideration; while in the study of planetary detail and what may be termed pictorial and chart photography, contrast is the quality to be chiefly considered.

The question of the resolving power of instruments has been considered for a number of general and special cases[1] by different writers. The theory of metrological power and contrast[2] has received much less attention. In the present paper it is proposed to investigate more fully the general conditions of metrological power and accuracy in the optical measurement of the relative or absolute position of points or lines.[3]

[1] RAYLEIGH, "Wave Theory," *Enc. Brit.*, 24, §§ 11, 12. 13, and 14. "Investigations in optics with reference to the spectroscope," *Phil. Mag.*, 8, 9, 1879–80; with reference to the microscope, *ibid.*, 42, 167, 1896; MICHELSON, *Phil. Mag.*, 31, 388, 1891; 34, 280, 1892; ASTROPHYSICAL JOURNAL, 1, 1. 1895; WADSWORTH, *ibid.*, 52, 1895; 3, 176 and 321; 4, 54, 1896; 6, 27, 1897; *Mem. Spet. Ital.*, 26, 2, 1897; *Phil. Mag.*, 43, 317, 1897; ASTROPHYSICAL JOURNAL, 16, 1, 1902.

[2] The investigation of the general subject of delineating power and contrast has been begun (ASTROPHYSICAL JOURNAL, 6, 119, 1897; 7, 70 and 77, 1898) and the results applied in detail to some special cases. ("Astronomical Photography," *A. N.*, 144, 97; "Photography of Planetary Surfaces," *Observatory*, 20, 333, 365, 404; "Visibility of Linear Markings on Planets," *A. J.*, 18, 41.) The concluding part of the general paper (ASTROPHYSICAL JOURNAL, 7, 70) and some other papers on additional special cases were in course of preparation when the work was interrupted as explained in the preliminary note. They will be taken up again as soon as possible.

[3] The case of the measurement of position of sources of considerable angular magnitude, such as the Sun, Moon, and major planets, involve different conditions of measurement, and will for that reason be considered separately. [See *M. N.*, 58, 288 (h).]

Let us denote the resolution of an optical instrument by R and its accuracy of measurement by A. These quantities are the reciprocals respectively of α, the limiting resolving power, *i. e.*, the angular distance between two points or lines that can just be resolved; and of ϵ, the limiting metrological power, or the smallest angular distance that can be measured with certainty. These four quantities are connected by the general relations[1]

$$\left.\begin{aligned} A &= aR, \\ \epsilon &= \frac{1}{a}\alpha \end{aligned}\right\} \qquad (1)$$

where a is a factor whose value is not constant but varies with the conditions of measurement. As has already been pointed out, a fundamental condition for attaining a maximum value of a is that the *scale* of the diffraction image of the source whose position is to be measured shall be *large* compared with the width of the reference cross-wires or points to which its position is referred.[2]

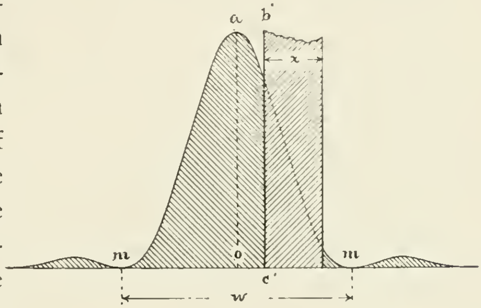


FIG. 1.

When the diffraction image is sufficiently broad with reference to the wire, x, as in Fig. 1, it is possible to locate the position of the former with reference to the latter with an error not exceeding one hundredth of the total width $mm = w$ of the diffraction image. Since the resolving power α for fine lines and points is (for rectangular aperture) equal to $\frac{1}{2}w$, it would follow that the maximum value of a might be as large as 50. Usually this degree of magnification of the image is not attainable, and the corresponding value of a is reduced. Experience, however, shows that under best conditions a value of from 10 to 15 can be attained. Thus, with a telescope having an aperture of 4 cm, the limiting resolving power of which is

[1] MICHELSON, "Measurement by Light Waves," *Am. Jour.*, 39, 115.

[2] *Phil. Mag.*, 44, 83; *Phys. Rev.*, 4, 96.

about 3″, it is possible under favorable conditions to measure angular differences of position as small as 0″.20 either micrometrically or heliometrically.[1] About the same order of accuracy is attained in setting on the images of spectral lines. Thus Jewell finds that his probable error of a single setting on a line in the solar spectrum obtained with Rowland's concave gratings is about 0.001 tenth-meter.[2] The limit of resolving power of the gratings is about 0.015 tenth-meters.[3] Similarly both Campbell and Frost find that their average errors of setting on the image of a line in the star spectra obtained with the Mills and Bruce spectrographs is about 0.0004 mm and 0.0003 mm respectively.[3] With the cameras of 406 and 449 mm focal length these linear errors correspond to an accuracy in angular measurement of about 0″.19 and 0″.14. The angular resolving power of the two camera objectives (apertures 37.4 mm and 51 mm) are only 2″.5 and 1″.8 respectively for the photographic region of the spectrum, $\lambda = 4500$.

In each of the above cases the accuracy of setting (on the image of the point or line) is about fifteen times the resolution obtainable with the instrument. For this class of measurements (*i. e.*, telescopic) we may therefore assume

$$\begin{aligned} a_{\text{max.}} &\cong 15 \\ \epsilon &\cong 0.07a \, . \end{aligned} \tag{2}$$

In order that these values of a and ϵ may represent a real and corresponding degree of accuracy in the determination of the position of the source itself (as distinguished from the position of the image) another very important condition is necessary. This is that the distribution in intensity in the diffraction pattern *shall be symmetrical about the position of the geometrical image of the source*. For if the distribution is unsymmetrical, as in Fig. 2, the tendency will be to place the measuring wires to

[1] *Jour. Franklin Inst.*, **138**, 1; July 1894.

[2] ROWLAND, *A. and A.*, **12**, 321, 1893. [3] *Phil. Mag.*, **43**, 320.

[3] From unpublished observations communicated by these observers. In each case the average error is that of the mean of four settings only. On the best solar plates Campbell states that his average error of setting is less than one-third of this, *i. e.*, only about 0.00012 mm.

one side of the position of the geometrical image, which is at *ao*, by an amount *oo'*, depending on the excess of illumination *amm'* on that side. The error thus introduced will correspondingly reduce the accuracy of setting, so that in general we shall have to write

$$a' = ka_{\text{max.}},$$

where k is less than unity by an amount depending on the asymmetry in the image. It becomes necessary, therefore, to investigate the various causes of such asymmetry and the effect produced by each.

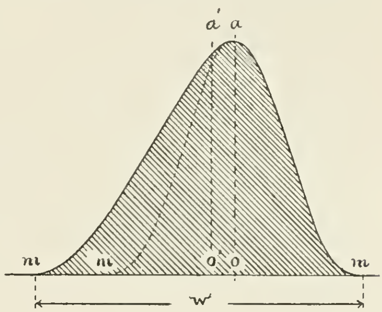


FIG. 2.

The general expression for the intensity, I^2, at any point, p, in the focal plane image is of the form[1]

$$I^2 = \left[\int\int \frac{1}{\lambda\rho} \sin\frac{2\pi}{\lambda}(at - \rho)\, i\, dx\, dy \right]^2. \qquad (3)$$

where ρ is the distance of the point p from the element of the wave-front, $dx\, dy$, whose amplitude of vibration is i. The integration is extended over the whole of the aperture through which the wave-front passes.

The expressions for the distribution in intensity in the image of a fine point which have been usually employed in discussing questions of resolving power are

$$I^2 = \frac{4}{\lambda^2 f^2}\left[\int_{-R}^{+R} \sqrt{R^2 - x^2} \cos\frac{2\pi\xi}{\lambda f} x\, dx\right]^2 = \frac{\pi^2 R^2}{\lambda^2 f^2} \cdot \frac{4 J_1^2\left(\frac{2\pi R\xi}{\lambda f}\right)}{\left(\frac{2\pi R\xi}{\lambda f}\right)^2} \qquad (4)$$

for a telescope with a circular aperture of radius R, and

$$I^2 = \frac{d^2}{\lambda^2 f^2}\left[\int_{-\frac{b}{2}}^{+\frac{b}{2}} \cos\frac{2\pi\xi}{\lambda f} x\, dx\right]^2 = \frac{d^2 b^2}{\lambda^2 f^2} \cdot \frac{\sin^2\left(\frac{\pi b\xi}{\lambda f}\right)}{\left(\frac{\pi b\xi}{\lambda f}\right)^2} = A\frac{\sin^2\frac{\pi}{\alpha_0}\alpha}{\left(\frac{\pi}{\alpha_0}\alpha\right)^2} \qquad (5)$$

for a telescope of rectangular aperture of width d and length b. These expressions represent respectively the distribution in

[1] RAYLEIGH, "Wave Theory," *Enc. Brit.*, 24, § 11; *Pop. Ast.*, 5, 534.

intensity at any distance $a = \frac{\xi}{f}$ from the center of the image along a line parallel to the length b of the diffracting aperture.

In deriving the expressions (4) and (5) from (3) the following assumptions are made :

1. That the amplitude of vibration i is uniform and constant over the entire wave-front within the diffracting aperture, and that the latter is symmetrical with respect to the line joining the geometrical centers of the source and the image, *i. e.*, with the line of collimation of the telescope.

2. That the wave-front passing the diffracting aperture is truly spherical and has its center at the center of the geometrical image. This amounts to the condition that there is no aberration.

3. That the wave-length λ is constant within the limits of integration, *i. e.*, either the light is strictly monochromatic or the telescope is strictly achromatic.

In practice not one of these assumptions is strictly correct and the expressions (4) and (5) are not therefore strictly accurate. In dealing with questions of resolving power the effect of variations from these theoretical conditions has been investigated in a number of cases and has generally been found to be small.[1] We cannot, however, assume that the same conclusion holds when we come to deal with questions involving the metrological power and accuracy, for we then have to consider quantities of a much smaller order of magnitude, and an amount of disturbance or imperfection, particularly asymmetry, in the image which is negligible in questions involving resolving power (or in many cases of contrast) will introduce an error of measurement considerably larger than the limit of accuracy ϵ attainable under the best conditions.

The various causes which produce distortion and asymmetry in the diffraction image may be divided into two general classes :

[1] For special cases in which the effect is of considerable importance see "Theory of the Objective Spectroscope," ASTROPHYSICAL JOURNAL, 4, 54; "Resolving Power of Telescopes and Spectroscopes for Lines of Finite Width," *Phil. Mag.*, 43, 317; also paper, "The Effect of Absorption on the Resolving Power of Prism Trains," to be published in the February number of the *Phil. Mag.*

(A) Those of a physical nature of a character already indicated (1), (2), and (3) (p. 272).

(B) Those of a more purely instrumental nature depending on peculiarities of form or size or operation of the instrument itself.

In considering both of these classes we shall in this general paper assume the following conditions, which may nearly always be fulfilled or at least closely approximated:

(*a*) That the aperture of the instrument is always rectangular and that one of the sides is parallel to the axis or line of measurement, this axis being likewise the line along which the distribution in intensity in the image has to be considered. The assumption of rectangular aperture gives in general simpler analytical expressions, and the results obtained, in most cases, lead to the same conclusions as would be reached if circular apertures were assumed.[1]

(*b*) That in addition to being parallel to one side of the rectangular aperture the axis of measurement is so chosen that the disturbing cause is symmetrical in respect to this axis. Specific cases in which this condition cannot be fulfilled will be treated separately.

Case A (1)—Illumination over the incident wave-front unsymmetrical.

The general expression (3) may be reduced to the form

$$I^2 = \frac{1}{\lambda^2 f^2}\left[\int\int \cos\frac{2\pi}{\lambda f}(\xi x + \eta y)\, i\, dx\, dy\right]^2 + \frac{1}{\lambda^2 f^2}\left[\int\int \sin\frac{2\pi}{\lambda f}(\xi x + \eta y)\, i\, dx\, dy\right]^2. \qquad (6)$$

Under the conditions (*a*) and (*b*) assumed above we obtain in this case for the intensity of the diffraction pattern at any point on the axis ξ, $(\eta = 0)$, the expressions

$$\begin{aligned} I_1^2 &= \frac{d^2}{\lambda^2 f^2}\left\{\left[\int f(x)\cos\frac{2\pi\xi}{\lambda f}x\, dx\right]^2 + \left[\int f(x)\sin\frac{2\pi\xi}{\lambda f}x\, dx\right]^2\right\} \qquad (7) \\ &= A\,(c^2 + s^2)\ , \end{aligned}$$

where $f(x) = i$ represents the amplitude of vibration at different portions of the wave-front.

[1] The case of circular apertures is taken up in detail in some special cases where the results differ appreciably from those obtained with rectangular apertures.

The only case of importance of this kind is that in which the wave-front, before reaching the diffracting aperture, has traversed an absorbing medium of varying density or varying thickness as in the case of the prism spectroscope. This latter case has been recently examined in connection with the question of resolving

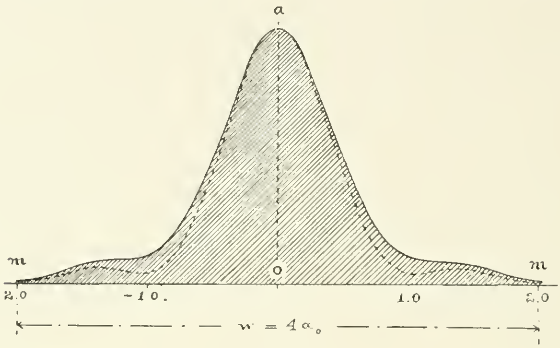


FIG. 3.

power of prism trains composed of very large or very dense prisms.[1]

The expression for I^2 which was found was

$$I^2 = \frac{d^2}{\lambda^2 f^2}\left[\frac{e^{Bb} + e^{-Bb} - 2\cos\frac{2\pi\xi b}{\lambda f}}{B^2 + \frac{2\pi\xi}{\lambda f}}\right], \tag{8}$$

where B is a constant depending on the coefficient of absorption of the glass composing the prism train, and is determined from the relation

$$\iota = i_0 e^{-Bx}, \tag{9}$$

i_0 being the intensity of light transmitted through the axis of the prism system.

The distribution in intensity represented by (8) is shown in Fig. 3 for the particular values of $B = \frac{1.386}{b}$ and $B = \frac{2.197}{b}$. For these values of B the intensity of the transmitted beam falls off 75 per cent. and 89 per cent., respectively, from one edge of the aperture to the other.

[1] "On the Effect of Absorption on the Resolving Power of Prism Trains," see footnote above.

The curve of intensities is symmetrical with respect to $\xi=0$, the position of the geometrical image. The unsymmetrical absorption therefore does not introduce any direct error of measurement at the focal plane.[1] In fact since the effect is in general to increase the apparent width w of the line, the accuracy of setting will, if anything, be slightly increased.

It may be similarly proved that an asymmetry in the form of the aperture itself will not affect the symmetry of the image about the geometrical center. Indeed, we may choose particular forms of aperture (other than rectangular) which will somewhat increase the resolving power and the accuracy of the measurement.[2]

CASE A (2).

Effect of asymmetry in the wave-front (aberration).—When the wave-front which forms the image is truly spherical its equation is

$$x^2+y^2+z^2=f^2 , \tag{10}$$

and the distance ρ from any point xy in this front from a point $\xi\eta$ in the focal plane is

$$\left.\begin{aligned}\rho &= \sqrt{f^2-2x\xi-2y\eta+\xi^2+\eta^2} \\ &\cong f\left[1-\frac{x\xi+y\eta}{f^2}\right] .\end{aligned}\right\} \tag{11}$$

If the wave-front is not spherical we may express the coördinates of the new surface with respect to the old as follows

$$\left.\begin{aligned}x' &= x+\beta x^2+\gamma x^3+\delta x^4+\cdots \\ y' &= y+\beta_1 y^2+\gamma_1 y^3+\delta_1 y^4+\cdots \\ z' &= z .\end{aligned}\right\} \tag{12}$$

The expression for the intensity I_1^2 at any point ξ, η, in the focal image will be found by substituting these new values for x', y', z', in (11) and (6), and performing the indicated integrations. But as before we considerably simplify the integrals with

[1] The effect of asymmetrical illumination on the displacement of an image out of the focal plane is considered under B.

[2] Discussion of this question will be taken up in a paper "On the Form of Telescope Aperture Best Adapted to the Resolution and Measurement of Close Double Stars," which is now in course of preparation.

which we have to deal by the assumption of conditions (a) and (b), p. 273. Then all terms in odd powers of y higher than the first disappear. Next, by changing f, equation (11), to a new value f^1 (*i. e.*, by readjusting the focus) we may cause the terms in x^2 and y^2 to disappear. Finally, since any disturbance which we need to consider is exceedingly small (as will appear later), terms in x^3 are relatively of much more importance than terms in x^4, y^4, or any higher powers, and all of the latter may therefore be neglected. Besides this any aberrational effect due to terms in x^4 will be symmetrical in its nature and will not therefore affect the *position of the center of area* of the diffraction image. The same would also be true of terms in x^2 and y^2 whose effect, however, can, as already seen, be rendered negligible by refocusing.

When we consider only terms in x^3 the expression for I^2 (6) (since sin x^3 like sin x is an uneven function), becomes for rectangular aperture

$$I_2^2 = A' \int_{-\frac{b}{2}}^{+\frac{b}{2}} \cos \frac{2\pi\xi}{\lambda f} \left[x + \gamma x^3 \right] dx \; . \tag{13}$$

This integral was first investigated by Airy[1] and subsequently more completely by Lord Rayleigh.[2] The latter has adapted Airy's results to the consideration of special cases for which the distortion of the wave-surface amounts to ¼, ½, and ¾ of a wave-length.

The values of I_1^2 for the value of γx^3 such that the extreme displacement of the wave-surface at the edge of the aperture is ¼ of a wave-length from the true spherical surface has been calculated from Airy's and Rayleigh's results and is given in column 2, Table I. The values of I^2 (from 5) are given in column 3, for comparison. The abscissæ are expressed in terms of a_0, the resolving power of the telescope of aperture b.

The form and relative position of the two curves I^2 and I_1^2 are shown in Fig. 4. The dotted curve represents the distribution in intensity in the focal plane image when there is no aber-

[1] *Camb. Phil. Trans.* **6**, 402, 1838. [2] *Phil. Mag.* (5), **8**, 404, 1879.

TABLE I.

a_0	I_1^2	I^2	a_0	I_1^2	I^2
−2.0	0.0086	0.0000	0.0	0.7093	1.0000
−1.8	.0061	.0108	+0.2	.9158	.8751
−1.6	.0012	.0358	+0.4	.9137	.5754
−1.4	.0004	.0468	+0.6	.6901	.2545
−1.2	.0031	.0243	+0.8	.3629	.0547
−1.0	.0018	.0000	+1.0	.0999	.0000
−0.8	.0017	.0547	+1.2	.0009	.0243
−0.6	.0399	.2545	+1.4	.0395	.0468
−0.4	.1714	.5754	+1.6	.1095	.0358
−0.2	.4159	.8751	+1.8	.1240	.0108
±0.0	.7093	1.0000	+2.0	.0762	.0000

ration (equation 5); the full curve, that when the aberration amounts to ¼ wave-length as assumed above. The effect of the aberration is threefold: (1) it diminishes the intensity at the center of the image; (2) it renders the image asymmetrical with respect to its center of area; and (3) it displaces very sensibly both the position of maximum brightness and of the center of area. The effect of (1) is not serious, but the effect of (2) and more particularly (3) on the accuracy of measurement is most important. The displacement of the point o' (Fig. 4) from the geometrical center o of the undistorted image is about $0.3\,a$, while the limiting accuracy of measurement, as we have already seen, is about $0.07\,a$. The displacement o-o' caused by an aberration amounting to only ¼ wave-length is therefore about four times the limit of accuracy ϵ attainable under best conditions.

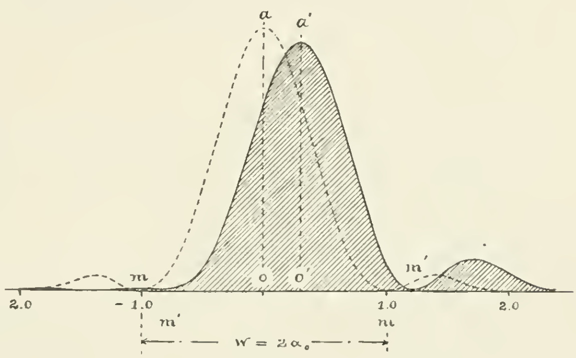


FIG. 4.

An examination of the curve representing a double source, Fig. 5, as viewed under the conditions assumed in Fig. 4, shows that neither the resolving power, which depends primarily on

the form and "width" *mm* of the images of each component, nor the contrasting power, which depends on the width *mm* and the value $o'a'$ of the maximum ordinate, are either of them affected greatly by an aberration of the amount (¼ wave-length) shown. Hence, it is generally assumed that the optical "definition" of an instrument (which, as usually defined, depends principally on resolution and contrast) is not injuriously affected by aberrations which introduce differences of phase in the wave-front of not more than ¼ period, and that therefore optical surfaces which reflect or transmit wave-fronts at nearly perpendicular incidence need to be accurate to ⅛ and ½ wave-lengths only. But as indicated above, this degree of perfection is not sufficient in instruments designed for accurate measurements.

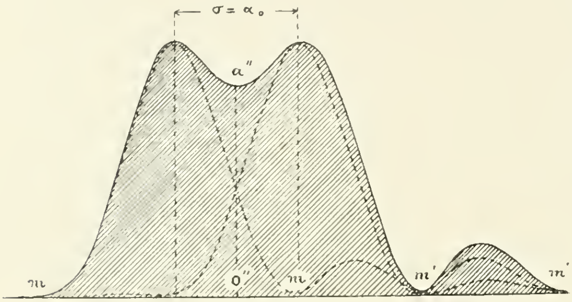


Fig. 5.

The conditions of optical measurement of position are such that we are concerned not so much with the absolute amount of aberration as with the constancy of its effect. For in the great majority of cases we are concerned with the simultaneous or successive determination of the separation or relative position of two images in the same focal field. Under such conditions, if the amount of aberration remained exactly the same for both images, the distortion and displacement of both would also be the same in amount, and in direction, and the measured distance between their centers would be practically unaltered. To avoid errors of measurement, therefore, we need only be certain that the *differential* effect of the wave disturbance on the two images is less in amount than the limit of accuracy ϵ already established. The degree of constancy required is readily determined from the results of the preceding investigation. The displacement of the center of the image in Fig. 4 is, as already stated,

about $0.3a$ for a phase difference of ¼ wave-length. This displacement is very nearly proportional to the tilting of the actual wave-front with reference to the true spherical wave-front, and this tilting in turn is very nearly proportional to the phase differences between center and edge. Hence, if the differential displacement of the two images is not to exeed the limiting value of ϵ ($0.07a$), the aberration must not vary by more than

$$\frac{1}{4} \cdot \frac{0.07}{0.30} \cong 0.06\lambda. \tag{14}$$

The amount of aberration (*i. e.*, the distortion of the wave-front forming the image) will vary at any instant of time: I, by reason of changes in position of an image in the focal field, involving a change in the inclination of the axis of the wave-front to the various optical surfaces and possibly a shift of the wave-front as a whole with reference to those surfaces; II, because of changes in the optical constants of the media traversed by the wave before reaching the focal plane. We will now examine the instrumental and operative conditions necessary to keep the variations due to either cause within the limits given.

I. (1) If the two images are separated by a considerable angular interval in the field of the observing telescope, the two wave-fronts by which they are formed will meet the optical surfaces of the objective at different angles, and the resultant distortion or curvature in each will vary by reason of this fact, even though they were precisely similar before incidence. The amount of aberration introduced in any given case is determined by calculation of the relative retardations of the extreme, with reference to the central rays, which meet at the focal plane.

Thus in Fig. 6 let abb' be the points at which the center and extreme edges of the given wave-front x_1x_2 meet the front surface of the optical system, and o', the best-defined center of curvature of the wave-front x_1x_2, after its passage through the optical system. Let hb be the distance from the point b to the instantaneous wave-front incident at a, and ah the distance from the instantaneous wave-front incident at b'. Then for the pri-

mary plane[1] the retardation of the extreme with reference to the central rays, *i. e.*, the amount of aberration for the two halves of the wave-front, will be represented by

$$\left.\begin{array}{l} ao' - (hb + bo') \\ \text{and} \quad ao' - (b'o' - h'a) \end{array}\right\} \qquad (15)$$

The determination of the value of these quantities in any given case demands a knowledge of the indices of refraction, radii of curvature, and thickness of the lenses forming the optical system, and is usually quite complicated. We can, however, draw certain conclusions from the conditions of the problem which are of more general interest than the detailed consideration of many individual cases.

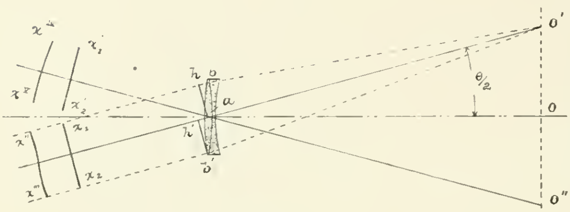


FIG. 6.

The displacement of the centers of two images o' o'' will be in opposite directions if they are on opposite sides of the principal optical axis ao, and will be equal in amount if they are at equal distances from this axis, and both the optical system and the incident wave-fronts, $x_1\, x_2\, x'_1 x'_2$, are symmetrical about their respective axes. Under such conditions of symmetry the distortion of the field itself is symmetrical with respect to its center o, and if these conditions could always be maintained, the proper corrections could be determined once for all and then always applied to the measurements. But the form of the incident wave-fronts is different at different times, as shown at $x''x'''$, $x^{iv}x^{v}$. These changes may be due to influences which are outside of and independent of the instrument itself; and the displacements and the required corrections will therefore vary by an amount which is indeterminate. Our only complete safeguard against errors of this kind is to confine our measurements to a field within which the aberrational distortion and displacement of the lens system itself is negligible. We can form a general idea of

[1] We confine our attention to the aberration at the primary focus because in the case of lines perpendicular to the xy plane the best-defined image is formed at that point.

what this field is in different cases without investigating each one in detail by the following considerations.

The requirements for obtaining the maximum practicable accuracy demand, as has been shown above, that the effect of aberration on the image shall not be more than one-fourth as great as that which would begin to sensibly affect good "optical definition". If we assume that the aberration due to a good lens system varies as the square of the distance from the center, and if we call θ the "field of good definition" (visual or photographic) of the given lens, then the "field of measurement," on the above assumptions, should not exceed $\frac{1}{2}\theta$.

The value of θ depends on the linear size of the objective, b, on its semi-angular aperture, $\beta = \frac{b}{2f}$, and on the type of optical construction. In general we may say that θ varies approximately as $\frac{1}{\sqrt{b}}$ and as $\frac{1}{B}$ for a given type of instrument. The maximum values of θ that have been so far attained with several types of lenses of different apertures are given in Table II.

TABLE II.

Type of Instrument	Size of Objective b	Angular Aperture β	Field θ	
			Visual	Photographic[1]
1. Photographic doublet (new form)[2]............	4 in.	1 : 5 = 0.20		about 15°
2. Photographic doublet (symmetrical construction)[2]	4	1 : 5 = 0.20		about 8°.5
3. Photographic triplet (symmetrical construction).....	2	1 : 8 = 0.12+	about 2°.5	about 3°
4. Ordinary achromatic (visual)...............	3	1 : 12 = 0.083	about 2°.0	
5. Ordinary achromatic (visual)...............	6	1 : 18 = 0.055	about 2°.0	
6. Ordinary achromatic (photographic)..........	15	1 : 15 = 0.067		about 1°.5
7. Ordinary achromatic (visual)................	40	1 : 18.5 = 0.054	about 0°.7	

[1] The available good field of an objective is always slightly larger photographically than visually on account of the somewhat less rigorous standard of test that can be applied.

[2] See *Annual Reports of the Allegheny Observatory*, 1899, p. 7; 1900, p. 19.

These values of θ are based on experiments and tests made by the writer and others on a large number of objectives of different sizes and types of the best modern construction. They are considerably smaller in some cases than those claimed by the makers of astronomical and photographic objectives, probably because the standard of "good definition" is higher. Many commercial photographic lenses, for example, "cover" fields of 40° or 50° with satisfactory results on an ordinary photographic standard. But when the rigorous tests of stellar photography are applied, the definition of these lenses breaks down only a few degrees from the axis; in fact, I have never found any of them which give results at all comparable with those designed by Hastings, for example, especially for astrophotographic work. A similar statement applies to the longer focus type of objectives (4) and (5). With a low-power eyepiece the visual definition is apparently quite good over fields considerably larger than those given, but with high powers and more careful tests the fields are more restricted.[1]

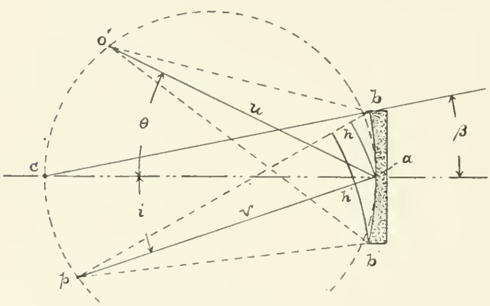


FIG. 7.

The available fields of measurement under the above standards vary therefore from a maximum of about 7° for (1) to about ⅓° for (7).

There is one special case which has recently been examined[2]

[1] The fields given above are those obtainable in photographic use on flat plates or in visual use without refocusing. These are the conditions that are generally imposed in metrological work. If we use curved plates (or in visual work curved micrometer slides) the field of good definition can be very greatly extended. Thus with a symmetrical doublet of 4½ in. aperture and 20 in. focal length ($\beta = 4.4$) it is possible by the use of a curved plate to cover sharply a field of about 33° diameter, or with a somewhat smaller angular aperture and a special construction of objective, a field of over 40° diameter. See *Annual Reports, Allegheny Observatory;* also "Efficiency of the Curved Plate for Charting Purposes," *Observatory*, **24**, 274 and 419, 1901.

[2] "Aberration of Mirrors and Concave Gratings at the Principal Focus," to be soon published in *Phil. Mag.*

the results of which are of particular interest. This is the case of the reflecting telescope and its associated spectroscopic instrument, the concave grating. The character and amount of aberration here depend on the form of the wave-front. We will consider first the case in which this is spherical as in the ordinary use of the concave grating. The general optical conditions involved in this problem are shown in Fig. 7.

Let p be the position of the radiant point, o', that of its image (either specular or spectral); and c, the center of curvature of the spherical surface[1] bab'. Put $pa = v$; $o'a = u$; $ac = \rho$; $pac = i$; and $cao' = \theta$.

Then from the general theory of optics we have always the relation

$$\frac{\cos^2 i}{v} + \frac{\cos^2 \theta}{u} = \frac{\cos i + \cos \theta}{\rho} . \tag{16}$$

With Rowland's mounting this relation is satisfied by making

$$u = \rho \cos \theta \quad \text{and} \quad v = \rho \cos i ,$$

and o', c, p, all lie on the circumference of a circle whose diameter is equal to ρ. The aberration at o' for this case has been calculated by Rowland[2] and Glazebrook.[3] The latter finds for the relative retardation for the central ray $(u + v)$ relatively to the edge ray $pb + bo'$,

$$\begin{aligned} E_1 = pb + bo' - (u + v) &= \rho \sin \beta (\sin i - \sin \theta) \\ &+ \frac{1}{8} \rho \sin^4 \beta (\sin \theta \tan \theta + \sin i \tan i) , \end{aligned} \tag{17}$$

where β is the semi-angular aperture of the surface bab' measured at the center of curvature C; therefore the angle bca. The first term of this expression is that defining the relation between the order of the spectrum m, the wave-length of the spectral line λ, and the number of lines on the grating surface N, *i. e.*,

$$\rho \sin \beta (\sin i - \sin \theta) = \frac{1}{2} N m \lambda . \tag{18}$$

The second term is the aberration of the upper half of the

[1] The case of spherical surfaces only is considered here, because that is the form of surface generally used for concave gratings. The case of parabolic and other forms of surfaces is, however, considered at length elsewhere. See paper above referred to.

[2] *Phil. Mag.* (5), 16, 210. [3] *Ibid.*, 377.

spherical surface ab. For the lower half of this surface ab' we find similarly

$$E_2 = (pb' + b'o) - (u + v) = \rho \sin \beta (\sin i - \sin \theta)$$
$$+ \frac{1}{8} \rho \sin^4 \beta (\sin \theta \tan \theta + \sin i \tan i) = E_1 \;. \qquad (19)$$

The aberration is therefore symmetrical, and its effect on the image is simply to broaden it symmetrically without displacing its center.

If the reflection is specular, $\theta = i$. Hence, if we make $u = v$ we always have

$$\rho = u \cos \theta = v \cos i \;.$$

The first term of (17) and (19) disappears, and the aberration (which is expressed by the second term) becomes

$$\frac{\rho}{4} \sin^4 \beta \sin \theta \tan \theta \;, \qquad (20)$$

which is likewise always symmetrical.

In case the images, either spectral or specular, are formed on the circle passing through the center of curvature of the mirror as above assumed, we can therefore use any extent of field desired without introducing any error of measurement due to the aberration of the spherical surface itself. The only limit to the available field is that imposed by condition of good definition, *i. e.*, that the aberration shall not exceed a quarter wave-length. This condition applied to (17), (19), and (20) gives

$$\frac{\sin^2 \theta}{\cos \theta} + \frac{\sin^2 i}{\cos i} \leqq \frac{2\lambda}{\rho \sin^4 \beta} \;. \qquad (21)$$

In the case of Rowland's large gratings $\rho \cong 650$ cm and $\beta = \frac{2\rho}{a} \cong 0.011$. For the maximum value of $i = 60°$ we have therefore

$$\frac{\sin^2 \theta}{\cos \theta} \cong 9.5 \quad \text{or} \quad \theta = 84° \, 15' .$$

For gratings and mirrors of large angular aperture, such as are sometimes used in stellar spectroscopic work, the maximum value of both θ and i are considerably reduced. Thus for a

grating of 150 cm radius, of curvature and linear aperture the same as before, $\beta=0.050$. We have then

$$\frac{\sin^2\theta}{\cos\theta}+\frac{\sin^2 i}{\cos i}=0.106 \ .$$

which shows at once that neither θ nor i can in this case exceed $18°20'$. If we use the Rowland mounting for which $\theta=0$ for the center of the field and make $i=15°$ we have in the above case

$$\theta_{\max}\cong 11° \ .$$

For the concave grating the maximum field is of course obtained by making $i=0$, and placing the photographic plate or eyepiece directly above the slit. Under such circumstances we can obtain good definition over a field 36° long (18° on each side of the center).

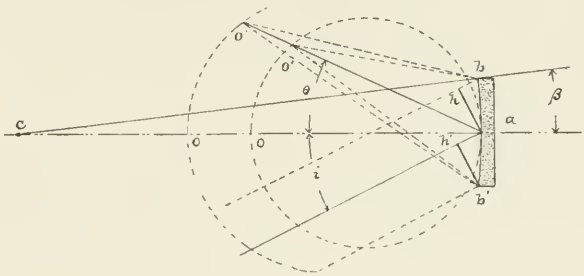


FIG. 8.

In the case of the reflecting mirror the maximum field is determined by making $\theta=i$ in (21). For a mirror of the dimensions last considered we have therefore

$$\theta_{\max}\cong 12°30' \ ,$$

or about ⅔ as large as obtainable with the concave grating.

Second case—incident wave-front plane. This case corresponds to the ordinary use of the reflecting telescope, and to the use of the grating as suggested by the writer in 1896.[1]

The optical conditions of this problem are shown in Fig. 8, the notation being the same as before.

The relative retardation of the central with reference to the extreme rays is in this case

$$\left.\begin{aligned} E_1 &= u-(bo'+bh) \text{ for the upper half of the surface,}\\ E_2 &= u-(bo'-ah') \text{ for the lower half of the surface.}\end{aligned}\right\} \quad (22)$$

[1] ASTROPHYSICAL JOURNAL, 3, 55–60. Since this article was written the concave grating has been used in the manner suggested, by Poor and Mitchell (*ibid.*, 7, 157; 10, 29), in stellar spectroscopic work, and by Jewell; Mohler & Daniel, (*ibid.*, 12, 361); Frost (*ibid.*, 12, 311), and by the writer (*Report Allegheny Observatory*, 1900, p. 23), in solar-eclipse work.

The general expressions for E_1, E_2 can be derived from the geometry of the triangles bao', $ab'o'$, bah, and $a'b'h'$. Using the same notation as before we find as general expressions for E_1 and E_2

$$\begin{aligned} E_1 &= \rho \sin\beta\,(\sin\theta - \sin i) \\ &+ \frac{1}{2}\rho \sin^2\beta \left\{ \cos\theta + \cos i - \frac{\rho}{u}\cos^2\theta) \right\} && = B \\ &+ \frac{1}{2}\frac{\rho^2}{u^2}\sin^3\beta \left\{ \sin\theta\cos\theta\,(u - \rho\cos\theta) \right\} && = C \\ &+ \frac{1}{2}\frac{\rho^2}{u^2}\sin^4\beta \left\{ \begin{array}{l} \rho\sin^2\theta\cos\theta - \frac{\rho}{u}\sin^2\theta\cos^2\theta - \frac{1}{4}u \\ + \frac{1}{4}u\cos^2\theta + \frac{1}{4}\frac{\rho^2}{u}\cos^4\theta - \frac{1}{2}\rho\cos\theta \end{array} \right\} && = D \qquad (23) \\ &+ \frac{1}{8}\rho\sin^4\beta\,(\cos\theta + \cos i) && = F \end{aligned}$$

and

$$E_2 = \rho\sin\beta\,(\sin i - \sin\theta) + B - C + D + F\ . \qquad (24)$$

In this case the aberration is unsymmetrical, the amount of asymmetry being expressed by the term in $\sin^3\beta$ (C). The value of this term depends on the manner in which the grating is used. Three cases may be distinguished, as has been already indicated:

(a) When the direction of reflection is parallel to the line of incidence pa.[1] Then $\theta = -i$, and from (16) we find for u, ($v = \infty$),

$$u_a = \frac{\rho}{2}\cos\theta\ . \qquad (25)$$

In this case we have for the term C

$$\rho\sin^3\beta\sin\theta\ . \qquad (26)$$

(b) When we examine the spectra in the neighborhood of the center of curvature. $\theta = 0$ for the center of the field, and we have for u[2]

$$u_b = \frac{\rho}{1 + \cos i}\ . \qquad (27)$$

Then for (C) we find for small values of θ

$$\rho\sin^3\beta\sin\theta\frac{1}{2}\cos i\,(1 + \cos i)\ . \qquad (28)$$

[1] Astrophysical Journal, 3, 58, 59. [2] *Ibid.*, pp. 54–57.

(*c*) When we make the incidence on the grating normal. Then $i = 0$ and

$$u_c = \frac{\rho \cos^2 \theta}{1 + \cos \theta} , \tag{29}$$

and for (C)

$$\rho \sin^3 \beta \sin \theta \frac{1}{2} \left(\frac{1 + \cos \theta}{\cos^2 \theta} \right) . \tag{30}$$

(*d*) Finally for regular specular reflection $\theta = i$, and

$$u = \frac{\rho}{2} \cos \theta , \tag{30}$$

just as in case (*a*).

As the aberration is in all these cases unsymmetrical, the available measurable field is found by equating (14) with (26), (28), and (30) (neglecting terms in $\sin^4 \beta$).

This gives us for cases (*a*), (*b*), and (*d*)

$$\sin \theta_a = \frac{\lambda}{16 \rho \sin^3 \beta} K . \tag{31}$$

In the case of Rowland's gratings, already discussed, for which $\rho = 650$ cm. and $\beta = 0.011$, the value of θ given by (31) is

$$\theta_{\max} \cong 2^\circ \, 20' ,$$

when the grating is used as in (*a*). When it is used as in (*b*), and we make the maximum angle of incidence $i = 60^\circ$, we have for θ.

$$\theta_{\max} = \theta \frac{2}{\cos i \, (1 + \cos i)} \cong 6^\circ .$$

Finally, if we use the same grating as in (*c*), we find for θ

$$\theta_{\max} = \theta_a \frac{2 \cos^2 \theta_a}{1 + \cos \theta_a} \cong 2^\circ 20' \text{ as before} .$$

Another case of great importance and interest is the use of mirrors as collimators or telescopes of spectroscopes. If the mirrors are of long focus, very satisfactory results are attained by their use. Thus in one of the instruments used by the writer the spherical mirror had an aperture of 6 cm, and a focal length of about 175 cm. For such a mirror we have $\rho = 350$, $\beta \cong \frac{1}{117}$.

With these values we get from (26)

$$\sin i = 0.016 \quad \text{or} \quad i = 55' .$$

The diameter of undistorted field is therefore about 1°.8. The diameter of the field of good definition (for qualitative work only) is of course about four times this, or about 7°.5. These results agree well with those actually found in practice,[1]

For large angular apertures the results are much less satisfactory. Thus for a surface such as was considered before for which $\rho = 150$ cm and $\beta = 0.05$ we have

$$\theta_a = \theta_d \cong \theta_c \cong 40'' \ .$$

By mounting the grating as in (b) and using a very large value of i ($i = 75°$) we may increase this field about six times, *i. e.*,

$$\theta_b \cong 2' \ .$$

We may likewise increase the field by diminishing the focal length (keeping β the same) in the proportion $\frac{1}{\rho}$. For a grating of only 1 inch aperture and 5 inches focal length $\beta = 0.05$ as before and $\rho = 12.5$cm. The value of θ_b for this case is

$$\theta_b \cong 25' \ .$$

The field of good definition in all the above cases is of course about four times as large as the field of negligible measurable distortions.

The above results show clearly that the use of spherical gratings or mirrors of very large angular aperture with parallel incident light is impracticable for either quantitative or qualitative work, except in the case of very small gratings used as in (b). If such mirrors or gratings are to be used of large size, it is absolutely necessary that their surfaces be specially figured for the particular use desired. This question is considered at greater length elsewhere.

I. (2) If the wave-fronts which form the two images are appreciably shifted in lateral position so as to fall upon different portions of the optical train, it is at once evident from (14) that the mechanical errors in the different surfaces involved must not exceed the amount that would introduce a difference of phase of 0.06λ. If t be the permissible mechanical error and ϕ the angle of incidence of the wave-front, we have at once

[1] "An Improved Form of Littrow Spectroscope," *Phil. Mag.* (5), 38, 137–142, 1894.

$$\left.\begin{aligned} t &= \frac{0.06\lambda}{2\cos i} \quad \text{for reflecting surfaces} \\ t &= \frac{0.6\lambda}{n\cos i' - \cos i} \quad \text{for refracting surfaces} \end{aligned}\right\}^{1} \tag{32}$$

Two cases of importance that need to be considered in this connection are those of the heliometer and the spectrograph. In the first instrument the two images which are brought to coincidence are formed by entirely different portions of the objective, and the optical surfaces of each portion must therefore be correct to the limit indicated in (32). Since i and i' are both nearly 90°, the two surfaces of the flint lens must each be optically right to within about $\frac{1}{10}\lambda$ and the two surfaces of the crown to within about $\frac{1}{8}\lambda$ each. The standard of workmanship required in a heliometer objective is therefore very high, and the difficulty of attaining the necessary degree of perfection is rendered vastly more difficult by the division of the finished object-glass into two parts. If there is the slightest strain or difference in density in the material, the glass is likely to spring slightly along the line of cutting, and this line is unfortunately parallel to the axis of measurement.[2]

In the case of the prism spectroscope and spectrograph the wave-fronts which form the images of different spectral lines are shifted laterally with reference to each other in passing through the train by the amount

$$\sum_{o}^{L} l\,\frac{d\theta}{d\lambda} = S\ , \tag{33}$$

where l is the length of path traversed by the wave-front between each successive refraction, beginning with the incident side of the first prism and ending with the camera or view telescope objective. For a prism train consisting of N similar

[1] RAYLEIGH, "On the Accuracy Required in Optical Surfaces," *Phil. Mag.* (5), **8**, 477.

[2] The form of prism heliometer which has recently been devised and constructed by the writer avoids this difficulty by rendering the division of the object-glass unnecessary. It can therefore be constructed of much larger size than has been possible with the older form. See *Allegheny Observatory Report* for 1900, p. 23; also U. S. Patents, Nos. 536493, 536494, 536555.

prisms with their corners in contact, we have for the shift from the central ray which passes at minimum deviation

$$S=\frac{1}{2}\frac{d\theta}{d\lambda}\left[\begin{array}{l}B_1+3B_2+5B_3+\ .\ .\ .\ .\ +(2N-1)\,B_N \\ \quad +2nB_1+4nB_2+6nB_3+\ .\ .\ .\ .\ +2Nn\dfrac{B_N}{2}\end{array}\right\}, \qquad (34)$$

where B_1B_2, etc., are the lengths of the bases of the prisms. For some of the larger instruments recently constructed the value of S is quite large. For the Bruce spectrograph of the Yerkes Observatory, for example,

$$N=3\,;\ n\cong 1.67\,;\ B_1=12\,\mathrm{cm}\,;\ B_2=13\,\mathrm{cm}\,;\ B_3=14\,\mathrm{cm}\ .$$

The angular separation of the extreme images measured on the plate is about 4°; therefore $\frac{d\theta}{d\lambda}$ (one refraction) $\cong \frac{2}{3}{}^\circ \cong 0.012$. We obtain therefore for S

$$S_{\max}\cong 2\,\mathrm{cm}\ .$$

The refracting angles of the prisms of this train are about $63^\circ\ 30'$, and μ for the central ray is about 1.673. Hence i and i' are about $61^\circ\ 45'$ and $31^\circ\ 45'$ respectively. For these values we have for t, from (32)

$$t_{\max}=\frac{0.06\lambda}{0.94}\cong 0.07\lambda\ ,$$

or the prism surfaces must be correct to within about $\frac{1}{13}$ of a wave over all those portions not common to the two beams, *i. e.*,

$$2\,\mathrm{cm}\div\cos 61^\circ\ 45'\cong 4\,\mathrm{cm}\ ,$$

at each end of the prism faces. This would mean that the prism faces should not depart from a perfect plane by more than 0.000003 cm in a length of 6.5 cm, or that the radius of curvature of the faces should be at least 7,000,000 cm. Such a degree of accuracy is very difficult to attain in surfaces whose length (13 cm) is so much in excess of their width (5.1 cm) as in this case, but it is even more difficult to maintain when once secured, if temperature conditions vary rapidly.

In case of the plane grating spectroscope the same portion of the grating surface may always be used, but the angle of incidence and reflection varies with the wave-length. As ordinarily

used with fixed collimator and observing telescope, the angles θ and i are connected by the relation

$$i + \theta = \gamma ,$$

where γ is the fixed angle between the axes of the two telescopes, and the angles θ and i are measured positively on opposite sides of the normal as before. In this case any irregularity of surface of height t introduces a retardation which is evidently

$$t(\cos i + \cos \theta) . \tag{35}$$

For setting on the central slit image we must have $\cos i = \cos \theta = \cos \frac{1}{2}\gamma$, and the error of phase is

$$2t \cos \frac{1}{2}\gamma . \tag{36}$$

For setting on any spectral line of wave-length λ we must revolve the grating through an angle δ such that

$$\sin \delta = \frac{Nm\lambda}{2a \cos \frac{\gamma}{2}} . \tag{37}$$

Also $\theta = \frac{\gamma}{2} - \delta$ and $i = \delta + \frac{\gamma}{2}$, and (35) reduces at once to the form

$$E = 2t \cos \delta \cos \frac{\gamma}{2} . \tag{38}$$

In order that the spectral image may not be displaced a measurable amount from its true position with reference to the central slit image, the condition is

$$2t \cos \frac{\gamma}{2} (1 - \cos \delta) \leqq 0.06 \lambda . \tag{39}$$

If the angle between the two telescopes is $30°$, $\cos \frac{\gamma}{2} = 0.966$, and for the image of the D lines in the second order spectrum of a grating with 5,000 lines to 1 cm, the value of δ is

$$\delta \cong 17° \, 45' ,$$

and for the fourth order spectrum

$$\delta \cong 37° \, 40' .$$

And from (39) we obtain at once for t

$$t_{\max} \lesseqqgtr \begin{cases} 0.6\lambda \text{ for the first case,} \\ 0.15\lambda \text{ for the second case,} \end{cases}$$

from which it appears that the irregularities in the surface of a plane grating large enough to introduce a measurable displacement of the image of a spectral line would probably manifest themselves by a deterioration of the optical definition.

II. The effect of temperature variations in the media traversed by the wave-fronts is more serious than those of irregularities of surface or varying obliquity which we have just considered. For the latter would be constant in amount and direction, provided the incident wave-fronts were constant in form, and under such circumstances could be corrected for. But the effect of temperature changes in the various optical parts of the instrument is to change continually the form of the passing wave-front. and thus not only render the corrections in question varying and uncertain, as has already been stated, but also introduce errors of displacement which vary with time alone. This latter class of errors can be eliminated only in two ways: first, by making the measurements on the two images absolutely simultaneous in time; or, second, by keeping the temperature variations within the limits which will render the resulting displacements less than the quantity ϵ. The first condition can be satisfied in certain cases by heliometric methods or by simultaneous photography of the two images, the measurements being afterward made on the photographic plate instead of directly in the focal plane. The elimination of the effect of what may be termed time aberrational displacements in this way is one of the advantages of the heliometric and photographic methods of measurement that does not seem to have been heretofore specifically recognized.

If the above time condition cannot be strictly fulfilled, then the temperature variation must not exceed an amount that would introduce an unsymmetrical aberration of more than 0.06λ (see (14)). We will next examine briefly what this condition of steadiness implies. The change in the optical density (index of refraction) of air and glass for $1°$ C. is

$$\left.\begin{aligned} \Delta n_a &= -1.1 \times 10^{-6} \text{ for air,} \\ \Delta n_g &= +4.0 \times 10^{-6} \text{ for flint glass.}^1 \end{aligned}\right\} \qquad (40)$$

[1] J. O. REED, "Einfluss der Temperatur auf die Brechung . . . . einiger Gläser," Inaugural Dissertation, Jena, 1897. Some varieties of glass examined by Dr. Reed,

The total difference in phase $\frac{\Delta\lambda}{\lambda}$ introduced in the wave-front in traversing a thickness L raised to a difference of temperature T will evidently be

$$\frac{\Delta\lambda}{\lambda} = \frac{L}{\lambda} T\Delta n \ . \qquad (41)$$

Under the condition that $\frac{\Delta\lambda}{\lambda}$ shall not exceed 0.06 we have at once

$$\left.\begin{aligned} LT &\leqq \frac{0.06\lambda}{1.1 \times 10^{-6}} \cong 2.5 \text{ for air,} \\ &\leqq \frac{0.06\lambda}{4 \times 10^{-6}} \cong 0.7 \text{ for flint glass.} \end{aligned}\right\} \qquad (42)$$

The focal length L_a of the largest heliometers now in use is about 250 cm, and the average thickness L_g of their objectives (aperture 16 cm) is about 1.5 cm. From (42) we obtain at once

$$\left.\begin{aligned} T_a &\leqq 0^\circ.01 \text{ C.} \\ T_g &\leqq 0^\circ.5 \text{ C.} \end{aligned}\right\} \qquad (43)$$

That is, in order to avoid an error of displacement greater than the limit of accuracy $\epsilon = 0.07a$ attainable with this instrument, the temperature of the two halves of the object-glass must remain constant to within ½° C. and that of the air in the two sides of the tube constant within 0°.01 C., in the interval between two successive sets of measurements.

In the case of the Bruce spectrograph considered above, L_g (for the prism train) is about 39 cm and the corresponding value of L_a ($39n$) is about 65 cm. If the temperature of either the air or glass changes alone, the limit of this temperature change must be

$$\left.\begin{aligned} T_a &\leqq 0^\circ.038 \text{ C.} \\ T_g &\leqq 0^\circ.018 \text{ C.} \end{aligned}\right\} \qquad (44)$$

If the two change together and by the same amount the limit of the change must be (40) and (41)

$$T \leqq \frac{0.06\lambda}{L_g \Delta n_g - L_a \Delta n_a} = 0^\circ.012 \text{ C.} \qquad (45)$$

notably the barium silicate crowns and the light barium flints, have much smaller temperature coefficients and would on that account be excellent for use in this connection.

For the camera objectives, focal lengths $L_a = 45$ cm and 60 cm the limiting values of T are

$$T_a \leqq 0^\circ.055 \text{ C.} \quad \text{and} \quad 0^\circ.04 \text{ C.} \tag{46}$$

The interval between measurements, which in this case is the interval between the star exposures and the exposure for the comparison spectrum, is so long and the temperature requirements so severe in the case of an instrument as large as the one considered that the only way of avoiding error would seem to lie in first keeping the temperature as constant as possible, perhaps within $0^\circ.1$, and exposing *continuously and simultaneously on both the star and the comparison source.* The method now generally used of making one exposure on the latter just before and another just after the exposure on the star is not the equivalent of a continuous exposure on both, because the temperature will not change uniformly in the glass, even if it does in the air, and the effect of an unsymmetrical or irregular change in the former is, as shown above, about twice as great as in the latter.

Case A (3). Effect of variations in the value of λ (non-achromatism of image).

When the value of λ in the general equation (3) varies, we have to take into account not only the resulting change in the *scale* of the diffraction pattern for each wave-length, but also several other effects as follows:

(*a*) The chromatic dispersion of the instrument and the relative inclination (if any) of the different incident wave-fronts corresponding to different values of λ.

(*b*) The spectral distribution in intensity in the different wave-fronts.

(*c*) The relative chromatic sensitiveness of the eye or photographic plate by means of which the position of the image is determined.

(*a*) The effect of chromatic dispersion of the instrument is to alter the inclination of the various wave-fronts passing through it by an amount depending on the wave-length λ and the optical form and constants of the media traversed.

Let us denote this varying inclination of the different wave-fronts by ϕ. Then in general

$$\left.\begin{aligned}\phi &= \frac{d\theta}{d\lambda}\,(\lambda-\lambda_0)=D\Delta\lambda \\ \text{and}\qquad d\phi &= D'\,d\lambda\ ,\end{aligned}\right\} \tag{47}$$

where λ_0 denotes the wave-length of the wave-front which is brought to focus on the axis of the geometrical image.

(b) The intensity of light in each wave-front will depend on the spectral distribution in intensity in the source of radiation itself, which we may denote by $\psi(\lambda)$, and on the coefficient of transmission for each wave-length, which we will denote by k_λ. The relative intensities in the different wave-fronts will therefore be proportional to

$$k_\lambda\psi(\lambda)\ . \tag{48}$$

(c) Finally, the relative effect of the image formed by each wave-front at the focal plane will depend on the "luminosity curve," either visual or actinic, which we may denote by $L(\lambda)$.

Taking into account all of these factors we have for the distribution in intensity in the spectral diffraction pattern of a point or line at the focal plane of an achromatic telescope of rectangular aperture the expression

$$I_s^2=\int_{-\infty}^{+\infty}k_\lambda\,\psi(\lambda)\,L(\lambda)\,(I_r^2)\,d\lambda\ , \tag{49}$$

where

$$I_r^2=A\,\frac{\sin^2\dfrac{\pi}{a_0}(a-\phi)}{\left[\dfrac{\pi}{a_0}(a-\phi)\right]^2}\cdot\frac{d^2}{f^2}\,\frac{\sin^2\dfrac{\pi b}{\lambda}(a-D\Delta\lambda)}{\pi^2(a-D\Delta\lambda)}\ . \tag{50}$$

from (5).[1]

In the evaluating of the integral I_s^2 of (49) and (50) we have to distinguish several cases:

1. When there is no chromatic dispersion, $D=0$, and if the different wave-fronts all fall concentrically on the diffracting aperture, the centers of the different spectral images will coin-

[1] See "General Theory of Telescopic Images for different forms of Radiating Sources," ASTROPHYSICAL JOURNAL, 6, 124; §§ A (b) and A (c).

cide, or more strictly, will all fall on the optical axis passing through the center of the geometrical image. Since each individual image is a symmetrical function of λ, the diffraction pattern resulting from their centrical superposition will also be symmetrical, and the result of the residual chromatic effects, (b) and (c), as well as any outstanding longitudinal chromatic aberration will be to change the *form*,[1] but not the position of the center of intensity of the physical image. This effect, like that of A (1), will not therefore affect the accuracy of measurement of positions.

2. When the chromatic dispersion is small the superposition of the different spectral images will not be quite exact, but the continuity of the image will be unbroken; *i. e.*, the chromatic resolution will not be sufficient to enable us to isolate or deal with any individual line or region of the spectrum. Under such circumstances there is considerable uncertainty as to just what part of the image corresponds in position to the geometrical center, and the error of measurement involved in the determination of the position of the source itself may be very appreciable. Such a case as is here supposed presents itself in micrometric and heliometric measurements of the relative positions of stars and small planets at low altitudes. In every position except exactly in the zenith (and even there under certain conditions of barometric pressure) the Earth's atmosphere acts as a prism, which not only results in the refraction, but also in the dispersion of the light coming from the star or planet. The necessity of considering the dispersion effect was early pointed out by Lee,[2] and was afterwards more fully discussed by Rambaut[3] and Gill[4], who do not, however, fully agree as to the magnitude of the effect involved. Their disagreement can be explained in

[1] As has already been indicated this change in *form* may fortunately be utilized as the basis of a possible method of measuring stellar temperatures. *Loc. cit.*, § A, p. 125.

[2] *Phil. Trans.*, 1815.

[3] "Effect of Atmospheric Dispersion on the Position of a Star." *M. N.*, 45, 123–145; *ibid.*, 48, 256–280.

[4] "Effect of Chromatic Dispersion of the Atmosphere on Parallaxes." *Ibid.*, 48, 53–76; *ibid.*, 415–425.

part by their failure to consider fully the effect of the factors k_λ, $\psi(\lambda)$ and $L(\lambda)$ of (*b*) and (*c*) above, which are quite as important in determining the apparent center of the image of the star as is the term $\phi = D\Delta\lambda$ which represents the atmospheric dispersion. Thus Rambaut shows that certain systematic differences in the measurement of the angular separation of the components of β *Cygni*, made at different zenith distances, may be explained if we take into account the factor $D\Delta\lambda = D(\lambda_0 - \lambda)$ only, by supposing that the value of λ_0 for the two stars (or, as he puts it, the mean refrangibility) differs by about 250 tenth-meters. Differences of this amount are easily possible between stars of such different types as the two components of the star in question, but the change in apparent separation of their images due to this difference in mean wave-length of maximum intensity is considerably less than Dr. Rambaut assumes, owing to the influence of the omitted factors k_λ, $\psi(\lambda)$, and $L(\lambda)$, in determining the apparent center of the image when the latter is observed either visually or photographically. This is very fortunate, since otherwise, as Dr. Gill points out, observations of parallactic displacement would be affected by errors or rather uncertain corrections of so great a magnitude as seriously to reduce the accuracy of such work. At the same time there remains a true apparent displacement of the center of intensity of the image which is, in many cases, greater than the limiting error of measurement ϵ. Whether this displacement of the *center of intensity* affects heliometer measurements to the same extent that it affects micrometer observations is, as Dr. Gill points out, a question that can be settled only by individual experiment.

The whole question is of vital interest not only in connection with parallax and double-star work, but also in connection with meridian work and almucantur observations and (under certain conditions as already indicated) zenith telescope measurements. I have therefore evaluated the integral (49) for a number of values of λ_0 and under a number of assumptions as to the form of the functions k_λ, $\psi(\lambda)$, and $L(\lambda)$ for both visual and photographic telescopes. This work would occupy too great a portion of the present paper and I will therefore present it in a separate communication.

3. When $\frac{d\theta}{d\lambda}$ is large, as in the case of the spectroscope, we no longer have to deal in general with the whole spectral image of the source, but only with individual portions of it, *i. e.*, with individual spectral lines. In practically all cases the effective "spectral width" $\Delta\lambda$ of individual lines is so narrow that over this range the dispersion coefficient $D=\frac{d\theta}{d\lambda}$ may be regarded as constant, and for the same reason the value of λ itself may be considered as constant with respect to the integration of (49) in the functions k_λ, $L(\lambda)$ and in the denominator of (50). Under such circumstances the expression for the distribution in the intensity in the image of an individual spectral line becomes

$$I_1^2 = A' \int_{-\infty}^{+\infty} \psi(\phi) \frac{\sin^2 \frac{\pi}{a_0}(a-\phi)}{\left[\frac{\pi}{a_0}(a-\phi)\right]^2} d\phi = A'' \psi_1(\psi, a_0, a) \,, \tag{51}$$

or if the source (in this case the slit of the spectroscope) has a finite width[1] σ_0,

$$I_2^2 = A'' \int_{-\frac{\sigma_0}{2}}^{+\frac{\sigma_0}{2}} \psi_1 \{\xi - \psi, a_0, a,\} \, d\xi \,. \tag{52}$$

In (51) and (52) $\psi(\phi)=\psi(\lambda)$ represents the spectral law of radiation not for the source as a whole, but for any individual element. In a normal source whose mean wave-length is λ_0 the distribution $\psi(\lambda)$ is generally assumed to be represented by the law

$$\psi(\lambda) = e^{-k(\lambda_0-\lambda)^2} \,, \tag{53}$$

which is symmetrical in form. For such sources I_1 is symmetrical and I_2 is therefore also symmetrical. This case has already been considered.[2]

When $\psi(\lambda)$ is unsymmetrical, as is generally the case when the source of radiation is subjected to the effect of unusual pressure or temperature conditions, or to abnormal magnetic or electrical disturbances, the form of I_2^2 and I_2^2 likewise become

[1] See *Phil. Mag.*, **43**, 330, 338; also ASTROPHYSICAL JOURNAL, **3**, 336.

[2] *Loc. cit.*, p. 330ff.

asymmetrical. Unfortunately our knowledge of the relations between the physical conditions of molecular and atomic vibrations and the resultant intensity of radiation are as yet too meager and unsatisfactory to enable us to express $\psi(\lambda)$ in definite mathematical form in such cases. We can only consider and endeavor to allow for the general effect produced, and this will be to render the measurement of the mean wave-length, λ_o, of a given line indeterminate, to a degree depending on physiological and psychological causes rather than on physical ones. That is, different observers will differ among themselves as to the setting of a cross-wire on the mean center of intensity of such an asymmetrically broadened image, but aside from this, this kind of asymmetry cannot be said to be a real source of error in the measurement of wave-lengths, since the latter can only be considered in reference to the line itself, and not, at least in the present state of our knowledge, in reference to the free or natural period of the vibrations which have produced it[1].

[1] In this connection see papers by JAUMANN, "Zur Kenntniss des Ablaufes der Lichtemission." *Wied. Ann.*, **53**, 832; **54**, 178; and by GALITZIN, "Zur Theorie der Verbreiterung der Spectrallinien," *ibid.*, **56**, 78. Further developments of the theory of radiation along the lines outlined in these papers seem probable.

(*To be concluded.*)

RESEARCHES ON THE ARC SPECTRA OF THE METALS.

VI. SPECTRUM OF MOLYBDENUM.[1]

By B. Hasselberg.

INTRODUCTION.

The comprehensive classical investigation of the solar spectrum by Rowland has not only furnished to spectroscopic research of our times a foundation of a relative accuracy hitherto unattained and to all appearances hardly to be surpassed for a long time, but has also by the simultaneous study of the spectra of the metals brought the question of the chemical constitution of the Sun very much nearer to a solution. Although our knowledge as to the source of the Fraunhofer lines has been decidedly increased, Rowland's catalogue of wave-lengths nevertheless itself gives the best testimony as to how much remains for research in this field before the chemistry of the Sun, and consequently that of the stars, can be brought to even a provisional conclusion. Of the seventy odd chemical elements, only about a half are represented in the absorbing envelope of the Sun, while a majority of the remainder have not permitted any identification, and a few in only a doubtful manner. There is, however, no reason to regard this condition of things as a permanent one, for continued researches in the field of pure spectroscopy will doubtless lead to a considerable extension of what has already been attained, particularly as Rowland's investigations of the spectra of the metals are certainly not to be regarded as equal in completeness to his researches on the solar spectrum, and further, the spectroscopic peculiarities of entire groups of metals, particularly the so-called rare elements, have been hitherto practically wholly unknown.

In this category of spectroscopically neglected metals belong the three elements, molybdenum, tungsten and uranium, which

[1] *Kongl. Svenska Vetenskaps-Akademiens Handlingar*, 36, No. 2, 1902.

on the basis of their chemical properties are included in the iron group. Rowland records the first of these as certainly present in the solar atmosphere, but the other two as doubtful. This statement, to which I shall recur below, is the more remarkable since the remaining members of the iron group—iron, nickel, cobalt, chromium, and manganese—are just the metals that appear to be the most numerously represented in the general solar spectrum. In order to obtain more definite information as to these conditions, which, moreover, also occur in the case of other groups of metals, and in order to complete my former investigations on the spectroscopy of the iron metals, I have considered a thorough investigation of the arc spectra of the above-named metals as particularly desirable, and this especially since these spectra have hitherto remained entirely unknown, aside from the small region between λ 3900 and λ 4000 studied by Lockyer.[1] The fact that the corresponding spark spectra have been investigated, first in general outline thirty years ago by Thalén,[2] and then for the violet and ultra-violet portions in a very commendable manner by Exner and Haschek,[3] and for uranium recently by Lohse,[4] can in my opinion only lend an increased interest to such an undertaking. Now that these investigations have been concluded within the limits prescribed by my apparatus and also by the present needs of astrophysics, for molebdenum and have been carried out to a considerable extent for tungsten, it seems a proper time to communicate the results obtained, and I therefore beg to lay before spectroscopists in the following pages the results for the first-named metal.

In my previous papers I have fully described the instrumental equipment employed, the method of obtaining and measuring the photographic plates as well as of eliminating foreign lines, and, as no important changes have been made in these respects, I may therefore omit further discussion of this matter. In reply to a suspicion recently expressed[5] that the grating I have used

[1] WATTS' *Index of Spectra*, 1889. [2] *Nova Acta Reg. Soc. Scient.* Upsala, 1868.

[3] *Wien. Sitz.-Ber.*, Mathem. naturwiss. Classe, **104**, 1895; **105**, 1896; **106**, 1897; **107**, 1898.

[4] *Berlin. Sitz.-Ber.*, 1897.

[5] *Wien. Sitz.-Ber.*, Mathem.-naturw. Classe, **110**, 986, 1901.

had a very slight dispersion, I would remark here that this grating, which Professor Rowland at the time had the kindness to select for me as the best among a considerable number, has a ruled surface of 8×5 sq. cm, with 14,438 lines to the inch, and that it constitutes in connection with an excellent Steinheil telescope of 85 mm aperture and 1.6 m focus as collimator and with a camera having an objective of the same dimensions, a spectrograph which in respect to its optical power is only surpassed by the large Rowland concave grating. For this reason I have hitherto employed this apparatus exclusively for my investigations of the arc spectra; but I have also recently mounted a large concave grating, of 6.3 m radius and 20,000 lines to the inch, in order to secure the necessary efficiency in the extreme ultra-violet, which cannot be reached by the present spectrograph on account of the absorption in the glass. Judging by the provisional tests of this grating, I have every reason to expect the best results from its use.

ELIMINATION OF FOREIGN LINES.

In my first attempt to obtain the arc spectrum of molybdenum I introduced fragments of molybdenum sulphide into the arc, at the suggestion of Baron Nordenskiöld, in the hope of thus obtaining the purest possible spectrum of molybdenum, since the mineral in question was said to contain in addition to molybdenum only sulphur, which is known to yield no spectrum in the arc. But my expectation was not confirmed, for a superficial examination of the plates exhibited so serious a contamination by foreign lines that the further use of this material had to be given up. I substituted for it pure metallic molybdenum from the chemical works of Merck in Darmstadt, and thus obtained a spectrum of decidedly greater purity, although in this case, as in all similar ones, chemistry is still far from being able to supply a spectroscopically pure preparation. The number of foreign lines therefore still remains not inconsiderable, in spite of the pains taken to suppress them in a spectrum having so many lines as the present one, but I nevertheless hope that most of the impurities have been removed from the catalogue of lines to be

given below. The comparisons made with previously known arc spectra, for accomplishing this elimination, give the following results :

MOLYBDENUM AND IRON.

On the first examination of the plates the majority of the easily recognizable iron lines were excluded from the provisional list of wave-lengths for molybdenum. The comparison with the iron spectrum of Kayser and Runge after the completion of the definitive catalogue yielded a considerable number of instances of approximate coincidence, the reality or unreality of which were tested, line for line, by the accurate investigation of plates exposed to the two spectra especially for this purpose. The results of this investigation are shown in the following table :

Mo λ	*Mo* *i*	*Fe* λ	*Fe* *i*	Remarks
3466.98	2	3467.05	5	Separated. λ *Mo* < λ *Fe*
81.95	1.2	81.94	6	*Fe* line lacking
3493.49	2−	3493.44	6	Separated. λ *Mo* > λ *Fe*
3504.55	2.3	3504.56	6	*Fe* line lacking.
24.35	2+	24.38	5	Separated. λ *Mo* < λ *Fe*
26.08	1+	26.12	5	Separated
52.57	1	52.62	6	Probably λ *Mo* < λ *Fe*. Rowland has { 52.57 .69
54.35	2−	54.28	4	Separated. λ *Mo* > λ *Fe*
66.57	1	66.50	5	Separated
71.42	1.2	71.38	5	Separated. λ *Mo* > λ *Fe*
74.05	2.3	74.04	3	Coinc. Other *Fe* lines in the vicinity are lacking in *Mo*
82.03	3.4	81.98	6	Separated. *Mo* line dupl.
90.90	2+	90.84	6	Coinc. or perhaps λ *Mo* < λ *Fe*
91.55	1.2	91.52	6	Separated. λ *Mo* > λ *Fe*. Rowland has { 91.50 .63
94.73	1	94.75	3	Separated. λ *Mo* < λ *Fe*
3595.87	2	3395.82	6	*Fe* line lacking
3603.86	1.2	3603.84	6	*Fe* line lacking
10.80	1+	10.87	3	Separated. *Mo* line exceedingly faint
14.87	1	14.79	6	Coinc.? *Fe* line diffuse
23.36	2+	23.34	3	Separated. λ *Mo* > λ *Fe*. Rowland gives 23.36
26.33	2.3	26.31	6	Coinc.
55.21	1.2	55.13	6	Separated. λ *Mo* > λ *Fe*
63.14	2	63.05	6	*Fe* line lacking
77.83	2−	77.78	4	No coinc. λ *Mo* > λ *Fe*
81.88	2−	81.80	6	*Fe* line lacking
86.72	2−	86.66	6	*Fe* line lacking
3690.30	1	3690.24	6	*Fe* line lacking
3702.67	2	3702.65	6	Separated. λ *Mo* > λ *Fe*. Rowland gives λ *Fe* 02.63
08.73	2−	08.74	6	Separated. λ *Mo* < λ *Fe*
18.66	1+	18.57	4	Separated. λ *Mo* > λ *Fe*
27.86	3	27.80	3	Separated. Δλ=0.09. Rowland has 27.78
3796.19	1.2	3796.14	6	*Fe* line lacking

Mo λ	Mo i	Fe λ	Fe i	Remarks
3801.14	1	3801.18	5	Trace of *Fe*, but not observable
02.00	2.3	01.95	6	Coinc.? λ *Mo* Δ λ *Fe*? Rowland gives λ *Fe* 01.95
06.15	2	06.15	6	*Fe* line lacking
10.99	1	10.92	4	Separated. λ *Mo* > λ *Fe*. Rowland has 10.90 *Fe*
14.64	1+	14.69	4	Widely separated
18.83	2−	18.80	6	*Fe* line lacking
21.82	1	21.75	6	*Fe* line lacking
24.34	1.2	24.27	6	Separated. λ *Mo* > λ *Fe*
25.63	1+	25.57	6	Coinc.? Perhaps λ *Mo* > λ *Fe*
29.04	3−	29.05	6	*Fe* line lacking
29.95	1.2	29.89	5	Not separable. Rowland gives ⊙ 29.91 but no [*Fe* line
30.98	2−	30.98	5	*Mo* line between two *Fe* lines
44.09	1.2	44.11	6	Separated. λ *Mo* < *Fe*
48.45	2+	48.45	6	Separated. λ *Mo* < *Fe*
64.25	10	64.19	6	Coinc.? *i Fe* > 6
3893.50	1+	3893.50	4	Separated. Rowland has 93.54
3903.07	10	3903.11	2	Coinc.? λ *Mo* ≤ λ *Fe*?. Both lines reversed
71.54	1.2	71.46	3	Separated. λ *Mo* > λ *Fe*
73.10	1+	73.05	6	Separated. λ *Mo* > λ *Fe*
3974.09	2	74.15	6	Separated
4000.67	2	4000.65	5	Separated. Rowland gives ⊙ 00.61 *Fe*. This line lies between *Mo* 00.67 and 00.55
06.85	1	06.79	5	Separated
20.59	1.2	20.62	6	Separated. λ *Mo* < *Fe*
32.65	1.2	32.62	6	*Fe* line uncertain
56.18	2	56.12	6	Coinc.? Trace of *Fe* line. *Mo* line between 56.22 *Cr* and 56.13 *Fe*
76.35	2	76.40	6	Separated. λ *Mo* < λ*Fe*. R. has ⊙ 76.38 *Fe*
86.16	2	86.14	6	Coinc.?
4093.32	1+	4093.36	6	*Fe* line lacking
4105.27	2+	4105.35	6	*Fe* line lacking. Rowland has ⊙ 05.32. The lines are separated
07.63	3	07.65	2	Separated. λ *Mo* < λ *Fe*
08.30	1.2	08.30	6	Coinc.? *Fe* line faint and diffuse
10.46	1	10.48	6	*Fe* line lacking
15.08	2−	15.05	5	Coinc.? *Fe* line diffuse. Rowland has ⊙ 15.09
19.12	2	19.07	5	Separated. λ *Mo* > λ *Fe*
23.83	2	23.88	4	Widely separated. Rowland λ *Fe* = 23.91
29.02	2−	28.98	6	?
35.55	1+	35.50	6	*Fe* line lacking
37.10	1+	37.13	2	Widely separated. Rowland λ *Fe* = 37.16
52.07	2−	52.11	5	Separated. Δλ > 0.04
57.59	2.3	57.53	6	*Fe* line lacking
60.44	1+	60.38	6	*Fe* line lacking
70.01	2−	69.97	6	Coinc.? *Fe* line diffuse
70.55	1	70.49	6	*Fe* line lacking
78.72	1	78.71	6	Perhaps separated and λ *Mo* > λ *Fe*
80.69	1	80.67	6	*Fe* line lacking
81.24	2	81.23	6	*Mo* line dupl. — *Fe* line coincides with the violet component
84.33	1	84.38	6	*Fe* line lacking
4190.17	2−	4190.14	6	Trace of *Fe*. λ *Mo* > λ *Fe*? K. and R. assign the line to *Mn*
4200.02	1	4200.03	6	*Fe* line lacking

Mo		*Fe*		Remarks
λ	*i*	λ	*i*	
4201.35	1+	4201.33	6	*Fe* line lacking
42.97	1.2	42.87	5	Separated
46.19	2.3	46.20	4	Separated
58.85	1+	58.77	5	*Fe* line concealed by a ghost
73.23	1.2	73.18	6	*Fe* line lacking
4277.38	3	4277.36	6	Trace of *Fe*. Coinc.?
4310.58	2−	4310.57	6	Separated. λ *Mo* > λ *Fe*
24.72	1	24.71	6	*Fe* line lacking
50.53	3−	50.48	6	*Fe* line lacking
69.23	2.3	69.23	6	*Fe* line lacking
75.07	1.2	75.11	6	*Fe* line lacking. K. and R. assign the line to *Mn*
75.21	1.2			
91.71	1.2	91.73	6	Separated.? The *Fe* line belongs to *Co*
4396.83	2	4396.83	6	Trace of *Fe* line. Coinc.?
4403.07	2+	4403.02	6	*Fe* line not visible, concealed by a ghost.
24.40	1	24.33	6	Separated
26.86	2.3	26.81	6	Separated. Δλ > 0.05
37.06	2−	37.11	5	Separated. λ *Mo* < λ *Fe*
44.21	1+	44.22	6	*Fe* line lacking
46.62	3−	46.54	6	*Fe* line concealed by a ghost
64.96	3−	64.95	4	*Mo* line on the red edge of the *Fe* line
68.46	3	68.51	6	*Fe* line lacking
4489.17	1.2	4489.15	6	Separated. λ *Mo* > λ *Fe*
4515.36	2−	4515.43	6	Separated
18.61	1+	18.69	6	Separated
28.77	2.3	28.85	1	Probably separated; lines broad and hard to separate
35.00	2+	35.01	6	*Fe* line lacking
60.32	2+	60.33	5	Coinc.? *Mo* line sharp, *Fe* line diffuse
79.92	1	80.00	6	Separated
4582.52	1+	4582.58	6	*Fe* line lacking
4611.36	2−	4611.45	2	Widely separated
26.67	3.4	20.72	6	Separated. λ *Mo* < λ *Fe*
27.70	2.3	27.72	6	Probably separated
30.20	2−	30.29	4	Widely separated
41.12	1−	41.19	6	Separated. λ *Mo* < λ *Fe*
51.25	2+	51.34	4	Widely separated
62.11	2.3	62.16	5	Separated. λ *Mo* < λ *Fe*
63.31	1+	63.32	6	Separated. λ *Mo* < λ *Fe*
4688.41	2.3	4688.46	6	Separated. λ *Mo* < λ *Fe*. *Fe* line broad, diffuse
4707.44	3.4	4707.52	2	Separated. λ *Mo* < λ *Fe*
31.64	3.4	31.67	6	Separated. λ *Mo* < λ *Fe*
34.34	1+	34.32	6	Separated. λ *Mo* > λ *Fe*
40.58	1	40.55	6	Separated. λ *Mo* > λ *Fe*
4749.61	1−	4749.56	6	*Fe* line lacking
4808.29	2−	4808.32	6	Separated
11.28	2.3	11.29	6	Coinc. Belongs to *Mo*
17.92	2−	17.97	6	Separated
4860.99	1	4860.99	6	*Fe* line lacking
4957.78	3	4957.87	2	Separated. Rowland gives for the *Fe* line 57.78, probably should be 57.88.
4964.63	2−	4964.72	6	Separated
5047.90	2	5047.92	6	*Fe* line lacking
5109.90	2+	5109.82	6	Separated

Mo λ	*i*	*Fe* λ	*i*	Remarks
5115.86	1−	5115.94	6	*Fe* line lacking
5232.58	1−	5232.55	6	*Fe* line lacking
5394.75	2	5394.81	6	Separated. λ *Mo* < λ *Fe*
5426.24	1	5426.21	6	Separated. λ *Mo* > λ *Fe*
5533.26	6	5533.17	5	Separated
34.85	1	34.94	6	*Fe* line lacking
5543.38	2+	5543.41	4	Coinc.? *Mo* line sharp. *Fe* line diffuse
5651.54	1+	5651.60	6	*Fe* line lacking
5672.35	1+	5672.39	6	*Fe* line lacking
5707.26	1.2	5707.22	5	Coinc.? Belongs to *V*
08.28	1+	08.32	5	Separated. λ *Mo* < λ *Fe*
12.05	2	12.09	5	Separated. λ *Mo* < λ *Fe.* K. and R. assign the line to *Ti*
27.30	2	27.27	6	Coinc.? Perhaps λ *Mo* > *Fe*
5771.33	1+	5771.35	6	*Fe* line lacking
5849.16	1.2	5849.14	6	*Fe* line lacking

In my former comparisons with the iron spectrum of Kayser and Runge I added the correction +0.07 tenth-meters in order to render their wave-lengths, which are based on Bell's earlier determinations for D_1, comparable with my measures, which are referred to the later system of Rowland. Nevertheless, I had at that time repeated occasion to doubt the accuracy of this correction, and this suspicion has been fully confirmed by a comparison of Kayser's recently issued list of standards in the arc spectrum of iron with the former measurements. If we group the differences appearing in such a comparison in sections of 100 tenth-meters and take the means, we obtain the following values of the correction:

λ3400 −	λ3500	Δλ = +0.07 tenth-meters
3500 −	3600	.04
3600 −	3700	.01
3700 −	3800	.02
3800 −	3900	.03
3900 −	4000	.05
4000 −	4100	.08
4100 −	4200	.07
4200 −	4300	.02
4300 −	4400	.05
4400 −	4500	.06

We see that the correction varies considerably in the different parts of the spectrum. On applying these new values to the

earlier measures of Kayser and Runge I was able to bring them into a far better agreement with my observations than formerly. Thus, for instance, cases occurred where at a very close coincidence of an iron line with a line of a different metal the use of the former correction led actually to a precisely opposite relative position of the two lines — the iron line being thus transposed, say, to the red edge of the other line, while on the double exposure of the two spectra just the reverse position could be distinctly observed. With the aid of the above corrections such contradictions have been disposed of almost without exception. On account of the small differences which still exist between the Kayser-Rowland iron standard lines, on the one hand, and the list of solar lines by Rowland, on the other, the comparison of my observations of the molybdenum lines with Kayser and Runge's iron lines cannot be carried out with the greatest rigor, but this is, however, sufficiently sharp for the portion of spectrum under consideration to render the results of the above table adequately reliable. For the remaining portion of the spectrum, between D and $\lambda 4500$, I have, of course, been obliged to employ the old correction.

We see from the table that in the great majority of approximate coincidences between the lines of iron and molybdenum the former are either lacking in my spectrum, or, if present, are clearly separated from the latter and consequently are independent. I would therefore express no definite opinion at present whether this absence of a number of the faintest lines of Kayser and Runge's iron spectrum is solely the consequence of a lower intensity of current employed by me, or indicates a greater purity of my sample of iron; but a certain probability is lent to the latter supposition by the fact that a quantity of impurities are actually present in the iron spectrum of Kayser and Runge. Moreover, since the cases are very rare in which a coincidence could be demonstrated without ambiguity between lines of molybdenum and the Sun, it was to be expected that the approximate coincidences with iron lines would in many cases prove only apparent, while, on the other hand, in the comparatively few doubtful cases such an explanation gains in proba-

bility. This is still more the case, since in such more or less certain coincidences the lines show a different character, those of molybdenum being always sharp, while those of iron are frequently diffuse.

MOLYBDENUM AND CHROMIUM.

The following table contains the approximate coincidences between lines of molybdenum and of chromium obtained by a comparison with my former measures of the latter spectrum; as will be seen, the investigation of the double exposures has shown these coincidences to be only apparent, almost without exception:

Mo λ	*Mo* *i*	*Cr* λ	*Cr* *i*	Remarks
3548.88	1	3548.95	1.2	Separated. *Cr* line very diffuse
64.45	1.2	64.44	1	Coinc. Belongs to *Mo*
66.20	2+	66.23	2.3	Coinc.? *Cr* line diffuse
3584.42	1.2	3584.45	2.3	Separated? λ *Mo* < λ *Cr*. *Cr* line diffuse
3603.86	1.2	3603.86	2	Coinc.? *Cr* line diffuse
08.52	2+	08.52	1.2	Coinc.
12.62	2—	12.70	1.2	Separated
13.80	1.2	13.78	1.2	λ *Mo* < λ *Co*. *Cr* line diffuse
81.88	2—	3681.81	1.2	λ *Mo* > λ *Cr*
43.98	1+	44.01	3	λ *Mo* < λ *Cr*
44.55	2—	44.63	2	Separated
47.37	2—	47.40	1.2	λ *Mo* < λ *Cr*
48.66	2+	48.73	2	
56.02	1.2	55.97	1	Coinc. Belongs to *Mo*?
68.78	1+	68.85	2	Separated. The *Cr* line coincides with *Mo* 68.92 ∴ λ *Cr* too small
3797.20	1.2	3797.28	2	
3808.04	1	3808.06	2	λ *Mo* < λ *Cr*
22.14	1	22.22	1	
25.50	1.2	25.54	2	The *Cr* line is between *Mo* 25.63 and 25.50
30.22	2—	30.17	3.4	Separated λ *Mo* > λ *Cr*
34.82	1.2	34.88	2.3	
3886.98	2.3	3886.94	3	λ *Mo* > λ *Cr*
3903.07	10	3903.02	3	λ *Mo* > λ *Cr*. Rowland gives 03.09 *Fe*, *Cr*
17.09	2—	17.15	1—	λ *Mo* < λ *Cr*
20.25	1—	20.25	1.2	Coinc.? Belongs to *Co*?
28.86	1.2	28.79	4	Separated. λ *Mo* > λ *Cr*
69.17	1	69.20	2	Separated. λ *Mo* < λ *Cr*
3994.06	2—	3994.10	1.2	Separated. λ *Mo* < λ *Cr*
4025.64	1.2	4025.60	1	Separated. λ *Mo* > λ *Cr*
43.91	1.2	43.85	1.2	Separated. λ *Mo* > λ *Cr*
56.18	2—	56.17	1.2	Separated. λ *Mo* < λ *Cr*. λ *Mo*=56.22 according to Rowland
67.88	1+	67.94	1	Widely separated

Mo λ	*i*	*Cr* λ	*i*	Remarks
4081.94	2—	4081.88	1	Coinc. λ *Cr* erroneous
4142.28	1+	4142.31	1.2	Separated. λ *Mo* < λ Cr
75.32	1	75.34	1.2	
4186.47	2—	4186.50	1.2	
4284.77	3	4284.84	1.2	Widely separated
4338.90	2—	4338.95	1.2	Separated. λ *Mo* < λ *Cr*
63.21	1—	63.25	2.3	λ *Mo* < λ *Cr*
76.87	1+	76.95	2	λ *Mo* < λ *Cr*
80.80	1.2	80.73	1	λ *Mo* > λ *Cr*
4397.48	2+	4397.40	2	λ *Mo* > λ *Cr*
4424.40	1	4424.40	2.3	Separated. λ *Mo* < λ *Cr*. $\Delta\lambda=0.07$. λ *Cr* erroneous; Rowland gives 24.46
42.37	2.3	42.43	1.2	Separated. λ *Mo* < λ *Cr*
4458.84	1.2	4458.75	2.3	Widely separated
4526.56	2+	4526.65	4	Widely separated
41.75	2—	41.70	2	Separated. λ *Mo* > λ *Cr*
86.25	1+	86.31	2	Separated. λ *Mo* < λ *Cr*
4588.33	1.2	4588.38	1.2	Separated. λ *Mo* < λ *Cr*
4610.07	3	4610.07	1.2	Coinc. Belongs to *Mo*
14.94	1	14.92	1.2	Separated? λ *Mo* > λ *Cr*
48.02	2	48.00	1.2	Separated. λ *Mo* < λ *Cr*
56.57	1	56.61	1.2	Separated. λ *Mo* < λ *Cr*
4673.24	1	4673.30	1.2	Widely separated
4700.71	2—	4700.77	2	?
06.25	2	06.25	1.2	Coinc. Belongs to *Mo*
23.27	1.2	23.28	2.3	Separated. λ *Mo* < λ *Cr*. $\Delta\lambda$ exceedingly small
4783.16	2.3	4783.16	1.2	Separated. λ *Mo* < λ *Cr*. *Cr* line diffuse

MOLYBDENUM AND COBALT.

As in the previous cases, the lines of approximately equal wave-lengths are here again practically wholly independent of each other, as is seen from the following table. In a few cases where a separation could not be effected, the molybdenum line is commonly too strong to be considered as impurity. In the region from D to F there occurs only a single pair of lines whose components differ from each other by less than 0.05 tenth-meters, namely, *Mo* 5431.27 and *Co* 5431.30, and this pair was not specially investigated.

Mo λ	*i*	*Co* λ	*i*	Remarks
3562.26	2—	3562.22	2.3	Somewhat separated. λ *Mo* > λ *Co*
3582.03	3.4	3582.00	2	Coinc.
3605.19	1+	3605.19	2	Coinc. Foreign Line?
08.52	2+	08.50	1.2	Separated. λ *Mo* > λ *Co*
39.71	1+	39.63	2.3	Widely separated
51.48	2	51.42	2	Widely separated
86.72	2—	86.63	1.2	Widely separated
3693.52	2—	3693.53	1.2	Coinc. Foreign Line?
3702.33	1+	3702.40	2.3	Separated λ *Mo* < λ *Co*
33.59	1+	33.62	2.3	Separated λ *Mo* < λ *Co*
3759.80	1.2	3759.83	1.2	Separated λ *Mo* < λ *Co*
3812.63	2	3812.57	1.2	Separated λ *Mo* > λ *Co*
14.64	1+	14.58	2	Coinc. ?
19.98	2.3	20.02	2	λ *Mo* < λ *Co*
66.87	1+	66.92	1	λ *Mo* < λ *Co*
73.30	1.2	73.25	4.5	λ *Mo* > λ *Co*
3893.50	1+	3893.44	1.2	Separated. λ *Mo* > λ *Ci*
3945.41	2—	3945.47	3	
47.33	2—	47.28	1.2	λ *Mo* > λ *Co*
66.52	2—	66.52	3	Coinc. *Mo* line too strong to come from *Co*
69.17	1	69.25	2.3	
3976.35	2	3976.28	2.3	?
4076.69	1.2	4076.74	2	Separated. λ *Mo* < λ *Co*
4207.75	1+	4207.77	1.2	Separated. λ *Mo* < λ *Co*
68.25	2—	68.18	1.2	λ *Mo* > λ *Co*
4292.34	3	4292.41	2	
4375.07	1+	4375.09	2.3	Separated λ *Mo* < λ *Co*
4391.71	1.2	4391.70	3	λ *Mo* < λ *Co*=91.74?
4490.37	2	4490.46	1.2	
4492.24	1	4492.23	1.2	λ *Mo* < λ *Co*
4517.30	3+	4517.28	3.4	Coinc. Impossible to separate. Rowland gives 17.32
4553.52	1.2	4553.51	1.2	λ *Mo* < λ *Co*
4651.25	2—	4651.28	1.2	λ *Mo* < λ *Co*
82.44	1—	82.53	4	
4686.01	2—	4686.05	1.2	Separated. λ *Mo* < λ *Co*
4776.54	3	4776.49	3.4	λ *Mo* > λ *Co*
85.34	2.3	85.26	2.3	*i Co* < 2.3
4792.96	2	4793.03	4	

MOLYBDENUM AND NICKEL.

The comparison of the spectra of these two elements has yielded only a small number of approximate coincidences, as appears from the following table. Since most of these are not real we may consider the molybdenum to be practically free from nickel.

Mo λ	*i*	*Ni* λ	*i*	Remarks
3533.87	2	33.89	1	*Cu* 33.90
3566.57	1	66.50	4.5	These lines have not been specially investigated
3793.75	2−	93.75	3	
3994.06	2−	94.13	2	
4006.23	2	06.30	2	
4075.72	2−	75.75	1.2	
4138.72	1.2	38.67	1	
4184.59	1.2	84.65	1.2	
4284.77	3	84.83	2.3	Widely separated
4325.44	1	25.49	1.2	Separated? $\lambda Mo > \lambda Ni$?
4423.24	1+	23.24	1.2	Coinc.
4553.40	1.2	53.37	1.2	Separated. $\lambda Mo > \lambda Ni$
4567.57	1−	67.59	1	Separated?
4618.15	1+	18.22	1.2	Coinc. Impurity?
4786.67	2	86.66	3	Rowland gives ⊙ 86.73 *Ni*. Separated
4792.96	2	92.98	1	Separated. $\lambda Mo < \lambda Ni$
4817.92	2−	17.97	1	Widely separated
5058.30	1	58.22	1	Separated
5080.23	2.3	80.16	1.2	Separated
5642.05	1−	42.08	1.2	Separated. $\lambda Mo < \lambda Ni$

MOLYBDENUM AND MANGANESE.

The following table shows that here also there were only a few isolated cases in which a separation of close pairs of lines in

Mo λ	*i*	*Mn* λ	*i*	Remarks
3669.50	2.3	3669.54	1	Separated. $\lambda Mo < \lambda Mn$
3680.36	1+	3680.32	1	Separated. $\lambda Mo > \lambda Mn$
3763.52	2	3763.51	2−	Coinc. ?
67.90	1+	67.84	2−	Separated. $\lambda Mo > \lambda Mn$
3776.73	1+	3776.70	1.2	Probably separated and $\lambda Mo > \lambda Mn$
3830.08	1+	3830.12	1+	Separated. $\lambda Mo < \lambda Mn$.
33.92	3	33.96	3.4	Separated. $\lambda Mo < \lambda Mn$.
44.09	1.2	44.10	3.4	Separated. $\lambda Mo < \lambda Mn$. $\Delta\lambda > 0.01$
3896.55	1.2	3896.48	1.2	Separated
3908.42	1.2	3908.34	1	Widely separated
11.24	1.2	11.27	1.2	Separated. $\lambda Mo < \lambda Mn$
3936.89	1+	3936.91	1+	Coinc. Rowland gives *Mn*
4008.21	1+	4008.19	1.2	Perhaps $\lambda Mo > \lambda Mn$
4075.43	2	4075.39	1.2	Coinc.
4132.41	2−	4132.45	1	Separated ? *Mo* line faint. ∴ *i Mo* var. Really *Mo*?
4148.88	1+	4148.94	2.3	Widely separated
4235.23	1.2	4235.28	3	Separated. $\lambda Mo < \lambda Mn$
4375.07	1+	4375.10	2	$\lambda Mo < \lambda Mn$
4381.82	4	4381.87	2−	$\lambda Mo < \lambda Mn$ *Mn* line lies between *Mo*: 75.21, .07
				$\lambda Mo < \lambda Mn$
4452.77	1.2	4452.73	1.2	$\lambda Mo > \lambda Mn$
4586.25	1+	4586.30	1+	Separated
4626.67	3.4	4626.74	2	
4838.35	1	4838.40	1	

the two spectra could not be effected. In view of the not inconsiderable intensity of these lines the assumption of an impurity by a third metal seems hardly probable.

MOLYBDENUM AND TITANIUM.

In the region of spectrum from D to F there is but a single pair whose components have very nearly the same wave-length, namely, *Mo* 5712.05 and *Ti* 5712.07. These are, however, independent of each other, since the *Mo* line lies at the edge of the solar line at 5712.09, which is certainly identical with the *Ti* line. The line is ascribed to iron by Rowland as well as by Kayser and Runge. The remaining pairs of lines in approximate agreement are given in the following table, from which it again appears that in almost every case the lines are independent of each other.

Mo λ	*Mo* *i*	*Ti* λ	*Ti* *i*	Remarks
3493.49	2—	3493.44	1	Separated. λ *Mo* > λ *Ti*
3510.93	1.2	3510.98	3	Widely separated
3566.20	2+	3566.16	1.2	Separated. λ *Mo* > λ *Ti*
3613.94	1.2	3613.89	2	
14.42	3	14.35	2	Separated
35.30	2	35.33	2	Separated. λ *Mo* < λ *Ti*
35.57	2.3	35.61	4.5	Separated. λ *Mo* < λ *Ti*
36.07	1	36.05	1	Coinc. Impurity?
54.73	2+	54.72	3	Nearly coinc., but λ *Mo* < *Ti*
62.30	1.2	62.37	2.3	Widely separated
63.83	1+	63.82	1	Coinc. Common impurity?
77.83	2—	77.90	1	Separated, but λ *Mo* > λ *Ti* = 77.80?
3688.12	1	3688.19	1	*Ti* line lacking
3702.33	1+	3702.42	2	Widely separated
35.80	1.2	35.84	1.2	Separated. λ *Mo* < λ *Ti*
61.93	2—	62.01	1	Widely separated
66.58	1	66.60	1	Separated. λ *Mo* < λ *Ti*
76.27	1+	76.20	1	Separated. λ *Mo* > λ *Ti*
88.42	2	88.44	1.2	*Ti* line lacking
3798.39	10	3798.47	1.2	Separated
3806.15	2	3806.19	1	*Ti* line lacking
14.56	1.2	11.56	1	Coinc. *Mo?*
14.64	1+	14.72	2	Widely separated
21.82	1	21.86	1	*Ti* line lacking
22.14	1	22.16	2.3	Separated. λ *Mo* < λ *Ti*
29.95	1.2	29.87	1.2	Separated
48.45	2+	48.48	1.2	Probably separated. λ *Mo* < *Ti*
74.34	1.2	74.32	2	Coinc. *Ti* line faint
3888.15	1—	3888.20	2	Separated. λ *Mo* < λ *Ti*. *Ti* line diffuse
3913.52	1.2	3913.58	2.3	Separated. λ *Mo* < λ *Ti*

Mo λ	*i*	*Ti* λ	*i*	Remarks
3934.41	1.2	34.37	1.2	Separated. λ *Mo* > λ *Ti*
3994.79	1	3994.84	1.2	Separated. λ *Mo* < λ *Ti*
4008.21	1+	4008.20	2	Coinc. *Mo* line sharp. *Ti* line diffuse
34.11	1	34.05	1.2	Separated. λ *Mo* > λ *Ti*
4057.77	1.2	4057.76	1.2	Coinc. Impurity?
4105.27	2+	4105.31	1.2	Separated. λ*Mo*<λ*Ti*. The line 05.31 belongs to *V*
64.26	1.2	64.27	1.2	Separated. λ *Mo* < λ *Ti*
4166.47	1.2	4166.45	2	Separated, but λ *Mo* < λ *Ti*
4224.93	1	4224.96	2	Separated. λ *Mo* < λ *Ti*
60.85	1.2	60.91	1	Separated
66.37	2—	66.37	1.2	Separated. λ *Mo* < λ *Ti*
80.17	1+	80.17	1.2	Separated. λ *Mo* < λ *Ti*. Δλ = 0.04
4291.39	2—	4291.32	2	Separated. λ *Mo* > λ *Ti*
4335.00	2	4334.98	1.2	Separated, but λ *Mo* < *Ti*
4417.40	1	4417.46	3	Separated. λ *Mo* < λ *Ti*
33.68	1.2	33.75	1.2	Widely separated
57.55	3.4	57.59	3.4	Separated. λ *Mo* < λ *Ti*
4489.17	1.2	4489.24	2.3	Separated. λ *Mo* < λ *Ti*
4501.44	2	4501.43	3	Separated? λ *Mo* > λ *Ti?*
35.00	2+	34.97	3.4	Separated. λ *Mo* > λ *Ti*
58.30	2.3	58.28	1.2	Coinc. *Mo?*
4599.35	2—	4599.40	2	Separated
4656.57	1	4656.60	3.4	Separated. λ *Mo* < λ *Ti*
4723.27	1.2	4723.32	2.3	
4808.68	1+	4808.70	2	
4811.28	2.3	4811.24	2	

MOLYBDENUM AND VANADIUM.

In the spectra of these two metals I have met with lines of the same, or nearly the same wave-lengths, in only a few isolated cases, which might be ascribed to the two metals on the basis of their ratios of intensity. The pairs are these :

Mo λ	*i*	*V* λ	*i*
3524.35	2+	3524.38	2+
4240.26	2	4240.25	2
4776.54	3	4776.54	3
5241.09	3	5241.06	2

As for the rest, the following table shows that the approximate coincidences found are not real.

After these comparisons with the arc spectra of the metals investigated by myself, I now proceed to the comparison of my

Mo λ	i	V λ	i	Remarks
3498.21	1+	3498.23	1	*V* line lacking
3504.55	2.3	3504.57	1.2	Separated ?
24.35	2+	24.38	2+	Coinc. ?
3562.26	2−	3562.32	1	Separated
3638.57	1.2	3638.57	1	Coinc. Foreign line ?
3669.50	2.3	3669.57	1.2	Widely separated
3732.91	3	3732.88	2	*Mo* line dupl.
34.56	1.2	34.59	1.2	Separated. λ *Mo* < λ *V*
70.66	2.3	70.68	1	*V* line lacking
75.82	1+	75.85	1.2	Separated. λ *Mo* < λ *V*
3776.27	1+	3776.31	1.2	Separated. λ *Mo* < λ *V*
3822.14	1	3822.14	2.3	Slightly separated. λ *Mo* < λ *V*
47.41	2+	47.46	2.3	Separated. λ *Mo* < λ *V*
70.77	1.2	70.72	1+	*V* line faint. λ *Mo* seems > λ *V*
3888.15	1−	3888.23	1+	Separated
3916.62	1+	3916.55	1.2	Separated
24.78	1	24.84	2.3	Coinc. close
31.57	1.2	31.50	2	Separated. λ *Mo* > λ *V*
35.33	1.2	35.28	2.3	
3950.40	1	3950.37	2−	
4003.62	1+	4003.70	1.2	
05.86	1	05.86	2+	Coinc. *V?* Other strong *V* lines are lacking in *Mo*
32.65	1.2	32.62	1.2	Separated. λ *Mo* < λ *V*
4067.88	1+	4067.90	1.2	Separated. λ *Mo* < λ *V*
4102.33	2.3	4102.32	3	Slightly separated. λ *Mo* < λ *V*
05.27	2+	05.32	3	Separated
07.63	3	07.64	1.2	Slightly separated. λ *Mo* < λ *V*
08.30	1.2	08.36	2	Widely separated
32.07	2−	32.13	3.4	Separated. *V* line reversed
4175.32	1	4175.30	1	Coinc. Foreign line ?
4240.26	2	4240.25	2	Coinc.
40.48	2	40.53	2+	Separated. λ *Mo* < λ *V*
77.08	3	77.12	3	Separated
79.19	1+	79.12	1.2	
91.39	2	91.46	2−	
4296.35	1.2	4296.28	2.3	
4369.23	2.3	4369.25	1+	Hard to separate. λ *Mo* < λ *V?*
4392.32	1.2	4392.24	2	Separated. λ *Mo* < λ *V*
4506.22	3	4506.30	2	Separated very slightly
4611.03	1+	4611.10	1.2	
26.67	3.4	26.67	2+	
30.20	2−	30.24	1	λ *Mo* < λ *V*
4648.02	2	4648.08	1	λ *Mo* < λ *V*
4706.25	2	4706.34	2.3	Widely separated
76.54	3	76.54	3	Coinc.
84.64	1	84.65	2	Separated. λ *Mo* < λ *V*
4786.68	2	4786.70	3	λ *Mo* < λ *V*
4833.13	1.2	4833.17	2	Δλ > 0.04
5014.80	1+	5014.83	3	Separated. λ *Mo* < λ *V*
5241.09	3	5241.06	2	Coinc. *i V* < 2
5437.97	2.3	5437.93	1.2	Separated. λ *Mo* > λ *V*
5632.74	4	5632.73	1+	Coinc ? Perhaps λ *Mo* < λ *V*
5734.32	2−	5734.26	2	Coinc ? *V?*
5747.93	1+	5747.98	1	Separated. λ *Mo* < λ *V*

observations with the arc spectra of other metals investigated by Kayser and Runge. For this purpose I first made out accurate lists of all those lines of molybdenum which agreed within the limits of error given by Kayser and Runge with lines measured by them. But nothing could be concluded from the approximate agreement of the positions, since these limits of error exceed from ten to a hundred times the uncertainty attaching to my determination of the molybdenum lines, on account of the diffuseness of the lines of lithium, sodium, potassium, caesium, antimony, mercury, copper, silver, and gold. For the present I therefore regard my spectrum of molybdenum as free from impurities due to these metals. This also holds good for the metals rubidium, magnesium, zinc, aluminium, indium, arsenic, thallium, and tin, for which no lines agreeing with molybdenum occur in the tables of Kayser and Runge. The remaining metals investigated by Kayser and Runge—calcium, strontium, cadmium, barium, lead, and bismuth—present in their arc spectra only isolated cases of approximate coincidence with the lines of molybedenum, as to the actual nature of which I can at present give no definite decision. In the summary of these lines given in the following table I have only taken into account those in which the limit of error assigned as possible by Kayser and Runge does not exceed the amount of 0.05 tenth-meters, since for greater values of this limit the close agreement of the wave-lengths is in my opinion too likely to be accidental.

	Kayser and Runge				Hasselberg	
	λ	Limits of Errors	*i*	Remarks	*Mo* λ	*i*
Bi	3888.34	0.03	6		88.36	2—
Pb	4062.30	0.03	4	reversed	62.24	2.3
Ba	3566.90	0.05	5	diffuse	66.91	1
Ba	4224.11	0.05	4		24.10	1
Ba	4291.32	0.05	4		91.39	2—
Ba	4325.38	0.05	5		25.44	1
Ba	4402.75	0.05	2	reversed	02.67	2—
Ba	4407.10	0.05	4	diffuse toward red	07.04	1+
Ba	4700.64	0.05	3	diffuse toward red	00.71	2—
Ca	3973.89	0.05	3	diffuse toward red	73.92	2+

The first of these lines may well belong to calcium, although it seems less probable on account of the diffuseness on one side of the lines in the spectrum of this metal. The first five of the barium lines should be separated from the corresponding molybdenum lines on account of the very considerable difference of the wave-lengths, while the last two should perhaps be removed from the molybdenum spectrum. I have, however, not removed the first or the last two lines of this table from the catalogue of molybdenum lines.

It now finally remains to compare the spectrum of molybdenum with the arc spectra of the metals of the platinum group recently investigated by Kayser. A considerable number of approximate coincidences was to be expected in advance on account of the extraordinary abundance of lines in these spectra, even if on account of the high accuracy of Kayser's measures the comparisons are limited to those cases in which the difference of wave-lengths does not exceed 0.05. This limitation appears fully justifiable, since in general Kayser's wave-lengths should be accurate to 0.01 and mine to 0.02 tenth-meters. I have collected the lines found to be of nearly the same wave-lengths in this comparison with the observations of Kayser, in the following table, in which I have reduced to my scale as well as I could his estimates of intensity, which range from 0 to 10:

	λ	i	*Mo* λ	*Mo* i	Probably belongs to
Pt	3818.83	3	18.83	2—	*Pt*
	3900.87	2.3	00.87	1	*Pt*
	4081.63	1—	81.62	3	*Mo*
	4201.37	1+	01.35	1+	?
	4269.41	1+	69.44	2.3	*Mo*
	5306.49	1——	06.49	1—	?
Pd	4170.00	3	70.01	2—	*Pd*
	4344.8	2.3	44.88	1.2	?
	4443.19	2	43.23	2—	?
	4458.79	1.2	58.84	1+	?
	4632.77	1	32.75	1+	?
	5117.16	1.2	17.18	4+	*Pd*
	5619.67	2—	19.63	5.6	*Pd*
	5739.88	1	39.92	2.3	*Pd*

	λ	i	Mo λ	Mo i	Probably belongs to
Ru	3463.75	1− −	63.78		?
	3480.29	1+	80.26	1.2	?
	3482.50	1+	82.55	2	?
	3514.91	1−	14.93	1+	?
	3640.79	2.3	40.76	2+	*Ru, Mo*
	3686.74	1−	86.72	2−	*Mo*
	3702.37	1+	02.33	1+	?
	3716.32	2−	16.27	2	?
	3722.46	1−	22.50	1−	?
	3730.75	2−	30.75	1	?
	3733.19	1+	33.22	2−	?
	3742.44	3	42.48	2.3	*Ru, Mo*
	3744.55	1+	44.55	2−	?
	3764.18	1−	64.20	1	?
	3765.94	1− −	65.92	1.2	*Mo*
	3782.89	1− −	82.86	1	?
	3831.95	2.3	31.95	1+	*Ru*
	3911.28	2−	11.24	1.2	?
	3921.06	2.3	21.09	1+	*Ru*
	3924.78	1+	24.78	1	?
	3939.27	1− −	39.30	1+	*Mo*
	4021.15	2−	21.19	2	?
	4032.65	1−	32.65	1.2	*Mo*
	4200.07	4+	00.02	1	*Ru*
	4207.80	1+	07.75	1+	?
	4245.00	2.3	44.96	1.2	*Ru*
	4287.21	2.3	87.26	2−	?
	4293.44	2.3	93.42	3	*Mo, Ru*
	4332.65	1+	32.68	1+	?
	4362.87	1−	62.87	1+	?
	4396.87	1− −	96.83	2	*Mo*
	4490.40	1+	90.37	2	?
	4617.83	1− −	17.82	1	?
	4833.16	1+	33.13	1.2	?
	5062.81	1−	62.76	1+	?
	5427.81	2.3	27.80	1	*Ru*
	5694.63	1+	94.64	1+	?
	5730.12	1+	30.17	2−	?
	5771.35	1+	71.33	1+	?
Ir	3468.75	1+	68.70	1	?
	3476.18	1− −	76.15	2	*Mo*
	3628.84	3	28.80	1.2	*Ir*
	3641.04	1−	41.08	1	?
	3661.87	3	61.91	2+	*Ir, Mo*
	3747.35	2.3	47.37	2−	*Ir, Mo*
	3768.82	1+	68.78	1+	?
	3800.24	1+	00.28	1+	?
	3817.38	1− −	17.37	1−	?
	4070.07	2.3	70.05	2+	*Ir, Mo*
	4200.03	1+	00.02	1	?
	4268.25	2.3	68.25	2−	*Ir, Mo*
	5364.51	1+	64.50	3.4	*Mo*

	λ	i	Mo λ	Mo i	Probably belongs to
Os	3490.46	1+	90.42	1	?
	3681.70	1+	81.69	1.2	?
	4003.65	1+	03.62	1+	?
	4124.76	1− −	24.72	2	*Mo*
	4201.54	1− −	01.50	1.2	*Mo*
	4233.63	1−	33.68	1.2	*Mo*
	4252.72	1− −	52.69	1+	*Mo*
	4294.10	1+	94.07	3	*Mo*
	4296.38	1− −	96.35	1.2	*Mo*
	4338.91	2	38.90	2−	?
	4692.22	1+	92.19	1+	?
Rh	3469.35	1− −	69.39	2	*Mo*
	3469.77	3	69.80	1	*Rh*
	3612.62	3	12.62	2−	*Rh*
	3639.68	3.4	39.71	1+	*Rh*
	3651.52	1+	51.48	2	?
	3713.59	2−	13.64	2−	?
	3755.21	1−	55.31	2	*Mo*
	3765.23	3	65.21	1+	*Rh*
	3775.86	1+	75.82	1+	?
	3812.60	1+	12.63	2	?
	3934.38	2.3	34.41	1.2	*Rh*
	3959.01	3	59.03	1	*Rh*
	4081.96	1+	81.94	2−	?
	4423.83	1−	23.79	2.3	*Mo*
	4506.82	1−	06.86	2	*Mo*
	4558.90	2.3	58.92	1.2	*Rh*
	4569.18	3.4	69.21	1+	*Rh*
	4608.29	1+	08.32	1	?
	5090.80	3	90.80	1	*Rh*
	5292.28	2.3	92.30	1.2	*Rh*
	5497.20	1− −	97.18	1.2	*Mo*
	5544.80	3.4	44.78	2.3	*Rh, Mo*

It appears hardly to be doubted from the large number of nearly coincident lines that on the one hand several of the faintest molybdenum lines of my catalogue are to be attributed to impurities of one or the other of the plantinum metals, while on the other hand several faint molybdenum lines occur in the spectra of the latter. On the assumption that the wave-lengths are actually identical, the origin of the lines can be given on the basis of the intensities with a pretty large degree of probability in about half the cases; but in the remaining cases where this basis of discrimination fails on account of nearly equal intensity there arises the possibility or indeed probability of contamination by a third metal. We see from the last column which

gives the probable origin of the lines according to this principle that of the molybdenum lines there are probably two due to platinum, four to palladium, five to ruthenium and ten to rhodium ; while there should be ascribed to molybdenum from platinum two, from ruthenium five, from rhodium five, from osmium five, and from indium two lines; and that finally in eight cases the lines may be simultaneously ascribed to two metals. All this, however, assumes an absolute coincidence. Since the decision of this question cannot be reached with entire rigor with the means at present at my disposal, in view of the exceedingly small quantities involved, I have preferred to provisionally retain the lines in my catalogue, in the possibility of investigating these details more accurately at some future time.

[*To be concluded.*]

THE SOLAR ATMOSPHERE

By Arthur Schuster.

The radiation received from different portions of the solar disk is known to diminish from the center toward the limb in a manner which is generally considered not to be consistent with the assumption of a uniformly absorbing solar atmosphere. The measurements recently published by Mr. Frank W. Very[1] allow us to examine this question in a more satisfactory manner than has hitherto been possible, and such examination leads to the conclusion that the difficulty which has been felt in explaining the law of variation of intensity across the Sun's disk, is easily removed. It is only necessary to place the absorbing layer sufficiently near the photosphere and to take account of the radiation which this layer, owing to its high temperature, must itself emit.

I introduce the following notation:

I = Intensity of the radiation, which is incident on the absorbing shell.

F = The radiation of a perfectly black body which is at the temperature of the shell.

A = the radiation leaving the absorbing layer in the direction of the Earth.

t = the length of path within the absorbing layer of rays which have traversed this layer.

t_0 = the thickness of the absorbing layer, *i. e.* the minimum value of t.

k = a constant on which the absorption depends, which may be different for different wave-lengths.

r = the perpendicular distance between any point on the Sun and the line drawn from the Sun's center towards the observer stationed on the Earth.

a = the Sun's radius.

Also for convenience of printing, write:

$$z = e^{-kt_0}; \ \gamma = \frac{r}{a}; \ \sigma = \frac{1}{\sqrt{1-\gamma^2}}.$$

The differential equation giving the law of variation of A with the distance t is:

$$\frac{dA}{dt} = -k(A-F),$$

[1] Astrophysical Journal, 16, 73, 1902.

and its appropriate solution is:

$$A = Ie^{-kt} + F(1 - e^{-kt}) ,$$

where

$$t = \left[(a + t_0)^2 - r^2\right]^{\frac{1}{2}} - \left[a^2 - r^2\right]^{\frac{1}{2}} .$$

If t_0 is small compared to a,

$$t = \frac{t_0}{\sqrt{1 - \gamma^2}} = \sigma t_0 ,$$

and the equation for the radiation may now be written:

$$A + (I - F)\, z^{\sigma} + F .$$

The three unknown quantities I, F, and z may be determined if the radiation is measured at three points of the solar disk. Let A_0, A_1, A_2 be these measured radiations and σ_0, σ_1, σ_2 the corresponding values of σ. By elimination of I and F we obtain:

$$(A_0 - A_1)\left(z^{\sigma_0 - 1} - z^{\sigma_2 - 1}\right) = (A_0 - A_2)\left(z^{\sigma_0 - 1} - z^{\sigma_1 - 1}\right) .$$

This determines z, which must be formed numerically by means of a series of systematic trials and interpolations. When z is determined I and F are calculated by the equations

$$I = \frac{A_0\left(1 - z^{\sigma_2}\right) - A_2\left(1 - z^{\sigma_0}\right)}{z^{\sigma_0} - z^{\sigma_2}} .$$

$$F = \frac{A_2\, z^{\sigma_0} - A_0\, z^{\sigma_2}}{z^{\sigma_0} - z^{\sigma_2}} .$$

If A_0 refers to the observation at the Sun's disk, it may be put equal to one, as in Mr. Very's observations, and I and F will then be referred to the same unit, which, however, it must be remembered will be different for different wave-lengths.

The solution of the equations may also be graphically carried out as follows:

For a particular value of z draw a family of curves satisfying the equation:

$$y = cz^{\sigma} ,$$

y being the ordinate, $\sigma = \sqrt{\frac{1}{1 - \gamma^2}}$ the abscissa and c a parameter which is varied step by step. A number of such families are drawn for different values of z. That curve is then chosen

which most nearly coincides with the curve of observed radiation, a displacement of the curve up and down being allowable. The amount of displacement necessary to bring the curves into coincidence will measure F, because y will for coincidence be equal to $A - F$. F being known, I will be determined from the corresponding relation $c = I - F$.

For convenience of reference I give in Table I Mr. Very's observations, on which the subsequent calculations are based. The wave-length for which the radiation is measured is denoted by λ and given in centimeters.

TABLE I.

λ	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 0.75$	$\gamma = 0.95$
15.0×10^{-6}	Rad. taken as one	0.959	0.950	0.856
10.1	"	0.943	0.894	0.765
7.81	"	0.941	0.885	0.749
6.15	"	0.948	0.845	0.681
5.50	"	0.933	0.831	0.587
4.68	"	0.902	0.764	0.462
4.16	"	0.858	0.744	0.471
$\sigma =$	1	1.155	1.512	3.202

By combining the observed values for $\gamma = 0$; $\gamma = 0.75$ and $\gamma = 0.95$ I have calculated z, I, and F for different wave-lengths; the results are given in Table II.

TABLE II.

λ	z	I	F	$I - F$
15.0×10^{-5}	0.581	1.15	0.793	0.36
10.1	0.366	1.46	0.736	0.72
7.81	0.355	1.51	0.720	0.79
6.15	0.312	1.76	0.654	1.11
5.50	0.442	1.62	0.505	1.12
4.68	0.360	2.07	0.399	1.67
4.16	0.315	2.25	0.426	1.82

To obtain an idea of the accuracy with which the formula satisfies the observations, I have calculated with the values of Table II, the radiation for the position defined by $\gamma = 0.5$; the

observations at these points not having been made use of in the computation of the constants. The comparison is given in Table III.

TABLE III.

$10^5 \lambda =$	15.0	10.1	7.81	6.15	5.50	4.68	4.16
Calculated radiation	0.983	0.962	0.958	0.943	0.941	0.912	0.906
Observed radiation	0.959	0.943	0.941	0.948	0.933	0.902	0.858

The observed radiations seem to be generally smaller than the calculated ones, but the difference probably falls within the errors of observations. The five individual measurements which Mr. Very in his paper gives for one set of observations range from 0.908 to 0.959 (a difference of 5 per cent.) while, with the exception of the number for the shortest wave-length, when the measurements are admittedly doubtful, the difference between the calculated and observed radiations exceeds 2 per cent. in only one case.

The coefficient of absorption depends on the quantity of z, which is the fraction of the incident light passing normally through the absorbing layer. Table II shows no decisive indication that this quantity varies in a regular manner with the wave-length. No doubt the coefficient of absorption seems decidedly less for the longest wave-length, but a reference to Table I will show that the whole variation of intensity for this wave-length is so small that the coefficient of absorption calculated from it is very uncertain. Our equations show that constancy of radiation for different distances from the Sun's limb may be due either, to smallness of the coefficient of absorption, in which case the radiation is everywhere equal to I, or to its greatness in which case the radiation is everywhere equal to F, or finally to the near equality of I and F. Table II shows that the difference between I and F diminishes considerably with increasing wave-lengths, and hence the coefficient of absorption has comparatively little influence on the results for the longest waves.

To confirm this, I have calculated the radiations transmitted for $\lambda = 15.0 \times 10^{-6}$, assuming a coefficient of absorption equal to its average value for the other wave-lengths, *i. e.*, $z = 0.358$.

The observations in that case may best be satisfied by taking $I = 1.249$, $F = 0.847$. The observed and calculated values now compare as follows:

	$\gamma=0.5$	$\gamma=0.75$	$\gamma=0.95$
Calculated radiation.................	0.970	0.932	0.862
Observed radiation..................	0.959	0.950	0.856

The disagreement of the numbers is not sufficient to allow us to state definitely that the coefficient of absorption has not the assumed value. That the absorption should be exactly the same for all wave-lengths is very unlikely, but the data we possess at present do not allow us to draw any conclusion as to the manner in which it varies.

I have confirmed in another manner that the available observations are not inconsistent with a constant coefficient of absorption. Assuming the constancy, the variation of the total radiation at different points of the Sun's disk should follow the same law as that of the separate portions of the spectrum. But the total radiation has been measured by Langley, and by Messrs. W. E. Wilson and Rambaut,[1] and the measurements being more easily made are probably more accurate than those of Mr. Very. Taking Mr. Wilson's observations and selecting again the same parts of the Sun's disk to determine the constants, *i. e.*, $\gamma = 1$, $\gamma = 0.75$, and $\gamma = 0.95$, I find that with $I = 1.486$, $F = 0.579$, $z = 0.465$, the numbers given in Table IV are obtained.

[1] *Proceedings Royal Irish Academy*, 2, No. 2, 1892.

TABLE IV.

$\gamma=$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Obser'd. rad.	0.999	0.996	9.988	0.973	0.953	0.925	0.887	0.829	0.749	0.451
Calcul'd rad.	0.999	0.994	0.985	0.973	0.954	0.927	0.890	0.832	0.736	0.579

With the exception of the radiation at the edge of the Sun which must be doubtful, the agreement is very close.

I return to the discussion of Table II. The justification of the view, that the radiation of the absorbing layer accounts for the discrepancy which has hitherto been found to exist between theory and observation, must be sought not so much in the

general agreement of the facts which is now arrived at (because our formula, having an additional constant to dispose of, can naturally be made to fit the facts better) as in the consistent fashion in which the calculated values of F give us numbers, such as we have *a priori* reason to expect. The quantity F is the radiation of a black body at the temperature of the absorbing layer. The curve of F drawn with the wave-length as abscissa should, if our supposition is correct, show a maximum, which, when compared to the radiation I of the photosphere, is displaced toward the red, and the highest ordinate should be less than the highest ordinate of I. To show that this is exactly what calculation gives us, I have made use of Professor Langley's observations, which give the observed intensity A_0 at the center of the disk for different wave-lengths. In the previous tables the value of A_0 was taken to be equal to 1 for each wave-length, and the values of I and F were therefore referred to a unit which changed with the wave-length. The second column in Table V gives the intensities of radiation for the wave-lengths given in the first column. The highest ordinate of the intensity curve is now taken to be the unit.

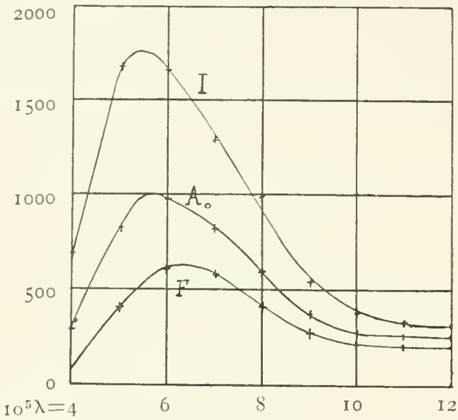
TABLE V.

$10^5 \times \lambda$	A^0	I	F
12	0.260	0.325	0.208
11	0.274	0.356	0.214
10	0.293	0.396	0.226
9	0.390	0.546	0.292
8	0.581	0.994	0.421
7	0.846	1.286	0.584
6	0.975	1.658	0.604
5	0.837	1.660	0.418
4	0.302	0.695	0.091

The third and fourth columns give the radiation (I) of photosphere before it traverses the absorbing layer and the radiation (F) of a black body at the temperature of the absorbing layer.

The numbers given in Table V are plotted in the figure, and show clearly that the radiation of the absorbing layer belongs to a lower temperature than that of the rays traversing it. The

maximum of the curve of I lies at about $\lambda = 5.4 \times 10^{-5}$ that of the curve of F at 6.4×10^{-5}. The ratio of these two numbers, or 0.84, should give the ratio of the absolute temperature of the absorbing layer to that of the original radiation. If we take the photosphere to be at a temperature of 10000°, this would give the temperature of the absorbing layer about 1500° lower. We may get another estimate of the ratio of the same temperatures by taking the ratio of the fourth roots of the two maxima of radiation. This ratio is 0.73, which would give a lower temperature of the absorbing layer. But all these calculations ought to be corrected for the absorption in our atmosphere, and I am perhaps trying to draw conclusions which the accuracy of the measurements do not as yet warrant. Sufficient has been said to show that important results may be expected, if Mr. Very's measurements are repeated and if a greater accuracy is obtained.



There is no reason to look for a different region in the Sun's surroundings for the cause of the observed diminution of radiation, than that which gives the Fraunhofer lines. The simplest supposition to make at present, and one consistent with our knowledge of spectra, is that the layer which gives the line absorption, absorbs also to some extent all wave-lengths extending from infra-red to violet, the diminution in the observed intensity of the solar radiation towards the edges of the disk would then simply be due to this absorption.

The principles which have been developed in this communication may find a wider application. Some observers have apparently been puzzled by the fact, that the radiation of the umbra of Sun-spots does not diminish as it nears the edge of the Sun in the same way as that of the luminous disk itself, but on the contrary remains nearly constant. Our tables show that in

the case of the solar disk only about half the radiation comes from the photosphere, the rest is made up by the radiation of the absorbing layer itself. If that absorption either by increased density or by greater thickness is increased four or five times, practically the whole of the radiation would come from the absorbing layer, and would be nearly constant for different portions of the solar disk. To see that this is the case it is only necessary to take the equation,

$$A = (I - F)\, e^{-kt} + F\ ,$$

and to substitute for e^{-kt} a value five times smaller than that given in Table II, or about 0.07. The whole radiation would then, even at the center, be sensibly equal to F.

MANCHESTER, ENGLAND,
November 19, 1902.

ON THE PROLONGATION OF SPECTRAL LINES.

By Theodore Lyman.

Investigators who have worked with concave diffraction gratings cannot have failed to observe the faint but sharp prolongations of strong lines which occur in the spectra produced by these instruments.

The cause of the difference in length of certain lines when a prism spectroscope is used is well known. Sir Norman Lockyer long ago made use of the phenomenon of "long and short lines" in his study of the chemistry of the Sun. He pointed out that when the vertical slit of the collimator is illuminated by the image of a light source formed by a lens, some of the lines in the resulting spectrum are longer than others. This of course arises from the fact that the portion of the source which illuminates the center of the slit possesses some vibration frequencies which are wanting in those portions which come to focus at the top and bottom of the slit. When a concave grating is used, the astigmatism renders this phenomenon less striking.

The prolongations of strong lines which are referred to above present a different appearance, however, from those obtained with a prism spectroscope. They are, in fact, due to a different cause. As the author had never seen any explanation of the matter, it seemed that an investigation on the subject might prove of interest.

In making some adjustment of a concave grating of twenty-one feet radius the principal image of the slit came into view. It was at once noticed that this image was prolonged into two narrow streamers, one vertical and the other horizontal, each quite distinct and sharp. This suggested that the vertical continuation of the slit image and the vertical continuation of strong spectral lines were due to the same cause. Observations were accordingly made upon the principal image of the source. In order, however, to simplify the diffraction

PLATE XVI.

FIG. 1.

FIG. 2.

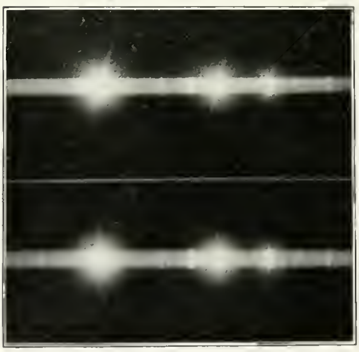


FIG. 3.

phenomena as much as possible a circular opening of about 0.01 cm diameter was substituted for the slit. In order to reduce spherical aberration to a minimum the light fell upon the grating at nearly normal incidence. In the final observations the polished surface was always covered. The first photograph was taken with the grating in its normal position and with all the ruled surface exposed. The result is an image of the pin hole accompanied by two sharp streamers, one vertical the other horizontal. The horizontal streamer is the longer and stronger of the two. The effect is shown in Plate XVI, Fig. 1.

If the ruled surface of the grating be protected by a screen having a rectangular aperture whose diagonal is less than the breadth of the ruled surface, the orientation of this aperture throws light upon the nature of the phenomenon. When the longest side of the opening is parallel to the longest side of the ruled surface, the image of the point source presents exactly the same appearance as when the whole grating surface is exposed. When, however, the aperture is set askew so that its sides make angles of 45° with the sides of the ruled surface, the effect on the image is striking. The result is shown in Fig. 2. The vertical streamer has revolved through 45° and the horizontal streamer has been broken into two parts. One of these parts has turned through 45° while the other and longer part has remained fixed. The streamers which accompany the image seem then to consist of two components, one a cross which turns as the sides which bound the ruled surface turned, the other a horizontal line or streamer which remains stationary. The idea that the movable cross is due to diffraction through a rectangular opening at once suggests itself. This theory may be further tested by covering the grating surface by a screen with a circular hole. The image of the source so obtained showed no vertical streamer, the horizontal line was still present though less intense. This result indicates that the explanation of the origin of the movable cross is the correct one.

When the point source was replaced by a slit the effects were of an exactly similar character.

It is easy to apply the results of these experiments to the vertical continuation of strong spectral lines. In order to demonstrate that the vertical streamers are due to the rectangular shape of the ruled surface, we have only to cover the surface with a screen whose rectangular aperture is set askew with respect to the direction of the lines of the grating. We obtain the effect shown in Fig. 3. Here the continuations of the strong lines have turned and broadened, though the lines themselves remain straight and sharp. The diffraction pattern from the rectangular grating aperture is no longer a cross with sharp vertical and horizontal arms. The cross has turned, and, since the aperture is no longer symmetrical with respect to the slit, the arms have broadened.

The horizontal band which does not turn as the aperture is rotated may be due to either of two causes or to a combination of them. The subordinate or secondary maxima which accompany every line in the spectrum furnish the first reason, irregularities in the grating ruling furnish the second. The author has shown that these irregularities may not only furnish a background, but, under favorable circumstances, may even produce sharp reproductions of real lines. The background or nearly continuous band can be noticed with almost every grating and can be best observed in that portion of the extreme ultra-violet where no real lines are obtained. Its intensity varies greatly with different gratings. In investigations where long exposures are necessary it often proves very inconvenient, for faint real lines are much obscured by its presence.

In many cases the horizontal band due to diffraction through a rectangular opening is much stronger and more troublesome than the band due to irregular ruling. In this case there is a remedy at hand. The ends of the ruled space may be covered with slanting pieces of black paper and the rectangular ruled space thus converted into a parallelogram. The effect of this arrangement is to revolve the horizontal streamer due to the shape of the opening, the

Fig. 4.

vertical streamer remaining fixed, with the result that the background of the spectrum is very materially cleared. The author can recommend this device to all those who investigate faint spectral lines, and to whom a clear field is a necessity.

JEFFERSON PHYSICAL LABORATORY,
Harvard University, November 1902.

Minor Contributions and Notes.

ON THE NUMBER OF STARS UPON A PHOTOGRAPHIC PLATE[1].

Since the foci for oblique rays if an objective can never lie in the same plane with axial focus, it must always happen that a part of the stars must form extra-focal images on a plane photographic plate. Only at that point where the plane of the plate cuts the geometrical position of the foci, will the image be focal, sharpest, and most intense; which will be the case, therefore, at the intersection of the plane of the plate with the focal surface. The greatest number of stars is, therefore, to be expected on the general average in this intersection curve, since the faintest stars are lost by defective focusing within it and without it. If the objective is not corrected for curvature of focal surface, the focal surface is a sphere with its center at the objective. If the plate is also perpendicular to the optical axis and pushed inside the axial focus, and if the center of the plate also coincides with the optical axis, the intersection curve will be a circle (focal circle), and the average number of stars will be solely a function of the distance from the center of the plate. If the plate is pushed along the optical axis, either inwards or outwards, the focal circle decreases or increases, and the number of stars varies accordingly. At what position of the plate shall we now obtain the greatest number of stars? This question may be solved on certain very general premises without knowledge of the complicated relations between light power and focusing as follows:

If we denote by δ the distance of any element of the plate from the focal sphere, we may assume that the optical effects depend only upon this δ, indifferently whether the element lies at a distance inside or outside the focal sphere, so long as the distances involved are small in comparison to the focal length. The number of stars on a plate can therefore be considered solely as a function of δ, which is related to the focal length f of the objective, and to the radius r of the focal circle, in the manner shown by the following simple geometrical considerations:

[1] Translated from a communication to the *K. Akademie in Wien*; Session of October 16, 1902, furnished by the author.

Outside the focal circle:

$$\delta_o = \frac{1}{2f}(r^2 - r_o^2) \ .$$

Inside the focal circle:

$$\delta_i = \frac{1}{2f}(r_o^2 - r^2) \ .$$

This formula is sufficiently accurate up to a field of view of 4°. The number of stars is, therefore, an unknown function of δ or $F(r^2 - r_o^2)$. Hence the number of stars outside the focal circle becomes

$$A_o = 2\pi \int_{r_o}^{R} F(r^2 - r_o^2)\, r\,dr = \Phi(R^2 - r_o^2) - \Phi(o) \ .$$

and inside the focal circle

$$A_i = 2\pi \int_{o}^{r_o} F(r_o^2 - r^2)\, r\,dr = \Phi(r_o^2) - \Phi(o) \ ,$$

where Φ is again an unknown function, and R is the distance from the center to the edge of the plate. The total number of stars is now

$$A = A_o + A_i = \Phi(R^2 - r_o^2) + \Phi(r_o^2) - 2\Phi(o) \ .$$

We see from the differential quotient

$$\frac{\partial A}{\partial r_o} = 2r_o\left[\Phi'(r_o^2) - \Phi'(R^2 - r_o^2)\right] \ ,$$

and from the obvious properties of the F- and Φ- functions that the number of stars is a maximum for the value $R^2 = 2r_o^2$ or $R^2\pi = 2r_o^2\pi$. We therefore obtain the greatest number of stars on a plate if we make the contents of the focal circle just half as great as the contents of the circular field of view to be photographed.

It follows from this rule that we must push in the plate from the axial focus by the amount

$$\delta_o = \frac{1}{2f} r_o^2 = \frac{R^2}{4f} = \frac{L2}{16f} \ ,$$

where L is the length of side of the square plate and f is the focal length. If we take $L = 165$ mm and $f = 3400$ mm for the Potsdam photographic refractor, the plate should be pushed in 0.47 mm in order to obtain the greatest possible number of stars. We see from the Potsdam plates for the *Carte celeste* that it was pushed in only 0.13 mm, whereby there has ensued on every plate an unnecessary loss, which may be estimated at least as 6 per cent. in the number of stars.

EGON VON OPPOLZER.

INNSBRUCK,
October 25, 1902.

REMARKS ON BIGELOW'S "ECLIPSE METEOROLOGY."

In the fourth chapter of his recently published work, Professor Bigelow also enters upon a discussion of the theory of Sun-spots for which I am sponsor. He does not appear to be quite favorably inclined to my theory, although I should not have expected that on the part of a meteorologist. There are two remarks in particular which force me to a reply.

1. In the section entitled "Remarks on the Computations by von Oppolzer and Fenyi," on p. 83, Mr. Bigelow writes: "Von Oppolzer, in his computation of the solar vertical temperature-gradient, uses $n = 27.6$, $(k-1):k = 1:3.5615$, $R = 420.552$, and finds $\frac{dt}{dh} = 18^\circ.96$ C. per 1000 m, which is 1463.4 times as large as my value ($0^\circ.0129562$ C.) given above. The difference comes from neglecting to evaluate the full value of n in the temperature and the gravity terms. For 1″ arc this gives a range of temperature 13667° C., which is an impossible result, and renders invalid the accompanying discussion." It is evidently clear that the discrepancy between his result and mine cannot be explained by any error of neglect, but only by one of principle. This result refers to the adiabatic decrease in temperature on the Sun, and by no means to the true decrease, as Mr. Bigelow seems to think, for this is at present unknown. The error is made by himself, and my computation is correct, and also my omissions are permissible. Mr. Bigelow believes simply that it is sufficient to be able to introduce in the formula for the terrestrial height of the barometer,

$$\log \frac{B_0}{B} = \frac{h - h_0}{K} = \frac{h - h_0}{RT} ,$$

the solar temperature 7535° C. in place of the temperature of the ground T, if there were air upon the Sun. He overlooks the fact that the barometric constant K, which is the height of the homogenous atmosphere, is inversely proportional to the surface gravity of the Sun. It is, however, instantly to be seen that with a greater force of gravity the height of the homogeneous atmosphere must under otherwise equal conditions become lower, and that the decrease in pressure must take place much more quickly, for if gravity is very slight the atmosphere will be high. Inasmuch as the surface gravity is 27.6 times greater on the Sun than on the Earth, our atmosphere would have on the Sun a homogeneous height of only 8000 m : 27.6 = 290 m.

According to Mr. Bigelow it would be of the same height, 8000 m. Such fundamental errors occur further in his formulæ. He believes that the adiabatic decrease in pressure can be obtained simply by introducing in the correction factor of the decrease of temperature of the ordinary barometric formula the adiabatic value, and then extends his computation up to the desired altitudes. He does not seem to be aware that the law of pressure can be obtained by a rigorous analytic process if the law of temperature is given. Thus on p. 83 he gives the integration of the equation

$$\frac{dp}{p} = -\frac{n}{RT}dh \quad \text{as} \quad \log\frac{p_0}{p} = -\frac{n}{RT}h\ ,$$

where n is a function of the temperature, and hence of h. Other remarkable statements are also found. If he means on p. 82 that the *large* barometer constant of 7296570 m which he incorrectly deduces for the Sun, has as a consequence a "rapid" decrease in pressure, then he is in error, for in this case it would be very slight. The smaller the constant, the more rapid is the decrease in pressure. I trust that these remarks will be sufficient to estimate the value of the considerations in this chapter, whereby, of course, I do not wish to extend my judgment to the remaining and surely valuable sections. Of course the tables for the decrease of pressure and temperature on the Sun are incorrect and valueless.

2. On p. 69 Mr. Bigelow writes with reference to my theory: "We cannot assume that the surface of the Sun would, like the Earth, appear dark if seen through a rift in the photosphere, and there is nothing to be gained on clearing the solar sky by descending currents." This also cannot apply in any way to my theory, since I expressly remarked that the upper layers of the photosphere are transparent, and partially protect the radiation of the lower layers. It would be a very unnatural assumption to regard the radiation as coming solely from the geometrical surface. An extreme reversal of temperature prevails over the nucleus of the Sun-spot, the existence of which I have doubtless sufficiently proven. Hence there lies over the nucleus an abnormally hot stratum which vaporizes the upper photospheric strata, consisting of the products of condensation (clouds), dissolves it, and hence produces clearness. *Above the nucleus of a spot there must therefore prevail a considerably less extinction than above the normal photosphere.* This also is shown in a convincing way by Wilson's measurements of the radiation of Sun-spots (*Proc. R. S.*, 55, 248). After Langley had called

attention to the phenomenon, Wilson showed with a perfect chain of evidence that the radiation of a Sun-spot is less when it is at the Sun's limb than when it is at the center of the disk, on account of the absorption in the solar atmosphere, but that this decrease is *slower* toward the limb than over the normal photosphere. In connection with the fact that the level of the spot is depressed, this clearly shows that a less extinction or a greater clearness of the solar atmosphere prevails over the nucleus. If it was absolutely clear above the nucleus the radiation of a spot would be found to be the same at the center and at the edge of the disk. Wilson's observation, therefore, shows that there are still certain effects of cloudiness (extinctions) present over the spots.

These last considerations may serve to further support my anticyclonic theory of Sun-spots, and at the same time to allay the apparent difficulty arising from Wilson's observation on the one hand, and the depression of the level of the Sun-spots on the other, which has led several observers (Frost, Hale, ASTROPHYSICAL JOURNAL, 4, 196; 6, 366) to somewhat far-fetched explanations.

EGON VON OPPOLZER.

INNSBRUCK,
September 25, 1902.

MISCELLANEOUS RESULTS, NO. 1.[1]

THE circulars hitherto published have subserved their purpose in furnishing a more prompt publication of results than would be secured by waiting for the issue of the volume of Annals which would contain them. Much material has, however, accumulated which might fill several circulars. To avoid the consequent delay, some of the results are given below in a briefer form.

VISIBILITY OF *EROS*.

A letter lately received from Professor Bailey states that *Eros* was photographed with the Bruce Telescope on July 8, 1902. The telescope was made to follow a star by means of an eyepiece adjoining the plate, while a motion was given to the latter equal to the computed motion of *Eros*. The stars accordingly leave trails having a direction and length corresponding to the motion of *Eros*, while the planet appears as a minute dot, the entire light being concentrated for the whole time of exposure upon a few silver particles. A second photograph confirmed this observation. So far as known, the first visual

observation of *Eros*, since its recent conjunction with the Sun, was obtained by Professor H. A. Howe at the Denver Observatory on August 2, 1902, as announced in the *Harvard Bulletin* of August 5, 1902. It will be remembered that the first photograph of *Eros* after its conjunction in 1899 was also obtained with the Bruce telescope on April 28, 1900, by the method described above. It was first seen on June 6, 1900, at the Denver Observatory, and later at the Arcetri Observatory on June 30, 1900. In like manner, it was followed after the opposition of 1900, until March 12, 1901, at Arcetri, until June 17, 1901 at Denver, and until September 9, 1901, at Arequipa, with the Bruce telescope.

WILLIAMS' NEW *ALGOL* VARIABLE, 13.1902.

In order to identify the *Algol* variable, 13.1902, recently announced by Mr. A. S. Williams (*A. N.*, **159**, 309), its position was marked on several of the Draper Memorial photographs. On one of these, taken 1893, July 11^d 18^h 20^m, G. M. T., the star appeared fainter than normal. On Mr. Williams' scale its magnitude was about 11.3, or a little nearer in brightness to his star *d*, than to his star *b*. It is also very nearly equal in brightness to the star which follows the variable about 45^s, north $6'.8$. This would indicate that the time of minimum preceded or followed the time of taking the photograph by about three hours, assuming the light curve described by Mr. Williams. The formula he gives indicates for $E = -918$, a time of minimum J. D. 2,412,656.720 $= 1893$, July 11^d 17^h 17^m, G. M. T., omitting his last three decimal places which appear to be indeterminate.

This photograph, therefore, gives a correction to the ephemeris of $+4^h$ or -2^h. An examination of the other plates of this region would distinguish between these two values, and determine the correct one with much greater accuracy. Probably we have at least one hundred photographs of this region, although the star may be too faint to appear on a portion of them. On one-tenth of the photographs the star should be below its normal brightness. A precise correction to the ephemeris can be found from each of these, if not too near maximum or minimum, when the photographic light curve has been found, as can now be readily done. Unfortunately, the pressure of other work will probably prevent the continuation of this research at present. As it is, the observation given above lengthens the period of observation from less than ten months to about nine years, and increases the

[1] *Harvard College Observatory Circular* No. 66.

accuracy with which the period can now be determined in nearly the same proportion.

EARLY OBSERVATIONS OF *NOVA PERSEI*, NO. 2.

As the *Nova* is diminishing in light it becomes necessary to use comparison stars so faint that they are not contained in the *Harvard Annals*, Volume XLV or other photometric catalogues. A sequence has accordingly been selected which is given in Table I. The designation is followed by the number, number of grades, difference in right ascension, and difference of declination from the *Nova*, taken from *Hagen's Second Catalogue*. The sixth column gives the photometric magnitude, found for the stars *b* to *f* from comparisons by Professor Wendell with star *a*. Constructing a curve with coördinates taken from the second and sixth columns, we have the means of converting the estimates of Hagen into magnitudes on the photometric scale. The seventh column gives the residual found by subtracting the magnitude given in the sixth column from the Hagen magnitude thus formed. Table II gives the photometric magnitude corresponding to various values of Hagen's grades. The fainter magnitudes found by interpolation are somewhat uncertain.

TABLE I.

SEQUENCE OF COMPARISON STARS.

Des.	H.	G.	$\Delta\alpha$	$\Delta\delta$	Magn.	H.-P.	Des.	H.	Gr.	$\Delta\alpha$	$\Delta\delta$	Magn.	H.-P.
			s	'						s	'		
a	20	67	−21	+4.8	9.10	+.03	*h*	65	178	+42	+4.8	..	..
b	32	84	+43	+8.7	9.71	+.19	*k*	66	179	+33	−4.1	..	..
c	34	92	−65	+3.4	10.57	−.32	*l*	68	185	+34	+1.5	..	..
	42	111	+35	+0.2	11.03	+.01	*m*	74	199	+21	+1.6	..	..
e	46	133	−82	+6.3	11.94	−.08	*n*	77	209	+13	+1.0	..	..
f	49	145	−24	−1.5	12.23	+.03	*o*	80	..	−3	−0.5	..	..
g	56	161	−17	+2.4	..	..							

Table II furnishes the means of determining the limiting magnitude below which the *Nova* must have been before its discovery. The last photograph taken before it was found on February 21, 1902, was obtained by Mr. A. S. Williams on February 20. It is reproduced in *Knowledge*, 24, 152. All stars included in the region covered by the print, and brighter than *Hagen* No. 42, are shown except Nos. 29, 34, 36, 38, and 39. The absence of 39 is perhaps due to the proximity of

No. 28. Stars of the magnitude 10.9 are therefore shown, and we may, hence, infer that the *Nova* was below this limit on February 20. Several defects appear on this print which cannot be distinguished from stars. The next photograph preceding this was obtained at this Observatory, and is shown in Fig. 1 of *Circular* No. 57. This print shows all stars brighter than No. 49, except No. 39. The *Nova* must therefore have been fainter than the magnitude 12.0 on that date.

TABLE II.

RELATION OF HAGEN'S GRADES TO MAGNITUDES.

Grades	Magnitudes	Grades	Magnitudes	Grades	Magnitudes	Grades	Magnitudes
0	6.56	50	8.39	100	10.59	150	12.42
10	6.90	60	8.82	110	11.00	160	12.72
20	7.25	70	9.27	120	11.39	170	13.00
30	7.61	80	9.72	130	11.76	180	13.25
40	7.99	90	10.16	140	12.10	190	13.48

About a year ago, Father Zwack of the Georgetown College Observatory called my attention to a faint star which appeared on one of our early photographs so near the position of the *Nova* that careful measurements were required to determine whether the positions are identical or not. A measurement of several of the early plates gave the magnitudes of this object on October 26, December 2, 1890, January 20, March 11, December 10, 1891, January 25, 1893, October 11, 1894, October 17, 1897, and March 7, 1900, as 12.95, 13.37 <13.7, <13.4, <13.7, 14.06, 13.15, 13.46, and 13.36, respectively. The images of this star on the photographs taken in 1890 are somewhat uncertain, but a comparison of the photographs taken in 1893 and 1894 leaves little doubt of the variability of this object. Measurements of its position were made on the last three photographs mentioned above. Taking the position in which the *Nova* appeared as an origin, the values of x for this star are $-1.''6$, $+1.''2$, and $+0.''2$, those of y, $+1.''3$, $-1.''0$, and $-0.''2$, respectively. These measures are based on the positions of the stars Nos. 20, 56, 77, 78, 79, 80, and 81, determined micrometrically at the Lick Observatory.

The object announced by M. S. Blajko, *A. N.*, 157, 193, which according to his measures, followed the *Nova* $0.^s31$, south $7''$, on January 30, 1899, is doubtless identical with this object. Hagen's double

star, No. 69, whose position is 40″ distant from the *Nova* does not appear on any of these photographs, although much fainter stars are shown.

We may therefore conclude that a star whose light varied from the thirteenth to the fourteenth magnitude was visible for several years within one or two seconds of arc of the *Nova*, the difference in position being less than the errors of measurement.

EDWARD C. PICKERING.

October 31, 1902.

COMMENT ON THE EARLY HARVARD PHOTOGRAPHS OF *NOVA PERSEI.*

IN *Harvard Circular* No. 66, Professor E. C. Pickering calls attention to a small star of the thirteenth or fourteenth magnitude, close to the present position of *Nova Persei*, which appears on a number of plates taken between October 26, 1890, and March 7, 1900. The photographs show this star to have been variable, ranging in light from the thirteenth to the fourteenth magnitude.

His measures of the position of the small star with respect to the place of the *Nova*, on three plates, viz:

$$\Delta\alpha - 1.''6, + 1.''2, + 0.''2 \ ;$$
$$\Delta\delta + 1.3, - 1.0, - 0.2 \ ;$$

give in the mean

$$\Delta\alpha - 0.''1$$
$$\Delta\delta \quad 0.''0 \ .$$

The measures above would indicate an exact identity of this small star with the *Nova*, as is implied in the last paragraph of the *Circular*. The fact that the star was variable would further strengthen this supposition. If we accept this, and it would appear to be almost certain, it would appear that the *Nova* formerly existed as a very small variable star not brighter than the thirteenth magnitude, whose variation was about one magnitude.

When the *Nova* has faded somewhat further, it will be possible to decide the question of identity, for the large telescopes will readily show if there are two stars at this place. The *Nova* is fading at the rate of about 0.2 magnitude per month and is now of about the tenth magnitude. Hence at this rate, by the first of the year 1904 the star will have reached the normal magnitude it had before the great outburst of light in 1901. It will probably not sink beyond the thirteenth

or fourteenth magnitude, when it will be interesting to see if it becomes variable again.

It is possible that the *Novæ* in expiring return to their original condition, and that such stars as *Nova Cygni* of 1876 existed previously as stars of the magnitude they now have. *Nova Coronae* of 1866 is the only one of these remarkable stars which was positively known previous to its outburst. In fading it went back to its original condition of brightness. Photographic records will in the future be of the utmost value in settling this question.

Professor Pickering speaks of the star announced by M. Ceraski in *A. N.*, 157, 193, as probably being the same as the small star shown on the Harvard photographs. This seems improbable unless we allow a very large error in the position assigned by M. Ceraski. His star was of the twelfth magnitude and was $0^{s}.31$ ($3''.4$) following and $7''$ south of the place of the *Nova Persei.* This would put it in Position Angle $154^{\circ}.1$, and Distance $7''.8$. I have looked for this star on a number of occasions since the *Nova* has become faint but have not seen it. It does not appear to have been shown on the plates of this region measured by Professor Perrine (*L. O. Bulletin* No. 23). Its possible identity with the Harvard star would be very interesting and can readily be proved if M. Ceraski will remeasure its position on the 1899 plate with reference to any of the stars used by Aitken (*L. O. Bulletin* No. 18), or by me in the measures in *A. N.*, 159, 49, 1902.

Should the small variable star shown on the Harvard photographs really prove to be the *Nova* the discovery will be only another of the many evidences of the extremely great value of the Harvard College Observatory photographs, which seem to be a veritable mine of the richest value, the thorough working of which would lead to many important discoveries yet unknown but which are there faithfully recorded on the photographic plates.

E. E. Barnard.

Yerkes Observatory
November 18, 1902.

NOTE ON REFLECTORS FOR ECLIPSE WORK.

During the course of some investigations in which I had occasion to use reflected light, I observed incidentally that silver begins rapidly to lose its reflecting power even in the visible violet, and sinks nearly to zero, for normal incidence, in the region of λ 3200, beyond which it again becomes a reflector, but only a very poor one in the rest of the photographic region.

Further investigations along this line were abandoned as soon as I became aware, through recent publications,[1] of the excellent work of Hagen and Rubens on metallic reflection. But while I have nothing to add, beyond the fact that our results with silver reflectors, obtained by different methods, are in perfect agreement, I wish to direct specific attention to some practical uses in eclipse work, and other places where economy of light is important, that should be made of their observations.

The following table, condensed from a much longer one in the original paper, contains the results of most value in this connection.

PRECENTAGE REFLECTED OF NORMALLY INCIDENT LIGHT.

For $\lambda =$	2510	2880	3050	3160	3260	3380	3570	3850	4200	4500	5000	5500	6000	6500	7000
By silver (fresh)	34.1	21.2	9.1	4.2	14.6	55.5	74.5	81.4	86.6	90.5	91.3	92.7	92.6	93.5	94.6
By speculum metal (62.2 *Cu* + 31.8 *Sn*)	29.9	37.7	41.7	....	....	....	51.0	53.1	56.4	60.0	63.2	64.0	64.3	65.6	66.8
By Schröder's alloy (66 *Cu* + 22 *Sn* + 12 *Zn*)	40.1	48.4	49.8	....	54.3	....	56.6	60.0	62.2	62.6	62.5	63.4	64.2	65.1	67.2
By Mach's magnalium (69 *Al* + 31 *Mg*)	67.0	70.6	72.2	....	75.5	....	81.2	83.9	83.3	83.4	83.3	82.7	83.0	82.1	83.3

The atmosphere, as is generally familiar, absorbs ultra-violet light to such an extent that but little is known of the solar spectrum beyond λ 3000, while even the light of considerably longer wave-lengths is still feeble. It therefore is a matter of the utmost importance in photographing this part of the spectrum during a total eclipse, when the exposures have to be very short, to economize its light as far as practicable.

In accordance with the above results I would suggest that, in eclipse work, spectrographs be used, if possible, without the aid of reflectors of any kind, and that in those cases where, because of the type of spectrograph, or for any other reason, a reflector is essential, it be selected according to the portion of the spectrum to be studied. For λ greater than 4000 a thick film of silver freshly deposited on glass is the best reflector known, while for much of the valuable ultra-violet it is all but worthless. However, where silver fails, Mach's magnalium succeeds, and seems to be the best reflector of ultra-violet light so far examined.

Evidently, therefore, when spectrographs, used in eclipse, or any other work, are supplied by reflected light, it would be best to use silver reflectors for wave-lengths greater than λ 4000, and Mach's

[1] *Annalen der Physik*, 8, 1–21, and 432–454, 1902.

magnalium, unless it should give too much trouble due to tarnishing, for the ultra-violet.

In those cases where much severe exposure is unavoidable, a Schröder reflector would probably be the best, since it is said to be but slightly affected by prolonged exposure to all sorts of weather, and besides, its coefficient of reflection, while much less than that of Mach's magnalium, is still distinctly better than that of speculum metal in the ultra-violet, and quite its equal for longer wave-lengths.

W. J. HUMPHREYS.

UNIVERSITY OF VIRGINIA,
Charlottesville, November 28, 1902.

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The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to uniformly employ the metric units of length and mass; the English equivalents may be added if desired.

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INDEX TO VOLUME XVI.

SUBJECTS.

INDEX TO VOLUME XVI.

AUTHORS.